Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.5-P-x-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [111]. This is test number [42].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (111)	% 0. (0)
Mathematica	% 92.79 (103)	% 7.21 (8)
Maple	% 100. (111)	% 0. (0)
Maxima	% 74.77 (83)	% 25.23 (28)
Fricas	% 72.07 (80)	% 27.93 (31)
Sympy	% 46.85 (52)	% 53.15 (59)
Giac	% 79.28 (88)	% 20.72 (23)

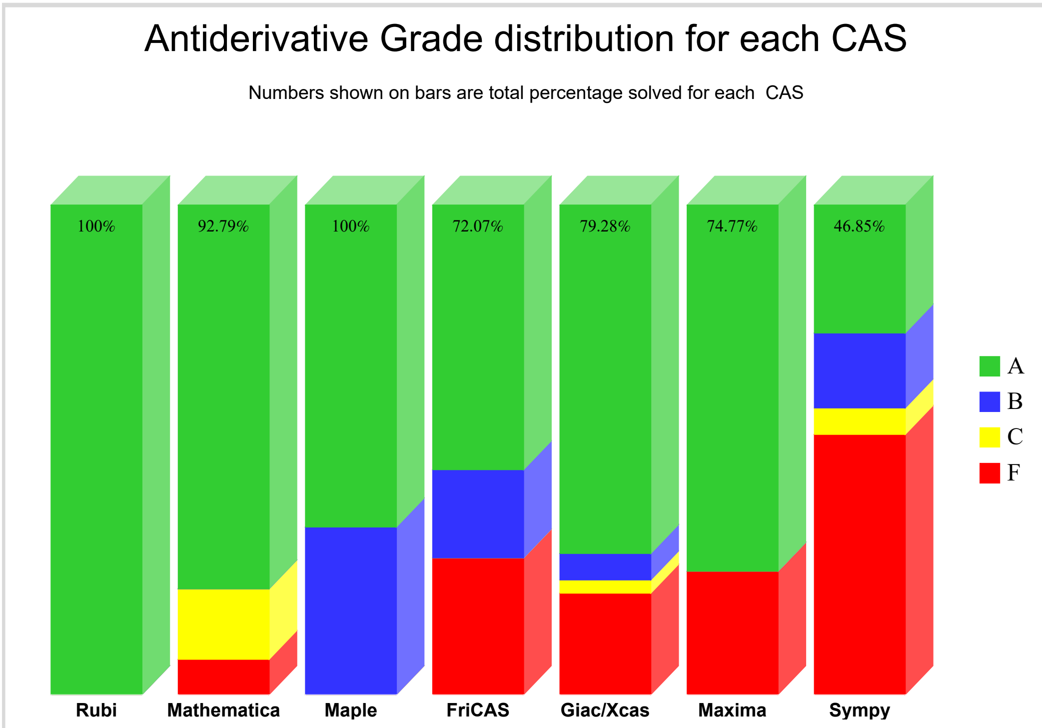
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

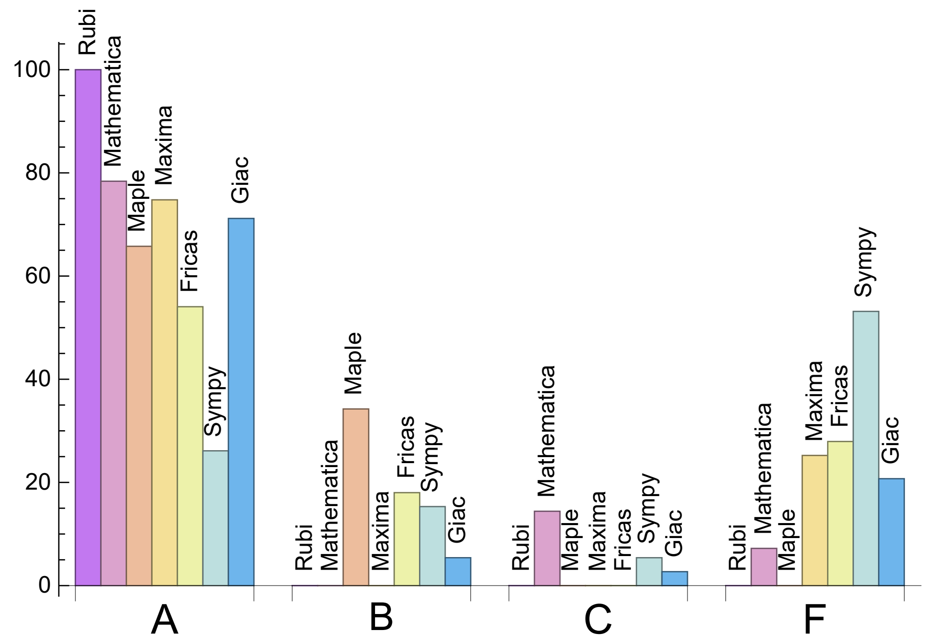
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	78.38	0.	14.41	7.21
Maple	65.77	34.23	0.	0.
Maxima	74.77	0.	0.	25.23
Fricas	54.05	18.02	0.	27.93
Sympy	26.13	15.32	5.41	53.15
Giac	71.17	5.41	2.7	20.72

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.68	217.75	1.	140.	1.
Mathematica	0.83	239.05	1.06	146.	1.01
Maple	0.03	883.14	2.36	186.	1.54
Maxima	1.07	136.18	1.18	119.	1.17
Fricas	8.08	464.49	3.68	282.	3.18
Sympy	16.22	986.25	9.76	233.5	1.38
Giac	1.94	680.84	3.04	142.5	1.32

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {15, 16, 17, 31, 32, 33, 34, 35, 47, 48, 49, 50, 51}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

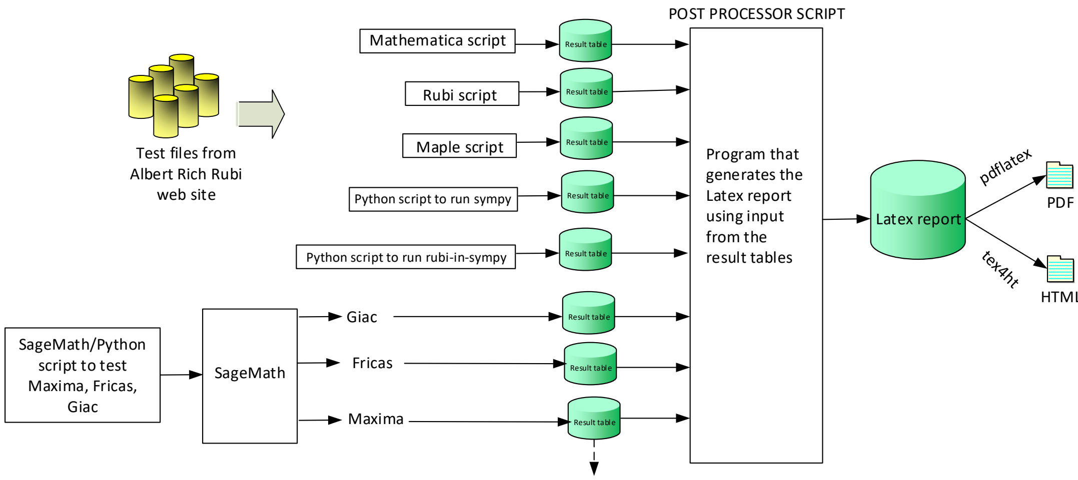
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade: { }

C grade: { 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 47, 48, 49, 50, 51, 105 }

F grade: { 103, 104, 106, 107, 108, 109, 110, 111}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 16, 17, 20, 26, 27, 28, 31, 32, 33, 34, 42, 43, 44, 47, 48, 49, 50, 51, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 108, 109, 110, 111 }

B grade: { 11, 12, 13, 14, 18, 19, 21, 22, 23, 24, 25, 29, 30, 35, 36, 37, 38, 39, 40, 41, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 103, 104, 106, 107 }

C grade: { }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111

B grade: { }

C grade: { }

F grade: { 20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 103, 104, 105, 106, 107 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 47, 48, 49, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 108, 109, 110, 111 }

B grade: { 26, 27, 28, 29, 30, 42, 43, 44, 45, 46, 50, 51, 93, 94, 95, 97, 98, 99, 100, 101 }

C grade: { }

F grade: { 20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 90, 96, 102, 103, 104, 105, 106, 107 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 85, 91, 97 }

B grade: { 10, 11, 20, 26, 27, 42, 43, 80, 81, 82, 83, 86, 87, 92, 93, 98, 99 }

C grade: { 15, 16, 31, 32, 47, 48 }

F grade: { 12, 13, 14, 17, 18, 19, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 84, 88, 89, 90, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade: { 40, 56, 108, 109, 110, 111 }

C grade: { 20, 21, 64 }

F grade: { 22, 23, 24, 25, 36, 37, 38, 39, 41, 52, 53, 54, 55, 57, 58, 59, 65, 66, 103, 104, 105, 106, 107 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	54	104	46	58
normalized size	1	1.	1.	0.82	1.08	2.08	0.92	1.16
time (sec)	N/A	0.041	0.002	0.007	1.33	1.462	0.064	1.079

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	77	161	65	86
normalized size	1	1.	1.	0.84	1.12	2.33	0.94	1.25
time (sec)	N/A	0.045	0.021	0.001	1.311	1.472	0.07	1.083

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	100	217	83	115
normalized size	1	1.	1.	0.85	1.14	2.47	0.94	1.31
time (sec)	N/A	0.073	0.019	0.	0.979	1.53	0.072	1.077

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	120	274	102	143
normalized size	1	1.	1.	0.86	1.14	2.61	0.97	1.36
time (sec)	N/A	0.095	0.035	0.	0.958	1.437	0.077	1.093

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	105	140	333	121	171
normalized size	1	1.	1.	0.86	1.15	2.73	0.99	1.4
time (sec)	N/A	0.111	0.039	0.002	0.978	1.474	0.08	1.094

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	97	95	127	252	116	143
normalized size	1	1.	0.87	0.85	1.13	2.25	1.04	1.28
time (sec)	N/A	0.126	0.051	0.001	0.952	1.48	0.083	1.094

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	186	385	165	212
normalized size	1	1.	1.	0.9	1.21	2.5	1.07	1.38
time (sec)	N/A	0.13	0.046	0.002	0.96	1.704	0.093	1.09

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	183	246	518	209	281
normalized size	1	1.	1.	0.93	1.26	2.64	1.07	1.43
time (sec)	N/A	0.168	0.059	0.	0.949	1.731	0.099	1.097

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	234	219	294	653	258	350
normalized size	1	1.	1.	0.94	1.26	2.79	1.1	1.5
time (sec)	N/A	0.238	0.088	0.002	0.937	1.732	0.104	1.086

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	58	58	143	515	69
normalized size	1	1.	1.11	1.29	1.29	3.18	11.44	1.53
time (sec)	N/A	0.032	0.018	0.039	0.958	1.905	2.175	1.094

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	58	86	69	170	2195	80
normalized size	1	1.	1.14	1.69	1.35	3.33	43.04	1.57
time (sec)	N/A	0.057	0.026	0.007	0.963	2.069	33.058	1.103

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	68	114	82	197	0	93
normalized size	1	1.	1.19	2.	1.44	3.46	0.	1.63
time (sec)	N/A	0.072	0.032	0.01	0.966	3.661	0.	1.144

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	81	145	97	234	0	108
normalized size	1	1.	1.27	2.27	1.52	3.66	0.	1.69
time (sec)	N/A	0.147	0.046	0.01	0.96	12.735	0.	1.07

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	98	179	119	279	0	130
normalized size	1	1.	1.29	2.36	1.57	3.67	0.	1.71
time (sec)	N/A	0.192	0.065	0.01	0.954	55.14	0.	1.104

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	98	92	88	212	923	90
normalized size	1	1.	1.07	1.	0.96	2.3	10.03	0.98
time (sec)	N/A	0.077	0.178	0.011	1.431	1.52	1.918	1.1

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	121	148	101	239	3589	104
normalized size	1	1.	1.16	1.42	0.97	2.3	34.51	1.
time (sec)	N/A	0.085	0.139	0.006	1.437	1.775	26.542	1.097

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	150	204	112	261	0	115
normalized size	1	1.	1.18	1.61	0.88	2.06	0.	0.91
time (sec)	N/A	0.101	0.485	0.004	1.473	2.986	0.	1.111

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	165	241	124	285	0	127
normalized size	1	1.	1.21	1.77	0.91	2.1	0.	0.93
time (sec)	N/A	0.14	0.601	0.004	1.49	11.104	0.	1.102

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	187	303	143	323	0	146
normalized size	1	1.	1.24	2.01	0.95	2.14	0.	0.97
time (sec)	N/A	0.176	0.642	0.007	1.464	39.42	0.	1.104

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	194	231	0	0	471	3996
normalized size	1	1.	1.03	1.22	0.	0.	2.49	21.14
time (sec)	N/A	0.211	0.273	0.089	0.	0.	93.814	1.972

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	9104
normalized size	1	1.	1.11	2.92	0.	0.	0.	43.15
time (sec)	N/A	0.24	0.226	0.024	0.	0.	0.	2.929

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	280	866	0	0	0	0
normalized size	1	1.	1.14	3.53	0.	0.	0.	0.
time (sec)	N/A	0.159	0.31	0.019	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	383	1132	0	0	0	0
normalized size	1	1.	1.32	3.9	0.	0.	0.	0.
time (sec)	N/A	0.725	0.559	0.03	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	441	1435	0	0	0	0
normalized size	1	1.	1.37	4.47	0.	0.	0.	0.
time (sec)	N/A	0.534	0.798	0.029	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	816	3835	0	0	0	0
normalized size	1	1.	1.5	7.04	0.	0.	0.	0.
time (sec)	N/A	4.213	1.614	0.046	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	90	122	112	446	604	126
normalized size	1	1.	0.96	1.3	1.19	4.74	6.43	1.34
time (sec)	N/A	0.052	0.055	0.017	0.937	2.493	2.651	1.1

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	112	182	143	585	2689	155
normalized size	1	1.	0.97	1.58	1.24	5.09	23.38	1.35
time (sec)	N/A	0.14	0.081	0.018	0.942	3.002	38.346	1.087

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	134	242	171	725	0	184
normalized size	1	1.	0.97	1.75	1.24	5.25	0.	1.33
time (sec)	N/A	0.154	0.054	0.018	0.941	4.617	0.	1.086

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	159	302	196	865	0	213
normalized size	1	1.	1.06	2.01	1.31	5.77	0.	1.42
time (sec)	N/A	0.214	0.077	0.019	0.972	13.663	0.	1.113

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	185	362	220	1007	0	242
normalized size	1	1.	1.14	2.23	1.36	6.22	0.	1.49
time (sec)	N/A	0.232	0.094	0.019	1.001	65.679	0.	1.083

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	146	146	130	416	952	135
normalized size	1	1.	1.04	1.04	0.93	2.97	6.8	0.96
time (sec)	N/A	0.098	0.494	0.02	1.434	1.582	2.771	1.091

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	165	165	186	214	162	547	4107	173
normalized size	1	1.	1.13	1.3	0.98	3.32	24.89	1.05
time (sec)	N/A	0.129	0.424	0.014	1.43	1.903	33.286	1.093

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	200	260	182	616	0	192
normalized size	1	1.	1.12	1.45	1.02	3.44	0.	1.07
time (sec)	N/A	0.141	0.434	0.013	1.457	3.159	0.	1.083

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	234	328	193	694	0	209
normalized size	1	1.	1.25	1.75	1.03	3.71	0.	1.12
time (sec)	N/A	0.167	0.629	0.014	1.504	10.495	0.	1.078

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	243	374	209	757	0	228
normalized size	1	1.	1.25	1.93	1.08	3.9	0.	1.18
time (sec)	N/A	0.197	0.669	0.015	1.467	48.782	0.	1.091

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	341	1237	0	0	0	0
normalized size	1	1.	1.03	3.75	0.	0.	0.	0.
time (sec)	N/A	0.745	0.875	0.122	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	398	1813	0	0	0	0
normalized size	1	1.	1.08	4.93	0.	0.	0.	0.
time (sec)	N/A	0.87	1.352	0.118	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	421	2310	0	0	0	0
normalized size	1	1.	1.09	5.98	0.	0.	0.	0.
time (sec)	N/A	0.49	1.559	0.122	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	489	1801	0	0	0	0
normalized size	1	1.	1.11	4.1	0.	0.	0.	0.
time (sec)	N/A	1.894	2.241	0.049	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	524	1917	0	0	0	8384
normalized size	1	1.	1.12	4.1	0.	0.	0.	17.91
time (sec)	N/A	1.118	2.556	0.034	0.	0.	0.	22.393

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	770	770	935	4570	0	0	0	0
normalized size	1	1.	1.21	5.94	0.	0.	0.	0.
time (sec)	N/A	7.835	6.566	0.07	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	128	186	163	869	668	166
normalized size	1	1.	0.9	1.3	1.14	6.08	4.67	1.16
time (sec)	N/A	0.076	0.098	0.019	0.941	2.077	2.794	1.132

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	161	278	209	1134	2822	212
normalized size	1	1.	0.92	1.59	1.19	6.48	16.13	1.21
time (sec)	N/A	0.224	0.129	0.018	0.954	2.323	43.34	1.093

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	193	370	254	1407	0	257
normalized size	1	1.	0.95	1.81	1.25	6.9	0.	1.26
time (sec)	N/A	0.252	0.091	0.022	0.946	4.319	0.	1.124

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	231	462	289	1671	0	302
normalized size	1	1.	1.03	2.06	1.29	7.46	0.	1.35
time (sec)	N/A	0.307	0.125	0.02	0.95	12.975	0.	1.136

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	261	554	321	1939	0	347
normalized size	1	1.	1.09	2.32	1.34	8.11	0.	1.45
time (sec)	N/A	0.345	0.142	0.021	0.988	59.828	0.	1.127

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	186	180	185	726	1103	177
normalized size	1	1.	1.01	0.97	1.	3.92	5.96	0.96
time (sec)	N/A	0.117	0.754	0.017	1.454	1.673	2.631	1.099

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	235	264	234	977	4498	231
normalized size	1	1.	1.05	1.18	1.05	4.38	20.17	1.04
time (sec)	N/A	0.215	0.59	0.018	1.431	2.213	36.753	1.114

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	243	243	259	322	270	1111	0	267
normalized size	1	1.	1.07	1.33	1.11	4.57	0.	1.1
time (sec)	N/A	0.227	0.749	0.016	1.427	4.001	0.	1.097

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	303	396	293	1278	0	308
normalized size	1	1.	1.15	1.51	1.11	4.86	0.	1.17
time (sec)	N/A	0.263	0.942	0.017	1.441	13.062	0.	1.128

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	325	454	309	1401	0	344
normalized size	1	1.	1.21	1.69	1.15	5.21	0.	1.28
time (sec)	N/A	0.286	1.099	0.017	1.47	59.004	0.	1.084

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	488	3725	0	0	0	0
normalized size	1	1.	1.03	7.86	0.	0.	0.	0.
time (sec)	N/A	2.193	2.301	0.24	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	625	7858	0	0	0	0
normalized size	1	1.	1.01	12.65	0.	0.	0.	0.
time (sec)	N/A	4.512	4.463	0.28	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	646	646	661	10222	0	0	0	0
normalized size	1	1.	1.02	15.82	0.	0.	0.	0.
time (sec)	N/A	3.299	5.195	0.285	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	845	3492	0	0	0	0
normalized size	1	1.	1.24	5.14	0.	0.	0.	0.
time (sec)	N/A	4.182	6.645	0.067	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	728	728	980	3824	0	0	0	16458
normalized size	1	1.	1.35	5.25	0.	0.	0.	22.61
time (sec)	N/A	2.733	6.851	0.046	0.	0.	0.	29.755

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1150	1144	1590	6026	0	0	0	0
normalized size	1	0.99	1.38	5.24	0.	0.	0.	0.
time (sec)	N/A	8.164	7.849	0.086	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	645	645	775	3107	0	0	0	0
normalized size	1	1.	1.2	4.82	0.	0.	0.	0.
time (sec)	N/A	3.367	6.14	0.061	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1177	1179	1649	6130	0	0	0	0
normalized size	1	1.	1.4	5.21	0.	0.	0.	0.
time (sec)	N/A	7.926	7.631	0.089	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	416	829	564	1152	503	645
normalized size	1	1.	1.	1.99	1.36	2.77	1.21	1.55
time (sec)	N/A	0.629	0.133	0.001	0.982	1.541	0.146	1.149

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	259	354	339	716	309	398
normalized size	1	1.	1.	1.37	1.31	2.76	1.19	1.54
time (sec)	N/A	0.332	0.054	0.002	0.966	1.574	0.113	1.118

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	161	186	385	165	212
normalized size	1	1.	1.	1.05	1.21	2.5	1.07	1.38
time (sec)	N/A	0.152	0.035	0.002	0.963	1.525	0.088	1.1

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	23
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.15
time (sec)	N/A	0.033	0.002	0.	0.951	1.688	0.08	1.187

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	9103
normalized size	1	1.	1.11	2.92	0.	0.	0.	43.14
time (sec)	N/A	0.318	0.254	0.014	0.	0.	0.	22.35

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	398	1813	0	0	0	0
normalized size	1	1.	1.08	4.93	0.	0.	0.	0.
time (sec)	N/A	0.923	1.427	0.095	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	625	7858	0	0	0	0
normalized size	1	1.	1.01	12.65	0.	0.	0.	0.
time (sec)	N/A	4.594	4.538	0.229	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	16	3	7
normalized size	1	1.	1.	1.25	1.25	4.	0.75	1.75
time (sec)	N/A	0.011	0.001	0.002	0.968	1.983	0.062	1.069

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	16	18	19	38	12	23
normalized size	1	1.	1.14	1.29	1.36	2.71	0.86	1.64
time (sec)	N/A	0.024	0.004	0.002	0.949	1.733	0.263	1.081

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	35	36	73	26	41
normalized size	1	1.	0.97	1.13	1.16	2.35	0.84	1.32
time (sec)	N/A	0.052	0.012	0.002	1.035	1.463	0.287	1.096

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	58	58	116	41	66
normalized size	1	1.	0.88	1.14	1.14	2.27	0.8	1.29
time (sec)	N/A	0.085	0.027	0.004	1.156	1.468	0.318	1.084

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	87	84	169	63	100
normalized size	1	1.	1.	1.28	1.24	2.49	0.93	1.47
time (sec)	N/A	0.117	0.023	0.003	1.032	1.517	0.342	1.072

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	92	122	113	231	88	142
normalized size	1	1.	1.	1.33	1.23	2.51	0.96	1.54
time (sec)	N/A	0.149	0.035	0.003	0.967	1.525	0.376	1.061

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	35	8	18
normalized size	1	1.	1.	1.09	1.36	3.18	0.73	1.64
time (sec)	N/A	0.01	0.003	0.004	0.964	1.516	0.097	1.086

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	23	29	30	59	29	35
normalized size	1	1.	1.05	1.32	1.36	2.68	1.32	1.59
time (sec)	N/A	0.021	0.007	0.004	0.972	1.529	0.252	1.07

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	45	39	80	44	45
normalized size	1	1.	1.03	1.55	1.34	2.76	1.52	1.55
time (sec)	N/A	0.05	0.013	0.005	0.969	1.485	0.594	1.065

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	69	61	120	66	66
normalized size	1	1.	0.94	1.47	1.3	2.55	1.4	1.4
time (sec)	N/A	0.068	0.02	0.006	0.963	1.509	0.885	1.11

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	98	84	170	94	93
normalized size	1	1.	1.02	1.48	1.27	2.58	1.42	1.41
time (sec)	N/A	0.085	0.024	0.006	0.973	1.519	1.506	1.081

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	91	134	113	230	122	131
normalized size	1	1.	1.01	1.49	1.26	2.56	1.36	1.46
time (sec)	N/A	0.107	0.041	0.006	0.977	1.48	2.535	1.079

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	26	68	19	30
normalized size	1	1.	1.	0.69	0.9	2.34	0.66	1.03
time (sec)	N/A	0.021	0.007	0.006	0.975	1.502	0.124	1.082

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	39	44	43	103	304	51
normalized size	1	1.	0.93	1.05	1.02	2.45	7.24	1.21
time (sec)	N/A	0.052	0.019	0.007	0.969	1.509	1.148	1.074

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	65	50	122	716	58
normalized size	1	1.	0.94	1.38	1.06	2.6	15.23	1.23
time (sec)	N/A	0.064	0.02	0.007	0.945	1.621	5.156	1.082

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	89	63	149	1389	72
normalized size	1	1.	0.96	1.56	1.11	2.61	24.37	1.26
time (sec)	N/A	0.079	0.026	0.007	0.959	1.608	26.093	1.09

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	120	84	196	2388	92
normalized size	1	1.	0.96	1.62	1.14	2.65	32.27	1.24
time (sec)	N/A	0.107	0.036	0.007	0.961	1.872	137.208	1.077

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	91	156	111	251	0	122
normalized size	1	1.	0.95	1.62	1.16	2.61	0.	1.27
time (sec)	N/A	0.137	0.049	0.009	0.978	2.151	0.	1.099

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	33	43	155	34	49
normalized size	1	1.	0.91	0.72	0.93	3.37	0.74	1.07
time (sec)	N/A	0.051	0.022	0.009	0.963	1.512	0.235	1.081

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	74	77	263	1188	89
normalized size	1	1.	0.93	1.04	1.08	3.7	16.73	1.25
time (sec)	N/A	0.174	0.039	0.01	0.986	1.683	5.553	1.097

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	77	110	92	335	4767	104
normalized size	1	1.	0.94	1.34	1.12	4.09	58.13	1.27
time (sec)	N/A	0.199	0.052	0.01	0.984	2.18	105.348	1.089

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	90	146	109	406	0	122
normalized size	1	1.	0.95	1.54	1.15	4.27	0.	1.28
time (sec)	N/A	0.221	0.047	0.011	0.988	6.179	0.	1.082

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	102	182	124	483	0	136
normalized size	1	1.	0.96	1.72	1.17	4.56	0.	1.28
time (sec)	N/A	0.266	0.064	0.012	0.98	37.171	0.	1.088

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	118	221	146	0	0	158
normalized size	1	1.	0.97	1.81	1.2	0.	0.	1.3
time (sec)	N/A	0.315	0.07	0.011	0.952	0.	0.	1.084

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	40	57	215	44	62
normalized size	1	1.	0.86	0.71	1.02	3.84	0.79	1.11
time (sec)	N/A	0.057	0.025	0.013	0.95	1.826	0.267	1.081

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	90	101	410	1255	115
normalized size	1	1.	0.9	1.01	1.13	4.61	14.1	1.29
time (sec)	N/A	0.26	0.051	0.011	0.959	1.948	5.75	1.078

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	134	123	529	5015	136
normalized size	1	1.	0.92	1.28	1.17	5.04	47.76	1.3
time (sec)	N/A	0.32	0.074	0.013	0.977	2.568	117.971	1.094

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	114	178	144	655	0	158
normalized size	1	1.	0.97	1.52	1.23	5.6	0.	1.35
time (sec)	N/A	0.246	0.06	0.013	0.945	7.256	0.	1.101

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	136	222	166	783	0	180
normalized size	1	1.	1.04	1.69	1.27	5.98	0.	1.37
time (sec)	N/A	0.28	0.069	0.015	0.958	34.496	0.	1.086

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	153	266	188	0	0	201
normalized size	1	1.	1.04	1.81	1.28	0.	0.	1.37
time (sec)	N/A	0.33	0.092	0.013	0.969	0.	0.	1.127

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	47	70	271	53	76
normalized size	1	1.	0.88	0.69	1.03	3.99	0.78	1.12
time (sec)	N/A	0.058	0.029	0.013	0.958	1.739	0.279	1.1

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	97	106	119	535	1032	132
normalized size	1	1.	0.92	1.01	1.13	5.1	9.83	1.26
time (sec)	N/A	0.196	0.087	0.014	0.957	1.848	5.594	1.101

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	158	146	698	5192	159
normalized size	1	1.	0.99	1.3	1.2	5.72	42.56	1.3
time (sec)	N/A	0.222	0.054	0.015	0.966	2.156	104.122	1.076

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	144	210	170	867	0	184
normalized size	1	1.	1.02	1.49	1.21	6.15	0.	1.3
time (sec)	N/A	0.253	0.079	0.014	0.97	6.231	0.	1.093

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	169	262	196	1049	0	209
normalized size	1	1.	1.07	1.66	1.24	6.64	0.	1.32
time (sec)	N/A	0.289	0.1	0.018	0.976	34.309	0.	1.085

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	195	314	220	0	0	234
normalized size	1	1.	1.1	1.77	1.24	0.	0.	1.32
time (sec)	N/A	0.343	0.119	0.016	0.985	0.	0.	1.084

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	717	717	0	3038	0	0	0	0
normalized size	1	1.	0.	4.24	0.	0.	0.	0.
time (sec)	N/A	0.596	0.	0.095	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	505	505	0	1585	0	0	0	0
normalized size	1	1.	0.	3.14	0.	0.	0.	0.
time (sec)	N/A	0.279	0.	0.026	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	526	453	0	0	0	0
normalized size	1	1.	1.47	1.26	0.	0.	0.	0.
time (sec)	N/A	0.159	1.422	0.023	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	447	447	0	1005	0	0	0	0
normalized size	1	1.	0.	2.25	0.	0.	0.	0.
time (sec)	N/A	0.273	0.	0.039	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	680	680	0	1395	0	0	0	0
normalized size	1	1.	0.	2.05	0.	0.	0.	0.
time (sec)	N/A	0.515	0.	0.051	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	0	18	23	39	0	95
normalized size	1	1.	0.	0.95	1.21	2.05	0.	5.
time (sec)	N/A	0.018	0.	0.006	1.126	1.217	0.	1.217

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	0	52	69	173	0	228
normalized size	1	1.	0.	0.91	1.21	3.04	0.	4.
time (sec)	N/A	0.067	0.	0.005	1.236	1.337	0.	1.17

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	0	53	66	171	0	221
normalized size	1	1.	0.	0.93	1.16	3.	0.	3.88
time (sec)	N/A	0.08	0.	0.004	1.133	1.31	0.	1.177

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	0	63	127	193	0	262
normalized size	1	1.	0.	0.91	1.84	2.8	0.	3.8
time (sec)	N/A	0.091	0.	0.004	1.155	1.306	0.	1.185

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [47] had the largest ratio of [0.6875]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	18	0.056
2	A	2	1	1.	23	0.043
3	A	2	1	1.	28	0.036
4	A	2	1	1.	33	0.03
5	A	2	1	1.	38	0.026
6	A	2	1	1.	20	0.05
7	A	2	1	1.	25	0.04
8	A	2	1	1.	30	0.033
9	A	2	1	1.	35	0.029
10	A	10	7	1.	18	0.389
11	A	9	7	1.	23	0.304
12	A	8	6	1.	28	0.214
13	A	10	7	1.	33	0.212
14	A	12	8	1.	38	0.21
15	A	15	8	1.	16	0.5
16	A	14	8	1.	21	0.381
17	A	15	7	1.	26	0.269
18	A	17	8	1.	31	0.258
19	A	19	9	1.	36	0.25
20	A	9	7	1.	20	0.35
21	A	8	7	1.	25	0.28
22	A	9	8	1.	30	0.267
23	A	11	9	1.	35	0.257
24	A	13	10	1.	40	0.25
25	A	13	10	1.	55	0.182
26	A	12	9	1.	18	0.5
27	A	11	9	1.	23	0.391
28	A	10	8	1.	28	0.286
29	A	10	8	1.	33	0.242
30	A	11	9	1.	38	0.237
31	A	17	10	1.	16	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	16	10	1.	21	0.476
33	A	15	9	1.	26	0.346
34	A	15	9	1.	31	0.29
35	A	16	10	1.	36	0.278
36	A	11	9	1.	20	0.45
37	A	10	9	1.	25	0.36
38	A	9	8	1.	30	0.267
39	A	9	8	1.	35	0.229
40	A	10	9	1.	40	0.225
41	A	13	11	1.	55	0.2
42	A	14	10	1.	18	0.556
43	A	13	9	1.	23	0.391
44	A	12	9	1.	28	0.321
45	A	12	10	1.	33	0.303
46	A	13	11	1.	38	0.29
47	A	19	11	1.	16	0.688
48	A	18	10	1.	21	0.476
49	A	17	10	1.	26	0.385
50	A	17	11	1.	31	0.355
51	A	18	12	1.	36	0.333
52	A	13	10	1.	20	0.5
53	A	12	9	1.	25	0.36
54	A	11	9	1.	30	0.3
55	A	11	10	1.	35	0.286
56	A	12	11	1.	40	0.275
57	A	11	9	0.99	55	0.164
58	A	11	10	1.	50	0.2
59	A	13	10	1.	50	0.2
60	A	2	1	1.	63	0.016
61	A	2	1	1.	63	0.016
62	A	2	1	1.	61	0.016
63	A	2	1	1.	63	0.016

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	9	8	1.	63	0.127
65	A	11	10	1.	63	0.159
66	A	13	10	1.	63	0.159
67	A	2	2	1.	26	0.077
68	A	3	2	1.	31	0.065
69	A	3	2	1.	36	0.056
70	A	3	2	1.	41	0.049
71	A	3	2	1.	46	0.043
72	A	3	2	1.	51	0.039
73	A	4	3	1.	21	0.143
74	A	4	3	1.	26	0.115
75	A	6	4	1.	31	0.129
76	A	6	4	1.	36	0.111
77	A	6	4	1.	41	0.098
78	A	6	4	1.	46	0.087
79	A	3	2	1.	16	0.125
80	A	3	2	1.	21	0.095
81	A	3	2	1.	26	0.077
82	A	3	2	1.	31	0.065
83	A	3	2	1.	36	0.056
84	A	3	2	1.	41	0.049
85	A	3	2	1.	26	0.077
86	A	3	2	1.	31	0.065
87	A	3	2	1.	36	0.056
88	A	3	2	1.	41	0.049
89	A	3	2	1.	46	0.043
90	A	3	2	1.	51	0.039
91	A	9	5	1.	21	0.238
92	A	9	5	1.	26	0.192
93	A	9	5	1.	31	0.161
94	A	3	2	1.	36	0.056
95	A	3	2	1.	41	0.049

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.	46	0.043
97	A	3	2	1.	16	0.125
98	A	3	2	1.	21	0.095
99	A	3	2	1.	26	0.077
100	A	3	2	1.	31	0.065
101	A	3	2	1.	36	0.056
102	A	3	2	1.	41	0.049
103	A	12	10	1.	32	0.312
104	A	10	10	1.	32	0.312
105	A	8	8	1.	32	0.25
106	A	7	7	1.	32	0.219
107	A	9	8	1.	32	0.25
108	A	1	1	1.	28	0.036
109	A	5	5	1.	31	0.161
110	A	5	5	1.	33	0.152
111	A	4	4	1.	36	0.111

Chapter 3

Listing of integrals

3.1 $\int (d+ex)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=50

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Rubi [A] time = 0.0412663, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1671}

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\int \left(ad+aex+bdx^2+bex^3+cdx^4+cex^5\right)\,dx$$
$$=adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Mathematica [A] time = 0.0017897, size = 50, normalized size = 1.

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2 + c*x^4),x]

[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Maple [A] time = 0.007, size = 41, normalized size = 0.8

$$adx+\frac{aex^2}{2}+\frac{bdx^3}{3}+\frac{bex^4}{4}+\frac{cdx^5}{5}+\frac{cex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+b*x^2+a),x)

[Out] a*d*x+1/2*a*e*x^2+1/3*b*d*x^3+1/4*b*e*x^4+1/5*c*d*x^5+1/6*c*e*x^6
```

Maxima [A] time = 1.32987, size = 54, normalized size = 1.08

$$\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bex^4+\frac{1}{3}\,bdx^3+\frac{1}{2}\,aex^2+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Fricas [A] time = 1.46244, size = 104, normalized size = 2.08

$$\frac{1}{6}x^6ec + \frac{1}{5}x^5dc + \frac{1}{4}x^4eb + \frac{1}{3}x^3db + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c + 1/5*x^5*d*c + 1/4*x^4*e*b + 1/3*x^3*d*b + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.064181, size = 46, normalized size = 0.92

$$adx + \frac{aex^2}{2} + \frac{bdx^3}{3} + \frac{bex^4}{4} + \frac{cdx^5}{5} + \frac{cex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x**3/3 + b*e*x**4/4 + c*d*x**5/5 + c*e*x**6/6
```

Giac [A] time = 1.07894, size = 58, normalized size = 1.16

$$\frac{1}{6}\,cx^6e + \frac{1}{5}\,cdx^5 + \frac{1}{4}\,bx^4e + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*a*x^2*e + a*d*x
```

3.2 $\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5)
/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rubi [A] time = 0.0446495, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5)
/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx\\ &= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}cfx^7\end{aligned}$$

Mathematica [A] time = 0.0209394, size = 69, normalized size = 1.

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5)
/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$adx + \frac{aex^2}{2} + \frac{\left(af + bd\right)x^3}{3} + \frac{bex^4}{4} + \frac{\left(bf + cd\right)x^5}{5} + \frac{cex^6}{6} + \frac{cfx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5+1/6*c*e*x
^6+1/7*c*f*x^7
```

Maxima [A] time = 1.31112, size = 77, normalized size = 1.12

$$\frac{1}{7}cfx^7 + \frac{1}{6}cex^6 + \frac{1}{4}bex^4 + \frac{1}{5}\left(cd + bf\right)x^5 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bd + af\right)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/4*b*e*x^4 + 1/5*(c*d + b*f)*x^5 + 1/2*a*e*x^2
 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 1.47249, size = 161, normalized size = 2.33

$$\frac{1}{7}x^7fc + \frac{1}{6}x^6ec + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{4}x^4eb + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f*c + 1/6*x^6*e*c + 1/5*x^5*d*c + 1/5*x^5*f*b + 1/4*x^4*e*b + 1/3*x
^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.070458, size = 65, normalized size = 0.94

$$adx + \frac{aex^2}{2} + \frac{bex^4}{4} + \frac{cex^6}{6} + \frac{cfx^7}{7} + x^5\left(\frac{bf}{5} + \frac{cd}{5}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*e*x**4/4 + c*e*x**6/6 + c*f*x**7/7 + x**5*(b*f/5 + c
*d/5) + x**3*(a*f/3 + b*d/3)
```

Giac [A] time = 1.0831, size = 86, normalized size = 1.25

$$\frac{1}{7}cfx^7 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/4*b*x^4*e + 1/3*b
*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

3.3 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=88

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x^8)/8
```

Rubi [A] time = 0.0733669, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1671}

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x^8)/8
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)\,dx = \int \left(ad+aex+(bd+af)x^2+(be+ag)x^3+(cd+bf)x^4+(ce+bg)x^5+c\right.$$

$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}(ce+bg$$

Mathematica [A] time = 0.0192433, size = 88, normalized size = 1.

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x^8)/8
```

Maple [A] time = 0., size = 75, normalized size = 0.9

$$adx + \frac{aex^2}{2} + \frac{\left(af + bd\right)x^3}{3} + \frac{\left(ag + be\right)x^4}{4} + \frac{\left(bf + cd\right)x^5}{5} + \frac{\left(bg + ce\right)x^6}{6} + \frac{cfx^7}{7} + \frac{cgx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(b*f+c*d)*x^5+1/6
*(b*g+c*e)*x^6+1/7*c*f*x^7+1/8*c*g*x^8
```

Maxima [A] time = 0.978737, size = 100, normalized size = 1.14

$$\frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{6}\left(ce + bg\right)x^6 + \frac{1}{5}\left(cd + bf\right)x^5 + \frac{1}{4}\left(be + ag\right)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}\left(bd + af\right)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*(c*e + b*g)*x^6 + 1/5*(c*d + b*f)*x^5 + 1/4
*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 1.53036, size = 217, normalized size = 2.47

$$\frac{1}{8}x^8gc + \frac{1}{7}x^7fc + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{4}x^4eb + \frac{1}{4}x^4ga + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}x^2ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*g*c + 1/7*x^7*f*c + 1/6*x^6*e*c + 1/6*x^6*g*b + 1/5*x^5*d*c + 1/5*x
^5*f*b + 1/4*x^4*e*b + 1/4*x^4*g*a + 1/3*x^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*
a + x*d*a
```

Sympy [A] time = 0.072304, size = 83, normalized size = 0.94

$$adx + \frac{aex^2}{2} + \frac{cfx^7}{7} + \frac{cgx^8}{8} + x^6\left(\frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*f*x**7/7 + c*g*x**8/8 + x**6*(b*g/6 + c*e/6) + x**5*
(b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Giac [A] time = 1.07714, size = 115, normalized size = 1.31

$$\frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*b*g*x^6 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b
*f*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*
e + a*d*x
```

3.4 $\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4\right)\,dx$

Optimal. Leaf size=105

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x^7)/7 + (c*g*x^8)/8
+ (c*h*x^9)/9
```

Rubi [A] time = 0.0948779, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {1671}

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x^7)/7 + (c*g*x^8)/8
+ (c*h*x^9)/9
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4\right)\,dx=\int \left(ad+aex+(bd+af)x^2+(be+ag)x^3+(cd+bf+ah)x^4+(ce+\right.$$

$$=adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5+$$

Mathematica [A] time = 0.035032, size = 105, normalized size = 1.

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x^7)/7 + (c*g*x^8)/8
+ (c*h*x^9)/9
```

Maple [A] time = 0., size = 90, normalized size = 0.9

$$adx+\frac{aex^2}{2}+\frac{(af+bd)x^3}{3}+\frac{(ag+be)x^4}{4}+\frac{(ah+bf+cd)x^5}{5}+\frac{(bg+ce)x^6}{6}+\frac{(bh+cf)x^7}{7}+\frac{cgx^8}{8}+\frac{chx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*c*g*x^8+1/9*c*h*x^9
```

Maxima [A] time = 0.957798, size = 120, normalized size = 1.14

$$\frac{1}{9}chx^9+\frac{1}{8}cgx^8+\frac{1}{7}(cf+bh)x^7+\frac{1}{6}(ce+bg)x^6+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{4}(be+ag)x^4+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*(c*f + b*h)*x^7 + 1/6*(c*e + b*g)*x^6 + 1/5
*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f
)*x^3 + a*d*x
```

Fricas [A] time = 1.43696, size = 274, normalized size = 2.61

$$\frac{1}{9}x^9hc + \frac{1}{8}x^8gc + \frac{1}{7}x^7fc + \frac{1}{7}x^7hb + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{5}x^5ha + \frac{1}{4}x^4eb + \frac{1}{4}x^4ga + \frac{1}{3}x^3db + \frac{1}{3}x^3fa + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*h*c + 1/8*x^8*g*c + 1/7*x^7*f*c + 1/7*x^7*h*b + 1/6*x^6*e*c + 1/6*x
^6*g*b + 1/5*x^5*d*c + 1/5*x^5*f*b + 1/5*x^5*h*a + 1/4*x^4*e*b + 1/4*x^4*g*
a + 1/3*x^3*d*b + 1/3*x^3*f*a + 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.077167, size = 102, normalized size = 0.97

$$adx + \frac{aex^2}{2} + \frac{cgx^8}{8} + \frac{chx^9}{9} + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*g*x**8/8 + c*h*x**9/9 + x**7*(b*h/7 + c*f/7) + x**6*
(b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**
3*(a*f/3 + b*d/3)
```

Giac [A] time = 1.09345, size = 143, normalized size = 1.36

$$\frac{1}{9}chx^9 + \frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{5}ahx^5 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/6*c
*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/5*a*h*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*
e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

3.5 $\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)dx$

Optimal. Leaf size=122

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+$$

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*g
+ b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rubi [A] time = 0.111316, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1671}

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*g
+ b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4+5x^5\right)dx=\int \left(ad+aex+(bd+af)x^2+(be+ag)x^3+(cd+bf+ah)x^4\right.$$

$$=adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+$$

Mathematica [A] time = 0.0393288, size = 122, normalized size = 1.

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*g
+ b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Maple [A] time = 0.002, size = 105, normalized size = 0.9

$$adx+\frac{aex^2}{2}+\frac{(af+bd)x^3}{3}+\frac{(ag+be)x^4}{4}+\frac{(ah+bf+cd)x^5}{5}+\frac{(ai+bg+ce)x^6}{6}+\frac{(bh+cf)x^7}{7}+\frac{(bi+cg)x^8}{8}+\frac{chx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*x^8+1/9*c*h*x^9+1/10
*c*i*x^10
```

Maxima [A] time = 0.977675, size = 140, normalized size = 1.15

$$\frac{1}{10}cix^{10}+\frac{1}{9}chx^9+\frac{1}{8}(cg+bi)x^8+\frac{1}{7}(cf+bh)x^7+\frac{1}{6}(ce+bg+ai)x^6+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{4}(be+ag)x^4+\frac{1}{2}aex^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*(c*g + b*i)*x^8 + 1/7*(c*f + b*h)*x^7 + 1
/6*(c*e + b*g + a*i)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^4
+ 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Fricas [A] time = 1.47442, size = 333, normalized size = 2.73

$$\frac{1}{10}x^{10}ic + \frac{1}{9}x^9hc + \frac{1}{8}x^8gc + \frac{1}{8}x^8ib + \frac{1}{7}x^7fc + \frac{1}{7}x^7hb + \frac{1}{6}x^6ec + \frac{1}{6}x^6gb + \frac{1}{6}x^6ia + \frac{1}{5}x^5dc + \frac{1}{5}x^5fb + \frac{1}{5}x^5ha + \frac{1}{4}x^4eb +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/10*x^10*i*c + 1/9*x^9*h*c + 1/8*x^8*g*c + 1/8*x^8*i*b + 1/7*x^7*f*c + 1/7
*x^7*h*b + 1/6*x^6*e*c + 1/6*x^6*g*b + 1/6*x^6*i*a + 1/5*x^5*d*c + 1/5*x^5*
f*b + 1/5*x^5*h*a + 1/4*x^4*e*b + 1/4*x^4*g*a + 1/3*x^3*d*b + 1/3*x^3*f*a +
 1/2*x^2*e*a + x*d*a
```

Sympy [A] time = 0.080023, size = 121, normalized size = 0.99

$$adx + \frac{aex^2}{2} + \frac{chx^9}{9} + \frac{cix^{10}}{10} + x^8\left(\frac{bi}{8} + \frac{cg}{8}\right) + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{ai}{6} + \frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*h*x**9/9 + c*i*x**10/10 + x**8*(b*i/8 + c*g/8) + x**
7*(b*h/7 + c*f/7) + x**6*(a*i/6 + b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*
d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Giac [A] time = 1.09444, size = 171, normalized size = 1.4

$$\frac{1}{10}cix^{10} + \frac{1}{9}chx^9 + \frac{1}{8}cgx^8 + \frac{1}{8}bix^8 + \frac{1}{7}cfx^7 + \frac{1}{7}bhx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}aix^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{5}ahx^5 + \frac{1}{4}ag$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/8*b*i*x^8 + 1/7*c*f*x^7 + 1/7
*b*h*x^7 + 1/6*b*g*x^6 + 1/6*a*i*x^6 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*
x^5 + 1/5*a*h*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 +
 1/2*a*x^2*e + a*d*x
```

3.6 $\int (d+ex)\left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=112

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2 + 2*a*c)*
d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 + (b*c*e*x^8)/4 + (c^2
*d*x^9)/9 + (c^2*e*x^10)/10
```

Rubi [A] time = 0.125778, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1671}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2 + 2*a*c)*
d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 + (b*c*e*x^8)/4 + (c^2
*d*x^9)/9 + (c^2*e*x^10)/10
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2d+a^2ex+2abdx^2+2abex^3+\left(b^2+2ac\right)dx^4+\left(b^2+2ac\right)ex^5+2bcdx^6+2bcex\right.$$

$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2+2ac\right)dx^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{2}{7}bcdx^7+$$

Mathematica [A] time = 0.0506573, size = 97, normalized size = 0.87

$$\frac{630a^2x(2d+ex)+42a\left(5bx^3(4d+3ex)+2cx^5(6d+5ex)\right)+42b^2x^5(6d+5ex)+45bcx^7(8d+7ex)+14c^2x^9(10d+9ex)}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (630*a^2*x*(2*d + e*x) + 42*b^2*x^5*(6*d + 5*e*x) + 45*b*c*x^7*(8*d + 7*e*x
) + 14*c^2*x^9*(10*d + 9*e*x) + 42*a*(5*b*x^3*(4*d + 3*e*x) + 2*c*x^5*(6*d
+ 5*e*x)))/1260
```

Maple [A] time = 0.001, size = 95, normalized size = 0.9

$$a^2dx+\frac{a^2ex^2}{2}+\frac{2\,abdx^3}{3}+\frac{abex^4}{2}+\frac{\left(2\,ac+b^2\right)dx^5}{5}+\frac{\left(2\,ac+b^2\right)ex^6}{6}+\frac{2\,bcdx^7}{7}+\frac{bcex^8}{4}+\frac{c^2dx^9}{9}+\frac{c^2ex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)*d*x^5+1/6
*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*d*x^9+1/10*c^2*e*x^1
0
```

Maxima [A] time = 0.951583, size = 127, normalized size = 1.13

$$\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{6}\,\left(b^2+2\,ac\right)ex^6+\frac{1}{2}\,abex^4+\frac{1}{5}\,\left(b^2+2\,ac\right)dx^5+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 + 1/6*(b^2
+ 2*a*c)*e*x^6 + 1/2*a*b*e*x^4 + 1/5*(b^2 + 2*a*c)*d*x^5 + 2/3*a*b*d*x^3 +
1/2*a^2*e*x^2 + a^2*d*x
```

Fricas [A] time = 1.47995, size = 252, normalized size = 2.25

$$\frac{1}{10}x^{10}ec^2 + \frac{1}{9}x^9dc^2 + \frac{1}{4}x^8ecb + \frac{2}{7}x^7dcb + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5dca + \frac{1}{2}x^4eba + \frac{2}{3}x^3dba + \frac{1}{2}x^2ea^2 + xda^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 1/4*x^8*e*c*b + 2/7*x^7*d*c*b + 1/6*x^6*e
*b^2 + 1/3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 1/2*x^4*e*b*a + 2/3*
x^3*d*b*a + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.08321, size = 116, normalized size = 1.04

$$a^2dx + \frac{a^2ex^2}{2} + \frac{2abdx^3}{3} + \frac{abex^4}{2} + \frac{2bcdx^7}{7} + \frac{bcex^8}{4} + \frac{c^2dx^9}{9} + \frac{c^2ex^{10}}{10} + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{2acd}{5} + \frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x**3/3 + a*b*e*x**4/2 + 2*b*c*d*x**7/7 +
 b*c*e*x**8/4 + c**2*d*x**9/9 + c**2*e*x**10/10 + x**6*(a*c*e/3 + b**2*e/6)
 + x**5*(2*a*c*d/5 + b**2*d/5)
```

Giac [A] time = 1.0943, size = 143, normalized size = 1.28

$$\frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{2}a^2x^2e + a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/6*b^2*x
^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 1/2*a*b*x^4*e + 2/3*
a*b*d*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.7 $\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Rubi [A] time = 0.130352, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int\left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\int\left(a^2d+a^2ex+a(2bd+af)x^2+2abex^3+\left(b^2d+2acd+2abf\right)x^4+\left(b^2+2ac\right)$$

$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2\right.$$

Mathematica [A] time = 0.0464454, size = 154, normalized size = 1.

$$a^2dx + \frac{1}{2}a^2ex^2 + \frac{1}{7}x^7\left(2acf + b^2f + 2bcd\right) + \frac{1}{5}x^5\left(2abf + 2acd + b^2d\right) + \frac{1}{6}ex^6\left(2ac + b^2\right) + \frac{1}{3}ax^3(af + 2bd) + \frac{1}{2}abex^4 +$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Maple [A] time = 0.002, size = 139, normalized size = 0.9

$$\frac{c^2fx^{11}}{11} + \frac{c^2ex^{10}}{10} + \frac{\left(2fbc + c^2d\right)x^9}{9} + \frac{bcex^8}{4} + \frac{\left(2bcd + f\left(2ac + b^2\right)\right)x^7}{7} + \frac{\left(2ac + b^2\right)ex^6}{6} + \frac{\left(d\left(2ac + b^2\right) + 2abf\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(2*b*c*f+c^2*d)*x^9+1/4*b*c*e*x^8+1/7*(
2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*(2*a*c+b^2)*e*x^6+1/5*(d*(2*a*c+b^2)+2*a*b*f
)*x^5+1/2*a*b*e*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.960336, size = 186, normalized size = 1.21

$$\frac{1}{11}c^2fx^{11} + \frac{1}{10}c^2ex^{10} + \frac{1}{4}bcex^8 + \frac{1}{9}\left(c^2d + 2bcf\right)x^9 + \frac{1}{6}\left(b^2 + 2ac\right)ex^6 + \frac{1}{7}\left(2bcd + \left(b^2 + 2ac\right)f\right)x^7 + \frac{1}{2}abex^4 + \frac{1}{5}(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*a*
b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x + 1
```

```
/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 1.70357, size = 385, normalized size = 2.5

$$\frac{1}{11}x^{11}fc^2 + \frac{1}{10}x^{10}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{4}x^8ecb + \frac{2}{7}x^7dcb + \frac{1}{7}x^7fb^2 + \frac{2}{7}x^7fca + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b + 1/4*x^8
*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a + 1/6*x^6*e*b^2 + 1/
3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*a + 1/2*x^4*e*b*a
 + 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.093461, size = 165, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{abex^4}{2} + \frac{bcex^8}{4} + \frac{c^2ex^{10}}{10} + \frac{c^2fx^{11}}{11} + x^9\left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^7\left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/7
+ 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b*
*2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A] time = 1.08962, size = 212, normalized size = 1.38

$$\frac{1}{11}c^2fx^{11} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*c
*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*x^6*e + 1/
3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e
 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.8 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=196

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+\right.$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*g)*x^6)/
6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)
/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11
+ (c^2*g*x^12)/12
```

Rubi [A] time = 0.167512, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1671}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*g)*x^6)/
6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)
/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11
+ (c^2*g*x^12)/12
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d+a^2ex+a(2bd+af)x^2+a(2be+ag)x^3+\left(b^2d+2acd+2abf\right)x^4\right.\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2acd+2a\right.\end{aligned}$$

Mathematica [A] time = 0.0586498, size = 196, normalized size = 1.

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*g)*x^6)/
6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)
/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11
+ (c^2*g*x^12)/12
```

Maple [A] time = 0., size = 183, normalized size = 0.9

$$\frac{c^2gx^{12}}{12}+\frac{c^2fx^{11}}{11}+\frac{\left(2gbc+ec^2\right)x^{10}}{10}+\frac{\left(2fbc+c^2d\right)x^9}{9}+\frac{\left(2bce+g\left(2ac+b^2\right)\right)x^8}{8}+\frac{\left(2bcd+f\left(2ac+b^2\right)\right)x^7}{7}+\frac{(e(2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/12*c^2*g*x^12+1/11*c^2*f*x^11+1/10*(2*b*c*g+c^2*e)*x^10+1/9*(2*b*c*f+c^2*
d)*x^9+1/8*(2*b*c*e+g*(2*a*c+b^2))*x^8+1/7*(2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*
(e*(2*a*c+b^2)+2*a*b*g)*x^6+1/5*(d*(2*a*c+b^2)+2*a*b*f)*x^5+1/4*(a^2*g+2*a*
b*e)*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.949298, size = 246, normalized size = 1.26

$$\frac{1}{12}c^2gx^{12}+\frac{1}{11}c^2fx^{11}+\frac{1}{10}\left(c^2e+2bcg\right)x^{10}+\frac{1}{9}\left(c^2d+2bcf\right)x^9+\frac{1}{8}\left(2bce+\left(b^2+2ac\right)g\right)x^8+\frac{1}{7}\left(2bcd+\left(b^2+2ac\right)f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/10*(c^2*e + 2*b*c*g)*x^10 + 1/9*(c^2*
d + 2*b*c*f)*x^9 + 1/8*(2*b*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + (b^
```

```
2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b*g + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + (b
^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x +
1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 1.73068, size = 518, normalized size = 2.64

$$\frac{1}{12}x^{12}gc^2 + \frac{1}{11}x^{11}fc^2 + \frac{1}{10}x^{10}ec^2 + \frac{1}{5}x^{10}gcb + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{4}x^8ecb + \frac{1}{8}x^8gb^2 + \frac{1}{4}x^8gca + \frac{2}{7}x^7dcb + \frac{1}{7}x^7fb^2 + \frac{2}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*g*c^2 + 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/5*x^10*g*c*b + 1/9*
x^9*d*c^2 + 2/9*x^9*f*c*b + 1/4*x^8*e*c*b + 1/8*x^8*g*b^2 + 1/4*x^8*g*c*a +
 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a + 1/6*x^6*e*b^2 + 1/3*x^6*e*
c*a + 1/3*x^6*g*b*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*a + 1/2*x
^4*e*b*a + 1/4*x^4*g*a^2 + 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 + 1/2*x^2*e*a^2 +
x*d*a^2
```

Sympy [A] time = 0.099348, size = 209, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2fx^{11}}{11} + \frac{c^2gx^{12}}{12} + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bc}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*f*x**11/11 + c**2*g*x**12/12 + x**10*(b*c*g
/5 + c**2*e/10) + x**9*(2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 + b**2*g/8 +
b*c*e/4) + x**7*(2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*b*g/3 + a*c*e/
3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/4 +
a*b*e/2) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A] time = 1.09694, size = 281, normalized size = 1.43

$$\frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{1}{5}bcgx^{10} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{4}acgx^8 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/5*b*c*g*x^10 + 1/10*c^2*x^10*e + 1/9*
c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x^8 + 1/4*b*c*x^8*e +
 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/3*a*b*g*x^6 + 1/6*b^2*x^
6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/4*a
^2*g*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e +
a^2*d*x
```

3.9 $\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4\right) dx$

Optimal. Leaf size=234

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2c(ah+bf)+b^2h+c^2d\right)+\frac{1}{7}x^7\left(2b(ah+cd)+2acf+b^2f\right)+\frac{1}{5}x^5\left(2abf+a(ah+2cd)+b^2d\right)+\frac{1}{8}$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*
g)*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*h))*x^7)/7 + ((2*b*c*e + b^2*g
 + 2*a*c*g)*x^8)/8 + ((c^2*d + b^2*h + 2*c*(b*f + a*h))*x^9)/9 + (c*(c*e +
2*b*g)*x^10)/10 + (c*(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13
)/13
```

Rubi [A] time = 0.238095, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1671}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2c(ah+bf)+b^2h+c^2d\right)+\frac{1}{7}x^7\left(2b(ah+cd)+2acf+b^2f\right)+\frac{1}{5}x^5\left(2abf+a(ah+2cd)+b^2d\right)+\frac{1}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*
g)*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*h))*x^7)/7 + ((2*b*c*e + b^2*g
 + 2*a*c*g)*x^8)/8 + ((c^2*d + b^2*h + 2*c*(b*f + a*h))*x^9)/9 + (c*(c*e +
2*b*g)*x^10)/10 + (c*(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13
)/13
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^2\left(d+ex+fx^2+gx^3+hx^4\right)\,dx=\int \left(a^2d+a^2ex+a(2bd+af)x^2+a(2be+ag)x^3+\left(b^2d+2abf+\right.\right.$$
$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2a\right.$$

Mathematica [A] time = 0.0877014, size = 234, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2h+2abf+2acd+b^2d\right)+a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2ach+b^2h+2bcf+c^2d\right)+\frac{1}{7}x^7\left(2abh+2acf+b^2f+2bcd\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f + a^2*h)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*
g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f + 2*a*b*h)*x^7)/7 + ((2*b*c*e + b^2
*g + 2*a*c*g)*x^8)/8 + ((c^2*d + 2*b*c*f + b^2*h + 2*a*c*h)*x^9)/9 + (c*(c*
e + 2*b*g)*x^10)/10 + (c*(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*
x^13)/13
```

Maple [A] time = 0.002, size = 219, normalized size = 0.9

$$\frac{c^2hx^{13}}{13}+\frac{c^2gx^{12}}{12}+\frac{\left(2\,bch+c^2f\right)x^{11}}{11}+\frac{\left(2\,gbc+ec^2\right)x^{10}}{10}+\frac{\left(\left(2\,ac+b^2\right)h+2\,fbc+c^2d\right)x^9}{9}+\frac{\left(2\,bce+g\left(2\,ac+b^2\right)\right)x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x)
```

```
[Out] 1/13*c^2*h*x^13+1/12*c^2*g*x^12+1/11*(2*b*c*h+c^2*f)*x^11+1/10*(2*b*c*g+c^2
*e)*x^10+1/9*((2*a*c+b^2)*h+2*f*b*c+c^2*d)*x^9+1/8*(2*b*c*e+g*(2*a*c+b^2))*
x^8+1/7*(2*a*b*h+f*(2*a*c+b^2)+2*b*c*d)*x^7+1/6*(e*(2*a*c+b^2)+2*a*b*g)*x^6
+1/5*(a^2*h+2*a*b*f+d*(2*a*c+b^2))*x^5+1/4*(a^2*g+2*a*b*e)*x^4+1/3*(a^2*f+2
*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.937189, size = 294, normalized size = 1.26

$$\frac{1}{13}c^2hx^{13}+\frac{1}{12}c^2gx^{12}+\frac{1}{11}\left(c^2f+2\,bch\right)x^{11}+\frac{1}{10}\left(c^2e+2\,bcg\right)x^{10}+\frac{1}{9}\left(c^2d+2\,bcf+\left(b^2+2\,ac\right)h\right)x^9+\frac{1}{8}\left(2\,bce+\left(b^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*(c^2*f + 2*b*c*h)*x^11 + 1/10*(c^2
*e + 2*b*c*g)*x^10 + 1/9*(c^2*d + 2*b*c*f + (b^2 + 2*a*c)*h)*x^9 + 1/8*(2*b
*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + 2*a*b*h + (b^2 + 2*a*c)*f)*x^7
 + 1/6*(2*a*b*g + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*
c)*d)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*
b*d + a^2*f)*x^3
```

Fricas [A] time = 1.73156, size = 653, normalized size = 2.79

$$\frac{1}{13}x^{13}hc^2 + \frac{1}{12}x^{12}gc^2 + \frac{1}{11}x^{11}fc^2 + \frac{2}{11}x^{11}hcb + \frac{1}{10}x^{10}ec^2 + \frac{1}{5}x^{10}gcb + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{9}x^9hb^2 + \frac{2}{9}x^9hca + \frac{1}{4}x^8ecb$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] 1/13*x^13*h*c^2 + 1/12*x^12*g*c^2 + 1/11*x^11*f*c^2 + 2/11*x^11*h*c*b + 1/1
0*x^10*e*c^2 + 1/5*x^10*g*c*b + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b + 1/9*x^9*h*b
^2 + 2/9*x^9*h*c*a + 1/4*x^8*e*c*b + 1/8*x^8*g*b^2 + 1/4*x^8*g*c*a + 2/7*x^
7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a + 2/7*x^7*h*b*a + 1/6*x^6*e*b^2 + 1
/3*x^6*e*c*a + 1/3*x^6*g*b*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*
a + 1/5*x^5*h*a^2 + 1/2*x^4*e*b*a + 1/4*x^4*g*a^2 + 2/3*x^3*d*b*a + 1/3*x^3
*f*a^2 + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.104479, size = 258, normalized size = 1.1

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2gx^{12}}{12} + \frac{c^2hx^{13}}{13} + x^{11}\left(\frac{2bch}{11} + \frac{c^2f}{11}\right) + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\left(\frac{2ach}{9} + \frac{b^2h}{9} + \frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*g*x**12/12 + c**2*h*x**13/13 + x**11*(2*b*c
*h/11 + c**2*f/11) + x**10*(b*c*g/5 + c**2*e/10) + x**9*(2*a*c*h/9 + b**2*h
/9 + 2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 + b**2*g/8 + b*c*e/4) + x**7*(2*
a*b*h/7 + 2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*b*g/3 + a*c*e/3 + b**
2*e/6) + x**5*(a**2*h/5 + 2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/
4 + a*b*e/2) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A] time = 1.08647, size = 350, normalized size = 1.5

$$\frac{1}{13}c^2hx^{13} + \frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{2}{11}bchx^{11} + \frac{1}{5}bcgx^{10} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{9}b^2hx^9 + \frac{2}{9}achx^9 + \frac{1}{8}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 2/11*b*c*h*x^11 + 1/5
*b*c*g*x^10 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/9*b^2*h*x
^9 + 2/9*a*c*h*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x^8 + 1/4*b*c*x^8*e + 2/7*b*
c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 2/7*a*b*h*x^7 + 1/3*a*b*g*x^6 + 1
/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^
5 + 1/5*a^2*h*x^5 + 1/4*a^2*g*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2
*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.10 $\int \frac{d+ex}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -(d*ArcTanh[x/2])/6 + (d*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 + (e*Log[4 - x^
2])/6
```

Rubi [A] time = 0.0324412, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1673, 12, 1093, 207, 1107, 616, 31}

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -(d*ArcTanh[x/2])/6 + (d*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 + (e*Log[4 - x^
2])/6
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
```

```
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{4-5x^2+x^4}\,dx &= \int \frac{d}{4-5x^2+x^4}\,dx+\int \frac{ex}{4-5x^2+x^4}\,dx\\
&= d\int \frac{1}{4-5x^2+x^4}\,dx+e\int \frac{x}{4-5x^2+x^4}\,dx\\
&= \frac{1}{3}d\int \frac{1}{-4+x^2}\,dx-\frac{1}{3}d\int \frac{1}{-1+x^2}\,dx+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)+\frac{1}{6}e\,\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)-\frac{1}{6}e\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x
\end{aligned}
$$

$$
= -\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)
$$

Mathematica [A] time = 0.0179092, size = 50, normalized size = 1.11

$$
\frac{1}{12}(-2(d+e)\log(1-x)+(d+2e)\log(2-x)+2(d-e)\log(x+1)-(d-2e)\log(x+2))
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e)*Log[1 - x] + (d + 2*e)*Log[2 - x] + 2*(d - e)*Log[1 + x] - (d -
 2*e)*Log[2 + x])/12
```

Maple [A] time = 0.039, size = 58, normalized size = 1.3

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}-\frac{\ln(x-1)\,d}{6}-\frac{\ln(x-1)\,e}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/12*ln(x-2)*d+1/
6*ln(x-2)*e-1/6*ln(x-1)*d-1/6*ln(x-1)*e
```

Maxima [A] time = 0.957648, size = 58, normalized size = 1.29

$$-\frac{1}{12}\,(d-2\,e)\log\left(x+2\right)+\frac{1}{6}\,(d-e)\log\left(x+1\right)-\frac{1}{6}\,(d+e)\log\left(x-1\right)+\frac{1}{12}\,(d+2\,e)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)*log(x - 1
) + 1/12*(d + 2*e)*log(x - 2)
```

Fricas [A] time = 1.90501, size = 143, normalized size = 3.18

$$-\frac{1}{12}\,(d-2\,e)\log\left(x+2\right)+\frac{1}{6}\,(d-e)\log\left(x+1\right)-\frac{1}{6}\,(d+e)\log\left(x-1\right)+\frac{1}{12}\,(d+2\,e)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)*log(x - 1
) + 1/12*(d + 2*e)*log(x - 2)
```

Sympy [B] time = 2.17518, size = 515, normalized size = 11.44

$$-\frac{(d-2e)\log\left(x+\frac{-35d^4e+\frac{51d^4(d-2e)}{2}-180d^2e^3-90d^2e^2(d-2e)+41d^2e(d-2e)^2-\frac{15d^2(d-2e)^3}{2}+320e^5-96e^4(d-2e)-80e^3(d-2e)^2+24e^2(d-2e)^3}{9d^5-160d^3e^2+256de^4}\right)}{12}+\frac{(d-e)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e)*log(x + (-35*d**4*e + 51*d**4*(d - 2*e)/2 - 180*d**2*e**3 - 90*d
**2*e**2*(d - 2*e) + 41*d**2*e*(d - 2*e)**2 - 15*d**2*(d - 2*e)**3/2 + 320*
e**5 - 96*e**4*(d - 2*e) - 80*e**3*(d - 2*e)**2 + 24*e**2*(d - 2*e)**3)/(9*
d**5 - 160*d**3*e**2 + 256*d*e**4))/12 + (d - e)*log(x + (-35*d**4*e - 51*d
**4*(d - e) - 180*d**2*e**3 + 180*d**2*e**2*(d - e) + 164*d**2*e*(d - e)**2
 + 60*d**2*(d - e)**3 + 320*e**5 + 192*e**4*(d - e) - 320*e**3*(d - e)**2 -
 192*e**2*(d - e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/6 - (d + e)*lo
g(x + (-35*d**4*e + 51*d**4*(d + e) - 180*d**2*e**3 - 180*d**2*e**2*(d + e)
 + 164*d**2*e*(d + e)**2 - 60*d**2*(d + e)**3 + 320*e**5 - 192*e**4*(d + e)
 - 320*e**3*(d + e)**2 + 192*e**2*(d + e)**3)/(9*d**5 - 160*d**3*e**2 + 256
*d*e**4))/6 + (d + 2*e)*log(x + (-35*d**4*e - 51*d**4*(d + 2*e)/2 - 180*d**
2*e**3 + 90*d**2*e**2*(d + 2*e) + 41*d**2*e*(d + 2*e)**2 + 15*d**2*(d + 2*e
)**3/2 + 320*e**5 + 96*e**4*(d + 2*e) - 80*e**3*(d + 2*e)**2 - 24*e**2*(d +
 2*e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/12
```

Giac [A] time = 1.09427, size = 69, normalized size = 1.53

$$-\frac{1}{12}\,(d-2\,e)\log\left(|x+2|\right)+\frac{1}{6}\,(d-e)\log\left(|x+1|\right)-\frac{1}{6}\,(d+e)\log\left(|x-1|\right)+\frac{1}{12}\,(d+2\,e)\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d - 2*e)*log(abs(x + 2)) + 1/6*(d - e)*log(abs(x + 1)) - 1/6*(d + e)
*log(abs(x - 1)) + 1/12*(d + 2*e)*log(abs(x - 2))
```

3.11 $\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 +
 (e*Log[4 - x^2])/6
```

Rubi [A] time = 0.0566029, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1673, 1166, 207, 12, 1107, 616, 31}

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```


```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 +
 (e*Log[4 - x^2])/6
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx &= \int \frac{ex}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2}{4-5x^2+x^4}\,dx \\
&= e\int \frac{x}{4-5x^2+x^4}\,dx - \frac{1}{3}(d+f)\int \frac{1}{-1+x^2}\,dx + \frac{1}{3}(d+4f)\int \frac{1}{-4+x^2}\,dx \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{6}e\,\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right) - \frac{1}{6}e\,\text{Subst}\left(\int \frac{}{-1} \right. \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) - \frac{1}{6}e\log\left(1-x^2\right) + \frac{1}{6}e\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0259607, size = 58, normalized size = 1.14

$$\frac{1}{12}(-2\log(1-x)(d+e+f) + \log(2-x)(d+2e+4f) + 2\log(x+1)(d-e+f) - \log(x+2)(d-2e+4f))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] + 2*(d - e + f)*Log
[1 + x] - (d - 2*e + 4*f)*Log[2 + x])/12
```

Maple [B] time = 0.007, size = 86, normalized size = 1.7

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}-\frac{\ln(2+x)\,f}{3}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(1+x)\,f}{6}+\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}+\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6
*ln(1+x)*f+1/12*ln(x-2)*d+1/6*ln(x-2)*e+1/3*ln(x-2)*f-1/6*ln(x-1)*d-1/6*ln(
x-1)*e-1/6*ln(x-1)*f
```

Maxima [A] time = 0.963057, size = 69, normalized size = 1.35

$$-\frac{1}{12}\left(d-2e+4f\right)\log(x+2)+\frac{1}{6}\left(d-e+f\right)\log(x+1)-\frac{1}{6}\left(d+e+f\right)\log(x-1)+\frac{1}{12}\left(d+2e+4f\right)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/6*(d + e
+ f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Fricas [A] time = 2.0685, size = 170, normalized size = 3.33

$$-\frac{1}{12}\left(d-2e+4f\right)\log(x+2)+\frac{1}{6}\left(d-e+f\right)\log(x+1)-\frac{1}{6}\left(d+e+f\right)\log(x-1)+\frac{1}{12}\left(d+2e+4f\right)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/6*(d + e
+ f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Sympy [B] time = 33.0575, size = 2195, normalized size = 43.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e + 4*f)*log(x + (-35*d**5*e + 51*d**5*(d - 2*e + 4*f)/2 - 820*d**4
*e*f + 90*d**4*f*(d - 2*e + 4*f) - 180*d**3*e**3 - 90*d**3*e**2*(d - 2*e +
4*f) - 4100*d**3*e*f**2 + 41*d**3*e*(d - 2*e + 4*f)**2 + 42*d**3*f**2*(d -
2*e + 4*f) - 15*d**3*(d - 2*e + 4*f)**3/2 - 432*d**2*e**2*f*(d - 2*e + 4*f)
 - 8000*d**2*e*f**3 + 240*d**2*e*f*(d - 2*e + 4*f)**2 - 240*d**2*f**3*(d -
2*e + 4*f) - 12*d**2*f*(d - 2*e + 4*f)**3 + 320*d*e**5 - 96*d*e**4*(d - 2*e
 + 4*f) + 720*d*e**3*f**2 - 80*d*e**3*(d - 2*e + 4*f)**2 - 1080*d*e**2*f**2
*(d - 2*e + 4*f) + 24*d*e**2*(d - 2*e + 4*f)**3 - 6400*d*e*f**4 + 492*d*e*f
**2*(d - 2*e + 4*f)**2 - 576*d*f**4*(d - 2*e + 4*f) + 30*d*f**2*(d - 2*e +
4*f)**3 + 512*e**5*f - 128*e**3*f*(d - 2*e + 4*f)**2 - 576*e**2*f**3*(d - 2
*e + 4*f) - 1472*e*f**5 + 320*e*f**3*(d - 2*e + 4*f)**2 - 480*f**5*(d - 2*e
 + 4*f) + 48*f**3*(d - 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 -
 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**
2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5
 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/12 + (d - e + f)*log(x + (-
35*d**5*e - 51*d**5*(d - e + f) - 820*d**4*e*f - 180*d**4*f*(d - e + f) - 1
80*d**3*e**3 + 180*d**3*e**2*(d - e + f) - 4100*d**3*e*f**2 + 164*d**3*e*(d
 - e + f)**2 - 84*d**3*f**2*(d - e + f) + 60*d**3*(d - e + f)**3 + 864*d**2
*e**2*f*(d - e + f) - 8000*d**2*e*f**3 + 960*d**2*e*f*(d - e + f)**2 + 480*
d**2*f**3*(d - e + f) + 96*d**2*f*(d - e + f)**3 + 320*d*e**5 + 192*d*e**4*
(d - e + f) + 720*d*e**3*f**2 - 320*d*e**3*(d - e + f)**2 + 2160*d*e**2*f**
2*(d - e + f) - 192*d*e**2*(d - e + f)**3 - 6400*d*e*f**4 + 1968*d*e*f**2*(
d - e + f)**2 + 1152*d*f**4*(d - e + f) - 240*d*f**2*(d - e + f)**3 + 512*e
**5*f - 512*e**3*f*(d - e + f)**2 + 1152*e**2*f**3*(d - e + f) - 1472*e*f**
5 + 1280*e*f**3*(d - e + f)**2 + 960*f**5*(d - e + f) - 384*f**3*(d - e + f
)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f
 - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 12
80*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f*
*4 + 576*f**6))/6 - (d + e + f)*log(x + (-35*d**5*e + 51*d**5*(d + e + f) -
```

```
820*d**4*e*f + 180*d**4*f*(d + e + f) - 180*d**3*e**3 - 180*d**3*e**2*(d +
 e + f) - 4100*d**3*e*f**2 + 164*d**3*e*(d + e + f)**2 + 84*d**3*f**2*(d +
e + f) - 60*d**3*(d + e + f)**3 - 864*d**2*e**2*f*(d + e + f) - 8000*d**2*e
*f**3 + 960*d**2*e*f*(d + e + f)**2 - 480*d**2*f**3*(d + e + f) - 96*d**2*f
*(d + e + f)**3 + 320*d*e**5 - 192*d*e**4*(d + e + f) + 720*d*e**3*f**2 - 3
20*d*e**3*(d + e + f)**2 - 2160*d*e**2*f**2*(d + e + f) + 192*d*e**2*(d + e
 + f)**3 - 6400*d*e*f**4 + 1968*d*e*f**2*(d + e + f)**2 - 1152*d*f**4*(d +
e + f) + 240*d*f**2*(d + e + f)**3 + 512*e**5*f - 512*e**3*f*(d + e + f)**2
 - 1152*e**2*f**3*(d + e + f) - 1472*e*f**5 + 1280*e*f**3*(d + e + f)**2 -
960*f**5*(d + e + f) + 384*f**3*(d + e + f)**3)/(9*d**6 + 45*d**5*f - 160*d
**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4
- 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 +
720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/6 + (d + 2*e + 4*
f)*log(x + (-35*d**5*e - 51*d**5*(d + 2*e + 4*f)/2 - 820*d**4*e*f - 90*d**4
*f*(d + 2*e + 4*f) - 180*d**3*e**3 + 90*d**3*e**2*(d + 2*e + 4*f) - 4100*d*
*3*e*f**2 + 41*d**3*e*(d + 2*e + 4*f)**2 - 42*d**3*f**2*(d + 2*e + 4*f) + 1
5*d**3*(d + 2*e + 4*f)**3/2 + 432*d**2*e**2*f*(d + 2*e + 4*f) - 8000*d**2*e
*f**3 + 240*d**2*e*f*(d + 2*e + 4*f)**2 + 240*d**2*f**3*(d + 2*e + 4*f) + 1
2*d**2*f*(d + 2*e + 4*f)**3 + 320*d*e**5 + 96*d*e**4*(d + 2*e + 4*f) + 720*
d*e**3*f**2 - 80*d*e**3*(d + 2*e + 4*f)**2 + 1080*d*e**2*f**2*(d + 2*e + 4*
f) - 24*d*e**2*(d + 2*e + 4*f)**3 - 6400*d*e*f**4 + 492*d*e*f**2*(d + 2*e +
 4*f)**2 + 576*d*f**4*(d + 2*e + 4*f) - 30*d*f**2*(d + 2*e + 4*f)**3 + 512*
e**5*f - 128*e**3*f*(d + 2*e + 4*f)**2 + 576*e**2*f**3*(d + 2*e + 4*f) - 14
72*e*f**5 + 320*e*f**3*(d + 2*e + 4*f)**2 + 480*f**5*(d + 2*e + 4*f) - 48*f
**3*(d + 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f**2
- 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**2*e**2*f**2 -
144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f
**2 - 2560*e**2*f**4 + 576*f**6))/12
```

Giac [A] time = 1.10251, size = 80, normalized size = 1.57

$$-\frac{1}{12}\left(d+4f-2e\right)\log\left(|x+2|\right)+\frac{1}{6}\left(d+f-e\right)\log\left(|x+1|\right)-\frac{1}{6}\left(d+f+e\right)\log\left(|x-1|\right)+\frac{1}{12}\left(d+4f+2e\right)\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 2*e)*log(abs(x + 2)) + 1/6*(d + f - e)*log(abs(x + 1)) - 1
/6*(d + f + e)*log(abs(x - 1)) + 1/12*(d + 4*f + 2*e)*log(abs(x - 2))
```

3.12 $\int \frac{d+ex+fx^2+gx^3}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - ((e + g)*Log[1 - x^2
])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi [A] time = 0.0722953, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1673, 1166, 207, 1247, 632, 31}

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + 4*f)*ArcTanh[x/2])/6 + ((d + f)*ArcTanh[x])/3 - ((e + g)*Log[1 - x^2
])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{4-5x^2+x^4}\,dx &= \int \frac{d+fx^2}{4-5x^2+x^4}\,dx + \int \frac{x\left(e+gx^2\right)}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx}{4-5x+x^2}\,dx, x, x^2\right) - \frac{1}{3}(d+f)\int \frac{1}{-1+x^2}\,dx + \frac{1}{3}(d+4f)\int \frac{1}{-4+x^2}\,dx \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{6}(-e-g)\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{1}{6} \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) - \frac{1}{6}(e+g)\log\left(1-x^2\right) + \frac{1}{6}(e+4g)\log\left(4-\right.
\end{aligned}$$

Mathematica [A] time = 0.032265, size = 68, normalized size = 1.19

$$\frac{1}{12}(-2\log(1-x)(d+e+f+g) + \log(2-x)(d+2e+4f+8g) + 2\log(x+1)(d-e+f-g) - \log(x+2)(d-2e+4f-$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x] + 2*(d -
e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)*Log[2 + x])/12
```

Maple [B] time = 0.01, size = 114, normalized size = 2.

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}-\frac{\ln(2+x)\,f}{3}+\frac{2\,\ln(2+x)\,g}{3}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(1+x)\,f}{6}-\frac{\ln(1+x)\,g}{6}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f+2/3*ln(2+x)*g+1/6*ln(1+x)*d-1/6
*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/12*ln(x-2)*d+1/6*ln(x-2)*e+1/3*ln(
x-2)*f+2/3*ln(x-2)*g-1/6*ln(x-1)*d-1/6*ln(x-1)*e-1/6*ln(x-1)*f-1/6*ln(x-1)*
g
```

Maxima [A] time = 0.966077, size = 82, normalized size = 1.44

$$-\frac{1}{12}\left(d-2\,e+4\,f-8\,g\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{6}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2\,e+4\,f+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(x + 1) - 1
/6*(d + e + f + g)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Fricas [A] time = 3.66056, size = 197, normalized size = 3.46

$$-\frac{1}{12}\left(d-2\,e+4\,f-8\,g\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{6}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2\,e+4\,f+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(x + 1) - 1
/6*(d + e + f + g)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14393, size = 93, normalized size = 1.63

$$-\frac{1}{12}\left(d+4f-8g-2e\right)\log\left(|x+2|\right)+\frac{1}{6}\left(d+f-g-e\right)\log\left(|x+1|\right)-\frac{1}{6}\left(d+f+g+e\right)\log\left(|x-1|\right)+\frac{1}{12}\left(d+4f+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + 1/6*(d + f - g - e)*log(abs(x
 + 1)) - 1/6*(d + f + g + e)*log(abs(x - 1)) + 1/12*(d + 4*f + 8*g + 2*e)*l
og(abs(x - 2))
```

3.13 $\int \frac{d+ex+fx^2+gx^3+hx^4}{4-5x^2+x^4} dx$

Optimal. Leaf size=64

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)+hx$$

```
[Out] h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x])/3 - ((e
+ g)*Log[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rubi [A] time = 0.147038, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1673, 1676, 1166, 207, 1247, 632, 31}

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)+hx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x])/3 - ((e
+ g)*Log[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{4-5x^2+x^4}\,dx &= \int \frac{x\left(e+gx^2\right)}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{e+gx}{4-5x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-4h+(f+5h)x^2}{4-5x^2+x^4}\right) dx \\
&= hx + \frac{1}{6}(-e-g)\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{1}{6}(e+4g)\,\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right) \\
&= hx - \frac{1}{6}(e+g)\log\left(1-x^2\right) + \frac{1}{6}(e+4g)\log\left(4-x^2\right) - \frac{1}{3}(d+f+h)\int \frac{1}{-1+x^2}\,dx + \frac{1}{3} \\
&= hx - \frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f+h)\tanh^{-1}(x) - \frac{1}{6}(e+g)\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.045697, size = 81, normalized size = 1.27

$$\frac{1}{12}(-2\log(1-x)(d+e+f+g+h)+\log(2-x)(d+2(e+2f+4g+8h))+2\log(x+1)(d-e+f-g+h)-\log(x+2)($$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x - 2*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2*f + 4*g + 8*h))*
Log[2 - x] + 2*(d - e + f - g + h)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h
)*Log[2 + x])/12
```

Maple [B] time = 0.01, size = 145, normalized size = 2.3

$$hx-\frac{\ln(2+x)d}{12}+\frac{\ln(2+x)e}{6}-\frac{\ln(2+x)f}{3}+\frac{2\ln(2+x)g}{3}-\frac{4\ln(2+x)h}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] h*x-1/12*ln(2+x)*d+1/6*ln(2+x)*e-1/3*ln(2+x)*f+2/3*ln(2+x)*g-4/3*ln(2+x)*h+
1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/6*ln(1+x)*h+1/12*
ln(x-2)*d+1/6*ln(x-2)*e+1/3*ln(x-2)*f+2/3*ln(x-2)*g+4/3*ln(x-2)*h-1/6*ln(x-
1)*d-1/6*ln(x-1)*e-1/6*ln(x-1)*f-1/6*ln(x-1)*g-1/6*ln(x-1)*h
```

Maxima [A] time = 0.960384, size = 97, normalized size = 1.52

$$hx-\frac{1}{12}\left(d-2e+4f-8g+16h\right)\log(x+2)+\frac{1}{6}\left(d-e+f-g+h\right)\log(x+1)-\frac{1}{6}\left(d+e+f+g+h\right)\log(x-1)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/6*(d - e + f - g + h
)*log(x + 1) - 1/6*(d + e + f + g + h)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8
*g + 16*h)*log(x - 2)
```

Fricas [A] time = 12.7352, size = 234, normalized size = 3.66

$$hx - \frac{1}{12}\left(d - 2e + 4f - 8g + 16h\right)\log\left(x+2\right) + \frac{1}{6}\left(d - e + f - g + h\right)\log\left(x+1\right) - \frac{1}{6}\left(d + e + f + g + h\right)\log\left(x-1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/6*(d - e + f - g + h
)*log(x + 1) - 1/6*(d + e + f + g + h)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8
*g + 16*h)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.06955, size = 108, normalized size = 1.69

$$hx - \frac{1}{12}\left(d + 4f - 8g + 16h - 2e\right)\log\left(|x+2|\right) + \frac{1}{6}\left(d + f - g + h - e\right)\log\left(|x+1|\right) - \frac{1}{6}\left(d + f + g + h + e\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] h*x - 1/12*(d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2)) + 1/6*(d + f - g +
h - e)*log(abs(x + 1)) - 1/6*(d + f + g + h + e)*log(abs(x - 1)) + 1/12*(d
+ 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2))
```

3.14 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4} dx$

Optimal. Leaf size=76

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}\log\left(1-x^2\right)(e+g+i)+\frac{1}{6}\log\left(4-x^2\right)(e+4g+16i)+hx$$

```
[Out] h*x + (i*x^2)/2 - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[
x])/3 - ((e + g + i)*Log[1 - x^2])/6 + ((e + 4*g + 16*i)*Log[4 - x^2])/6
```

Rubi [A] time = 0.191523, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1673, 1676, 1166, 207, 1663, 1657, 632, 31}

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}\log\left(1-x^2\right)(e+g+i)+\frac{1}{6}\log\left(4-x^2\right)(e+4g+16i)+hx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x + (i*x^2)/2 - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[
x])/3 - ((e + g + i)*Log[1 - x^2])/6 + ((e + 4*g + 16*i)*Log[4 - x^2])/6
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+14x^5}{4-5x^2+x^4}\,dx = \int \frac{x\left(e+gx^2+14x^4\right)}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{4-5x^2+x^4}\,dx$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx+14x^2}{4-5x+x^2}\,dx,x,x^2\right) + \int \left(h+\frac{d-4h+(f+5h)x^2}{4-5x^2+x^4}\right)dx$$

$$= hx+\frac{1}{2}\operatorname{Subst}\left(\int \left(14-\frac{56-e-(70+g)x}{4-5x+x^2}\right)dx,x,x^2\right) + \int \frac{d-4h+(f+5h)x}{4-5x^2+x^4}$$

$$= hx+7x^2-\frac{1}{2}\operatorname{Subst}\left(\int \frac{56-e-(70+g)x}{4-5x+x^2}\,dx,x,x^2\right) - \frac{1}{3}(d+f+h)\int \frac{1}{-1+x^2}$$

$$= hx+7x^2-\frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f+h)\tanh^{-1}(x)-\frac{1}{6}(-224-$$

$$= hx+7x^2-\frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f+h)\tanh^{-1}(x)-\frac{1}{6}(14+e$$

Mathematica [A] time = 0.0651369, size = 98, normalized size = 1.29

$$\frac{1}{12}\left(-2\log(1-x)(d+e+f+g+h+i)+\log(2-x)(d+2e+4(f+2g+4h+8i))+2\log(x+1)(d-e+f-g+h-i)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x + 6*i*x^2 - 2*(d + e + f + g + h + i)*Log[1 - x] + (d + 2*e + 4*(f
+ 2*g + 4*h + 8*i))*Log[2 - x] + 2*(d - e + f - g + h - i)*Log[1 + x] - (d
- 2*(e - 2*f + 4*g - 8*h + 16*i))*Log[2 + x])/12
```

Maple [B] time = 0.01, size = 179, normalized size = 2.4

$$-\frac{\ln(2+x)\,d}{12}+\frac{\ln(2+x)\,e}{6}+\frac{\ln(1+x)\,d}{6}-\frac{\ln(1+x)\,e}{6}+\frac{\ln(x-2)\,d}{12}+\frac{\ln(x-2)\,e}{6}-\frac{\ln(x-1)\,d}{6}-\frac{\ln(x-1)\,e}{6}+\frac{8\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)*d+1/6*ln(2+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/12*ln(x-2)*d+1/
6*ln(x-2)*e-1/6*ln(x-1)*d-1/6*ln(x-1)*e+8/3*ln(x-2)*i-1/6*ln(x-1)*i+8/3*ln(
2+x)*i-1/6*ln(1+x)*i+2/3*ln(2+x)*g-1/6*ln(1+x)*g+2/3*ln(x-2)*g-1/6*ln(x-1)*
```

```
g-4/3*ln(2+x)*h+1/6*ln(1+x)*h+4/3*ln(x-2)*h-1/6*ln(x-1)*h+1/3*ln(x-2)*f-1/6
*ln(x-1)*f-1/3*ln(2+x)*f+1/6*ln(1+x)*f+1/2*i*x^2+h*x
```

Maxima [A] time = 0.953738, size = 119, normalized size = 1.57

$$\frac{1}{2}ix^2 + hx - \frac{1}{12}\left(d - 2e + 4f - 8g + 16h - 32i\right)\log\left(x+2\right) + \frac{1}{6}\left(d - e + f - g + h - i\right)\log\left(x+1\right) - \frac{1}{6}\left(d + e + f + g +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/6
*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f + g + h + i)*log(x - 1
) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Fricas [A] time = 55.1404, size = 279, normalized size = 3.67

$$\frac{1}{2}ix^2 + hx - \frac{1}{12}\left(d - 2e + 4f - 8g + 16h - 32i\right)\log\left(x+2\right) + \frac{1}{6}\left(d - e + f - g + h - i\right)\log\left(x+1\right) - \frac{1}{6}\left(d + e + f + g +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/6
*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f + g + h + i)*log(x - 1
) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10385, size = 130, normalized size = 1.71

$$\frac{1}{2}ix^2 + hx - \frac{1}{12}\left(d + 4f - 8g + 16h - 32i - 2e\right)\log\left(|x+2|\right) + \frac{1}{6}\left(d + f - g + h - i - e\right)\log\left(|x+1|\right) - \frac{1}{6}\left(d + f + g + h \cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d + 4*f - 8*g + 16*h - 32*i - 2*e)*log(abs(x + 2))
+ 1/6*(d + f - g + h - i - e)*log(abs(x + 1)) - 1/6*(d + f + g + h + i + e)
*log(abs(x - 1)) + 1/12*(d + 4*f + 8*g + 16*h + 32*i + 2*e)*log(abs(x - 2))
```

3.15 $\int \frac{d+ex}{1+x^2+x^4}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(d*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - (d*Log[1 - x + x^2]
)/4 + (d*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.0768899, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1673, 12, 1094, 634, 618, 204, 628, 1107}

$$-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4),x]
```

```
[Out] -(d*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - (d*Log[1 - x + x^2]
)/4 + (d*Log[1 + x + x^2])/4
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{d+ex}{1+x^2+x^4}\,dx = \int \frac{d}{1+x^2+x^4}\,dx + \int \frac{ex}{1+x^2+x^4}\,dx$$
$$= d\int \frac{1}{1+x^2+x^4}\,dx + e\int \frac{x}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}d\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}d\int \frac{1+x}{1+x+x^2}\,dx + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{4}d\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}d\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}d\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}d\int \frac{1+2x}{1+x+x^2}\,dx - e\,\text{Subst}$$
$$= \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}d\log\left(1-x+x^2\right) + \frac{1}{4}d\log\left(1+x+x^2\right) - \frac{1}{2}d\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x$$
$$= -\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}d\log\left(1-x+x^2\right) + \frac{1}{4}d\log\left(1+x+x^2\right)$$

Mathematica [C] time = 0.178098, size = 98, normalized size = 1.07

$$\frac{1}{6}i\left(\sqrt{6-6i\sqrt{3}}d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right) - \sqrt{6+6i\sqrt{3}}d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right) + 2i\sqrt{3}e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4),x]
```

```
[Out] (I/6)*(Sqrt[6 - (6*I)*Sqrt[3]]*d*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[6 + (6
*I)*Sqrt[3]]*d*ArcTan[((I + Sqrt[3])*x)/2] + (2*I)*Sqrt[3]*e*ArcTan[Sqrt[3]
/(1 + 2*x^2)])
```

Maple [A] time = 0.011, size = 92, normalized size = 1.

$$\frac{d\ln\left(x^2+x+1\right)}{4} + \frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \frac{d\ln\left(x^2-x+1\right)}{4} + \frac{d\sqrt{3}}{6}\arctan\left(\frac{(2x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arc
tan(1/3*(1+2*x)*3^(1/2))*e-1/4*d*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)
```

```
*3^(1/2))*d+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e
```

Maxima [A] time = 1.43112, size = 88, normalized size = 0.96

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

Fricas [A] time = 1.51995, size = 212, normalized size = 2.3

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

Sympy [C] time = 1.918, size = 923, normalized size = 10.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(-d/4 - sqrt(3)
*I*(d + 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(-d/4 - sqrt(3)*I*(d + 2*e)/
```

```
12) + 60*d**2*e*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**2 + 72*d**2*(-d/4 - sqrt(3
)*I*(d + 2*e)/12)**3 + 4*e**5 + 24*e**4*(-d/4 - sqrt(3)*I*(d + 2*e)/12) + 4
8*e**3*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**2 + 288*e**2*(-d/4 - sqrt(3)*I*(d +
 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (-d/4 + sqrt(3)*I*(d +
2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/12) - 15*d
**2*e**3 - 18*d**2*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12) + 60*d**2*e*(-d/4 +
 sqrt(3)*I*(d + 2*e)/12)**2 + 72*d**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**3 +
4*e**5 + 24*e**4*(-d/4 + sqrt(3)*I*(d + 2*e)/12) + 48*e**3*(-d/4 + sqrt(3)*
I*(d + 2*e)/12)**2 + 288*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**3)/(3*d**5 -
 8*d**3*e**2 - 16*d*e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/12)*log(x + (-7*d**
4*e + 6*d**4*(d/4 - sqrt(3)*I*(d - 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(
d/4 - sqrt(3)*I*(d - 2*e)/12) + 60*d**2*e*(d/4 - sqrt(3)*I*(d - 2*e)/12)**2
 + 72*d**2*(d/4 - sqrt(3)*I*(d - 2*e)/12)**3 + 4*e**5 + 24*e**4*(d/4 - sqrt
(3)*I*(d - 2*e)/12) + 48*e**3*(d/4 - sqrt(3)*I*(d - 2*e)/12)**2 + 288*e**2*
(d/4 - sqrt(3)*I*(d - 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (d
/4 + sqrt(3)*I*(d - 2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(d/4 + sqrt(3)*I*(
d - 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/12) +
 60*d**2*e*(d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 72*d**2*(d/4 + sqrt(3)*I*(d
- 2*e)/12)**3 + 4*e**5 + 24*e**4*(d/4 + sqrt(3)*I*(d - 2*e)/12) + 48*e**3*(
d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 288*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/12)*
*3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4))
```

Giac [A] time = 1.09963, size = 90, normalized size = 0.98

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

3.16 $\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - ((d - f
)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.0848895, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - ((d - f
)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \int \frac{ex}{1+x^2+x^4}\,dx + \int \frac{d+fx^2}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}\int \frac{d-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d+(d-f)x}{1+x+x^2}\,dx + e\int \frac{x}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-d+f)\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}(c$$
$$= -\frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+x^2\right) - e\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) + \frac{1}{2}(-$$
$$= -\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d$$

Mathematica [C] time = 0.138965, size = 121, normalized size = 1.16

$$\frac{\left(2id+\left(\sqrt{3}-i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{\left(\left(\sqrt{3}+i\right)f-2id\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{\sqrt{6-6i\sqrt{3}}} - \frac{e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)}{\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]] - (e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/Sqrt[3]
```

Maple [A] time = 0.006, size = 148, normalized size = 1.4

$$\frac{d\ln\left(x^2+x+1\right)}{4} - \frac{\ln\left(x^2+x+1\right)f}{4} + \frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) + \frac{\sqrt{3}f}{6}\arctan\left(\frac{(1+2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/
2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(1+2*x)
*3^(1/2))*f+1/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x
```

```
-1)*3^(1/2))*d+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan
(1/3*(2*x-1)*3^(1/2))*f
```

Maxima [A] time = 1.43683, size = 101, normalized size = 0.97

$$\frac{1}{6}\sqrt{3}\left(d-2e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f\right)\log\left(x^2+x+1\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

Fricas [A] time = 1.77454, size = 239, normalized size = 2.3

$$\frac{1}{6}\sqrt{3}\left(d-2e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f\right)\log\left(x^2+x+1\right)-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

Sympy [C] time = 26.5424, size = 3589, normalized size = 34.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(-d/4
 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/4
- sqrt(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4 - s
qrt(3)*I*(d + 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 - sqrt(
3)*I*(d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e +
 f)/12) + 72*d**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*d**2*e
**2*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2
*e*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*f**3*(-d/4 + f/
4 - sqrt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2
*e + f)/12)**3 + 4*d*e**5 + 24*d*e**4*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)
/12) + 15*d*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)
**2 - 54*d*e**2*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 288*d*e**2
*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*
(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/4 + f/4 - sqrt
(3)*I*(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/1
2)**3 - 8*e**5*f - 96*e**3*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 +
 36*e**2*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*
f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4 - sq
rt(3)*I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/
12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6
*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f +
 40*d*e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2*f**4 + 3*f**6)) + (-d/4
+ f/4 + sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(-d/4 + f/4
 + sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/4 + sqrt
(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4 + sqrt(3)
*I*(d + 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 + sqrt(3)*I*(
d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12
) + 72*d**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*d**2*e**2*f*
(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2*e*f*(
-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*f**3*(-d/4 + f/4 + sq
rt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f
)/12)**3 + 4*d*e**5 + 24*d*e**4*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) +
 15*d*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 -
54*d*e**2*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 288*d*e**2*(-d/4
 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*(-d/4
+ f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/4 + f/4 + sqrt(3)*I*
(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3
- 8*e**5*f - 96*e**3*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*e*
*2*f**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(
-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4 + sqrt(3)*
I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3
)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6*d**3*
f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*
e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 -
 sqrt(3)*I*(d - 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 - sqrt(
```

```
3)*I*(d - 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(d/4 - f/4 - sqrt(3)*I*(d
- 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*
e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f
)/12)**2 - 42*d**3*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 72*d**3*
(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 + 108*d**2*e**2*f*(d/4 - f/4 -
sqrt(3)*I*(d - 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2*e*f*(d/4 - f/4 - sq
rt(3)*I*(d - 2*e + f)/12)**2 - 12*d**2*f**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e
 + f)/12) - 144*d**2*f*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 + 4*d*e*
*5 + 24*d*e**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 15*d*e**3*f**2 +
48*d*e**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 - 54*d*e**2*f**2*(d/4
 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 288*d*e**2*(d/4 - f/4 - sqrt(3)*I*(d
 - 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*(d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)**2 + 36*d*f**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) - 72*
d*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*(
d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 + 36*e**2*f**3*(d/4 - f/4 - sqrt
(3)*I*(d - 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)**2 - 6*f**5*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 144*f*
*3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4
*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2
*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**
4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)*
log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 25*d
**4*e*f + 18*d**4*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) - 15*d**3*e**3
 - 18*d**3*e**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) - 25*d**3*e*f**2 +
 60*d**3*e*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 - 42*d**3*f**2*(d/4
- f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 72*d**3*(d/4 - f/4 + sqrt(3)*I*(d - 2
*e + f)/12)**3 + 108*d**2*e**2*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) +
 20*d**2*e*f**3 - 144*d**2*e*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2
- 12*d**2*f**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) - 144*d**2*f*(d/4 -
 f/4 + sqrt(3)*I*(d - 2*e + f)/12)**3 + 4*d*e**5 + 24*d*e**4*(d/4 - f/4 + s
qrt(3)*I*(d - 2*e + f)/12) + 15*d*e**3*f**2 + 48*d*e**3*(d/4 - f/4 + sqrt(3
)*I*(d - 2*e + f)/12)**2 - 54*d*e**2*f**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e +
 f)/12) + 288*d*e**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**3 - 20*d*e*f
**4 + 180*d*e*f**2*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 + 36*d*f**4*
(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) - 72*d*f**2*(d/4 - f/4 + sqrt(3)*I
*(d - 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*(d/4 - f/4 + sqrt(3)*I*(d - 2*
e + f)/12)**2 + 36*e**2*f**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 11*
e*f**5 - 48*e*f**3*(d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)**2 - 6*f**5*(d/
4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12) + 144*f**3*(d/4 - f/4 + sqrt(3)*I*(d
- 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3
*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4 + 16
*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2*f**4 + 3*f**6
))
```

Giac [A] time = 1.09702, size = 104, normalized size = 1.

$$\frac{1}{6}\sqrt{3}\left(d+f-2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+f+2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f\right)\log\left(x^2+x+1\right)-\frac{1}{4}\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
f + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

3.17 $\int \frac{d+ex+fx^2+gx^3}{1+x^2+x^4} dx$

Optimal. Leaf size=127

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}}{2\sqrt{3}}$$

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3
]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4 + (g*Log[1
 + x^2 + x^4])/4
```

Rubi [A] time = 0.101051, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1673, 1169, 634, 618, 204, 628, 1247}

$$-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] -((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3
]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4 + (g*Log[1
 + x^2 + x^4])/4
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
```

```
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{1+x^2+x^4}\,dx = \int \frac{d+fx^2}{1+x^2+x^4}\,dx + \int \frac{x\left(e+gx^2\right)}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}\int \frac{d-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d+(d-f)x}{1+x+x^2}\,dx + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{1+x+x^2}\,dx,x,x^2\right)$$
$$= \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-d+f)\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}(d+f)\int \frac{1}{1-x+x^2}\,dx + \frac{1}{4}(d$$
$$= -\frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+x^2\right) + \frac{1}{4}g\log\left(1+x^2+x^4\right) + \frac{1}{2}(-d-f)$$
$$= -\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}(d-f)\log\left(1-\right.$$

Mathematica [C] time = 0.484633, size = 150, normalized size = 1.18

$$\frac{2\left(\sqrt{2+2i\sqrt{3}}\left(\left(\sqrt{3}+i\right)f-2id\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)+(2g-4e)\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)+\sqrt{3}g\log\left(x^4+x^2+1\right)\right)+2\sqrt{2-2i\sqrt{}}}{8\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2 - (2*I)*Sqrt[3]]*((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[
3])*x)/2] + 2*(Sqrt[2 + (2*I)*Sqrt[3]]*((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[
((I + Sqrt[3])*x)/2] + (-4*e + 2*g)*ArcTan[Sqrt[3]/(1 + 2*x^2)] + Sqrt[3]*g
*Log[1 + x^2 + x^4]))/(8*Sqrt[3])
```

Maple [A] time = 0.004, size = 204, normalized size = 1.6

$$\frac{d\ln\left(x^2+x+1\right)}{4} - \frac{\ln\left(x^2+x+1\right)f}{4} + \frac{\ln\left(x^2+x+1\right)g}{4} + \frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)*g+1/6*d*arctan(1/3*(1+2
*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/6*3^(1/2)*
```

```
arctan(1/3*(1+2*x)*3^(1/2))*f+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/4
*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+1)+1/4*ln(x^2-x+1)*g+1/6*3^(1/2)*arctan(1/3*(
2*x-1)*3^(1/2))*d+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))*f-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g
```

Maxima [A] time = 1.47307, size = 112, normalized size = 0.88

$$\frac{1}{6}\sqrt{3}\left(d-2e+f+g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f-g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f+g\right)\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + 2*e + f - g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Fricas [A] time = 2.98584, size = 261, normalized size = 2.06

$$\frac{1}{6}\sqrt{3}\left(d-2e+f+g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f-g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f+g\right)\log\left(x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + 2*e + f - g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11059, size = 115, normalized size = 0.91

$$\frac{1}{6}\sqrt{3}\left(d+f+g-2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+f-g+2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\left(d-f+g\right)\log\left(x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + f - g + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

3.18 $\int \frac{d+ex+fx^2+gx^3+hx^4}{1+x^2+x^4} \, dx$

Optimal. Leaf size=136

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{(2e-g}{}$$

```
[Out] h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f - 2*h
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sq
rt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x
^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rubi [A] time = 0.139987, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1673, 1676, 1169, 634, 618, 204, 628, 1247}

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{(2e-g}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```

```
[Out] h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f - 2*h
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sq
rt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x
^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{1+x^2+x^4}\,dx = \int \frac{x\left(e+gx^2\right)}{1+x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx}{1+x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-h+(f-h)x^2}{1+x^2+x^4}\right) dx$$
$$= hx + \frac{1}{4}(2e-g)\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}g\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) +$$
$$= hx + \frac{1}{4}g\log\left(1+x^2+x^4\right) + \frac{1}{2}\int \frac{d-h-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d-h+(d-f)x}{1+x+x^2}\,dx + \frac{1}{2}($$
$$= hx + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}g\log\left(1+x^2+x^4\right) + \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}($$
$$= hx + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+x^2\right) +$$
$$= hx - \frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f-2h)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [C] time = 0.601022, size = 165, normalized size = 1.21

$$\frac{1}{24}\left(4\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}+3i\right)d+\left(\sqrt{3}-3i\right)f-2\sqrt{3}h\right)+4\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(\left(\sqrt{3}-3i\right)d+\left(\sqrt{3}+3i\right)f-2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```

```
[Out] (24*h*x + 4*((3*I + Sqrt[3])*d + (-3*I + Sqrt[3])*f - 2*Sqrt[3]*h)*ArcTan[(
(-I + Sqrt[3])*x)/2] + 4*((-3*I + Sqrt[3])*d + (3*I + Sqrt[3])*f - 2*Sqrt[3
]*h)*ArcTan[((I + Sqrt[3])*x)/2] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)]
+ 4*Sqrt[3]*g*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 6*g*Log[1 + x^2 + x^4])/24
```

Maple [B] time = 0.004, size = 241, normalized size = 1.8

$$hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\sqrt{3}e}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] h*x+1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)*g+1/6*d*arctan(1/3*
(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/6*3^(1
/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g
-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h+1/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x
+1)+1/4*ln(x^2-x+1)*g+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+1/3*3^(1/2)
*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/
6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))*h
```

Maxima [A] time = 1.48999, size = 124, normalized size = 0.91

$$\frac{1}{6}\sqrt{3}\left(d-2e+f+g-2h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f-g-2h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g - 2*h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + 2*e + f - g - 2*h)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Fricas [A] time = 11.1041, size = 285, normalized size = 2.1

$$\frac{1}{6}\sqrt{3}\left(d-2e+f+g-2h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f-g-2h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g - 2*h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + 2*e + f - g - 2*h)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10218, size = 127, normalized size = 0.93

$$\frac{1}{6}\sqrt{3}\left(d+f+g-2h-2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\left(d+f-g-2h+2e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}\left(d-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + f - g - 2*h + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

3.19 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4} dx$

Optimal. Leaf size=151

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{\tan^{-1}}{}$$

```
[Out] h*x + (i*x^2)/2 - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (
(d + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g - i)*ArcTa
n[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d -
f)*Log[1 + x + x^2])/4 + ((g - i)*Log[1 + x^2 + x^4])/4
```

Rubi [A] time = 0.176035, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1676, 1169, 634, 618, 204, 628, 1663, 1657}

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```


```
[Out] h*x + (i*x^2)/2 - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (
(d + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g - i)*ArcTa
n[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d -
f)*Log[1 + x + x^2])/4 + ((g - i)*Log[1 + x^2 + x^4])/4
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+19x^5}{1+x^2+x^4}\,dx = \int \frac{x\left(e+gx^2+19x^4\right)}{1+x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+19x^2}{1+x+x^2}\,dx, x, x^2\right) + \int \left(h+\frac{d-h+(f-h)x^2}{1+x^2+x^4}\right)dx$$
$$= hx + \frac{1}{2}\text{Subst}\left(\int \left(19-\frac{19-e+(19-g)x}{1+x+x^2}\right)dx, x, x^2\right) + \int \frac{d-h+(f-h)x^2}{1+x^2+x^4}$$
$$= hx + \frac{19x^2}{2} + \frac{1}{2}\int \frac{d-h-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d-h+(d-f)x}{1+x+x^2}\,dx - \frac{1}{2}\text{Subst}$$
$$= hx + \frac{19x^2}{2} + \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-d+f)\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}(19$$
$$= hx + \frac{19x^2}{2} - \frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+x^2\right) - \frac{1}{4}(19-g$$
$$= hx + \frac{19x^2}{2} - \frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f-2h)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{(19-2}{}$$

Mathematica [C] time = 0.64167, size = 187, normalized size = 1.24

$$\frac{1}{12}\left(\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(2\sqrt{3}d-\left(\sqrt{3}+3i\right)f-\left(\sqrt{3}-3i\right)h\right)+\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\left(-2i\sqrt{3}d+\left(3\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

```
[Out] (6*x*(2*h + i*x) + (1 + I*Sqrt[3])*(2*Sqrt[3]*d - (3*I + Sqrt[3])*f - (-3*I
 + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2] + (I + Sqrt[3])*((-2*I)*Sqrt[3]
*d + (3 + I*Sqrt[3])*f + I*(3*I + Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2] -
 2*Sqrt[3]*(2*e - g - i)*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 3*(g - i)*Log[1 + x^
2 + x^4])/12
```

Maple [B] time = 0.007, size = 303, normalized size = 2.

$$\frac{ix^2}{2}+hx+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{4}+\frac{\ln\left(x^2+x+1\right)g}{4}-\frac{\ln\left(x^2+x+1\right)i}{4}+\frac{d\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x)
```

```
[Out] 1/2*i*x^2+h*x+1/4*d*ln(x^2+x+1)-1/4*ln(x^2+x+1)*f+1/4*ln(x^2+x+1)*g-1/4*ln(
x^2+x+1)*i+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*3^(1/2)*arctan(1/3
*(1+2*x)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/6*3^(1/2)*a
rctan(1/3*(1+2*x)*3^(1/2))*g-1/3*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h+1/6*
3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*i+1/4*ln(x^2-x+1)*g-1/4*ln(x^2-x+1)*i+1
/4*ln(x^2-x+1)*f-1/4*d*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*
d+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/6*3^(1/2)*arctan(1/3*(2*x-1)*
3^(1/2))*f-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g-1/3*3^(1/2)*arctan(1/3
*(2*x-1)*3^(1/2))*h-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*i
```

Maxima [A] time = 1.46354, size = 143, normalized size = 0.95

$$\frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}\left(d-2\,e+f+g-2\,h+i\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\left(d+2\,e+f-g-2\,h-i\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*log(
x^2 - x + 1)
```

Fricas [A] time = 39.4204, size = 323, normalized size = 2.14

$$\frac{1}{2}\,ix^2+\frac{1}{6}\,\sqrt{3}\left(d-2\,e+f+g-2\,h+i\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\left(d+2\,e+f-g-2\,h-i\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*log(
x^2 - x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10442, size = 146, normalized size = 0.97

$$\frac{1}{2}\,ix^2 + \frac{1}{6}\,\sqrt{3}\left(d + f + g - 2\,h + i - 2\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x + 1)\right) + \frac{1}{6}\,\sqrt{3}\left(d + f - g - 2\,h - i + 2\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x - 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d + f + g - 2*h + i - 2*e)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/6*sqrt(3)*(d + f - g - 2*h - i + 2*e)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*log(
x^2 - x + 1)
```

3.20 $\int \frac{d+ex}{a+bx^2+cx^4} dx$

Optimal. Leaf size=189

$$\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*d*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqr
t[b^2 - 4*a*c]
```

Rubi [A] time = 0.210849, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1673, 12, 1093, 205, 1107, 618, 206}

$$\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*d*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqr
t[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \int \frac{d}{a+bx^2+cx^4}\,dx + \int \frac{ex}{a+bx^2+cx^4}\,dx$$
$$= d\int \frac{1}{a+bx^2+cx^4}\,dx + e\int \frac{x}{a+bx^2+cx^4}\,dx$$
$$= \frac{(cd)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(cd)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - e\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c\right.$$
$$= \frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.272544, size = 194, normalized size = 1.03

$$\frac{\frac{2\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}} + e\left(\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] - (2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]] + e*(Log[-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]))/(
2*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.089, size = 231, normalized size = 1.2

$$-\frac{e}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)+2\,\frac{c\sqrt{-4\,ac+b^2}d\sqrt{2}}{\left(8\,ac-2\,b^2\right)\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\text{Artanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4\,ac+b^2}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+2*c*(
-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))+(-4*a*c+b^2)^(1/2)/(8
*a*c-2*b^2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c*(-4*a*c+b^2)^(1/2)/(8*a*
c-2*b^2)*d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex+d}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [B] time = 93.8135, size = 471, normalized size = 2.49

$$\mathrm{RootSum}\Big(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(32a^2ce^2-8ab^2e^2-16abcd^2+4b^3d^2\right)+t\left(-16acd^2e+4b^2d^2e\right)+ae^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(32*a**
2*c*e**2 - 8*a*b**2*e**2 - 16*a*b*c*d**2 + 4*b**3*d**2) + _t*(-16*a*c*d**2*
e + 4*b**2*d**2*e) + a*e**4 + b*d**2*e**2 + c*d**4, Lambda(_t, _t*log(x + (
-512*_t**3*a**4*c**2*e**2 + 256*_t**3*a**3*b**2*c*e**2 - 128*_t**3*a**3*b*c
**2*d**2 - 32*_t**3*a**2*b**4*e**2 + 64*_t**3*a**2*b**3*c*d**2 - 8*_t**3*a*
b**5*d**2 - 64*_t**2*a**3*b*c*e**3 - 64*_t**2*a**3*c**2*d**2*e + 16*_t**2*a
**2*b**3*e**3 + 4*_t**2*a*b**4*d**2*e - 32*_t*a**3*c*e**4 + 8*_t*a**2*b**2*
e**4 + 24*_t*a**2*b*c*d**2*e**2 - 16*_t*a**2*c**2*d**4 - 6*_t*a*b**3*d**2*e
**2 + 12*_t*a*b**2*c*d**4 - 2*_t*b**4*d**4 - 4*a**2*b*e**5 + 20*a**2*c*d**2
*e**3 - 5*a*b**2*d**2*e**3 + 8*a*b*c*d**4*e - b**3*d**4*e)/(16*a**2*c*d*e**
4 + 8*a*b*c*d**3*e**2 - 4*a*c**2*d**5 + b**2*c*d**5))))
```

Giac [C] time = 1.97199, size = 3996, normalized size = 21.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*(sqrt(a*c)*b^2*c + 4*sqrt(a*c)*a*c^2 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b
*c)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 4*(sqrt(a*c)*b^2*c + 4*sqrt(a*c
)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(sqrt(a*c)*
b^2*c - 4*sqrt(a*c)*a*c^2 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c - 4*(a*c^3)^(1/4)*a*c^2 + (a
*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) - ((a*c^3)^(1/4)*b^2*c - 4*(a*c^3)^(1/4)*a*c^2 + (a*c^3)^(1/4)*sqrt(
b^2 - 4*a*c)*b*c)*d*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-(
(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c
)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^2 -
 4*a^2*c^3) + 1/2*(2*(sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*
```

```
c)*sqrt(a*c)*b*c)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 4*(sqrt(a*c)*b^2*
c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 2*(sqrt(a*c)*b^2*c + 4*sqrt(a*c)*a*c^2 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*
cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c - 4*(a*c^3)^(1
/4)*a*c^2 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*cosh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c - 4*(a*c^3)^(1/4)*a*c^2 + (a*c^
3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))))*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))) - x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
)/(a*b^2*c^2 - 4*a^2*c^3) + 1/4*((sqrt(a*c)*b^2*c + 4*sqrt(a*c)*a*c^2 + sqr
t(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)^2*e + (sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*
b*c)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*(sqrt(a*c)*b^2*
c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) - 2*(sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sq
rt(a*c)*b*c)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (sqrt(a*c)*b^2*c - 4*sqrt(a*c)
*a*c^2 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(5/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c + 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c
)*sqrt(a*c)*b*c)*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c
^3)^(1/4)*b^2*c - 4*(a*c^3)^(1/4)*a*c^2 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b
*c)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(1/4)*b^2*c -
 4*(a*c^3)^(1/4)*a*c^2 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*cos(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^2 - 4*a^2*c^3
) + 1/4*((sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)
*b*c)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cos
```

```
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e + (sqrt(a*c)*b^2*c
 - 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + 2*(sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 -
sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*
c)*b^2*c + 4*sqrt(a*c)*a*c^2 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c))))) + (sqrt(a*c)*b^2*c - 4*sqrt(a*c)*a*c^2 + sqrt(b^2 - 4*a*
c)*sqrt(a*c)*b*c)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + (sqr
t(a*c)*b^2*c + 4*sqrt(a*c)*a*c^2 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c)*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(1/4)*b^2*c - 4*(a*c
^3)^(1/4)*a*c^2 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c))))) + ((a*c^3)^(1/4)*b^2*c - 4*(a*c^3)^(1/4)*a*c^2 +
 (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c)*d*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^2 - 4*a^2*c^3)
```

3.21 $\int \frac{d+ex+fx^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.239978, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \int \frac{ex}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2}{a+bx^2+cx^4}\,dx$$
$$= e\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{a+bx+}\right.$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - e\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-}\right.$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.226158, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} + e\log\left(\sqrt{b^2-4ac}-b-2cx^2\right) - e\log\left(\sqrt{b^2-\right.}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [B] time = 0.024, size = 616, normalized size = 2.9

$$-\frac{e}{8\,ac-2\,b^2}\sqrt{-4\,ac+b^2}\ln\left(-2\,cx^2+\sqrt{-4\,ac+b^2}-b\right)-2\,\frac{c\sqrt{2}fa}{\left(4\,ac-b^2\right)\sqrt{\left(\sqrt{-4\,ac+b^2}-b\right)c}}\mathrm{Artanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4\,ac+b^2}-\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c
/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*a+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*
b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c*(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*
a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+ex+d}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 2.92854, size = 9104, normalized size = 43.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
```

```
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(
a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) - 4*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*s
qrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3
 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2
- 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
in(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(
1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^
2)*d*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4
*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1
/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/
```

```
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4
)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a
*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sq
rt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3
+ sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*
pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) - 1/
4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)
*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
```

```
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a
*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*
f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)
^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqr
t(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)
*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2*e - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4
*a*c)*sqrt(a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c
^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^
3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (sqr
t(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 + (sqrt(a*c)*b^2*c^2 + 4*sq
rt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*e*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3
 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1
```

```
/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
)*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
 + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a
*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3
)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^3 - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*
sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e - (sqrt
(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(sqrt(a*c)*b^2*c^2 - 4*s
qrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) - 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(
a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c
```

```
)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 -
4*a*c)*sqrt(a*c)*b*c^2)*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c^3 - 4*a^2*c^4)
```

3.22 $\int \frac{d+ex+fx^2+gx^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=245

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+\right.}{4c}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*
g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*L
og[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.159176, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1673, 1166, 205, 1247, 634, 618, 206, 628}

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*
g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*L
og[a + b*x^2 + c*x^4])/(4*c)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{a+bx^2+cx^4}\,dx = \int \frac{d+fx^2}{a+bx^2+cx^4}\,dx + \int \frac{x\left(e+gx^2\right)}{a+bx^2+cx^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{a+bx+cx^2}\,dx, x, x^2\right) + \frac{1}{2}\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{g\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+c}\right.}{4c}$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{g\log\left(a+bx^2+c\right.}{4c}$$
$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{(2ce-bg)\tanh^{-1}}{2c\sqrt{b^2-4a}}$$

Mathematica [A] time = 0.31004, size = 280, normalized size = 1.14

$$\frac{\frac{2\sqrt{2}\sqrt{c}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} + \frac{2\sqrt{2}\sqrt{c}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}} + \left(g\left(\sqrt{b^2-4ac}-b\right)+2ce\right)\log\left(\sqrt{b^2-\right.}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + (2*Sqr
t[2]*Sqrt[c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]] + (2*c*e + (-b
+ Sqrt[b^2 - 4*a*c])*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] + (-2*c*e + (
b + Sqrt[b^2 - 4*a*c])*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[b
```

```
^2 - 4*a*c])
```

Maple [B] time = 0.019, size = 866, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g*a-1/4/(4*a*c-b^2)/c*ln(-2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g*b^2+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*a+1/2/
(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
d+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g*a-1/4/(4*a*c-b^2)/c*ln(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g*b^2-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(
2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*g+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.23 $\int \frac{d+ex+fx^2+gx^3+hx^4}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=290

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tan}{2c\sqrt{b^2}}$$

```
[Out] (h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h -
2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a
+ b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.725255, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1673, 1676, 1166, 205, 1247, 634, 618, 206, 628}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tan}{2c\sqrt{b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h -
2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a
+ b*x^2 + c*x^4])/(4*c)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{a+bx^2+cx^4}\,dx &= \int \frac{x\left(e+gx^2\right)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+hx^4}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx}{a+bx+cx^2}\,dx, x, x^2\right) + \int \left(\frac{h}{c} + \frac{cd-ah+(cf-bh)x^2}{c\left(a+bx^2+cx^4\right)}\right) dx \\
&= \frac{hx}{c} + \frac{\int \frac{cd-ah+(cf-bh)x^2}{a+bx^2+cx^4}\,dx}{c} + \frac{g\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{(2ce-bg)\operatorname{Subst}\left(\int \frac{1}{a+bx+\cdots}\right.}{4c} \\
&= \frac{hx}{c} + \frac{g\log\left(a+bx^2+cx^4\right)}{4c} - \frac{(2ce-bg)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c} + \frac{\left(cf-bh\cdots\right.}{\cdots} \\
&= \frac{hx}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(cf-bh-\frac{2c^2d-bcf+b^2h-2ach}{\sqrt{b^2-4ac}}\right)\cdots}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b\cdots}}}
\end{aligned}$$

Mathematica [A] time = 0.558978, size = 383, normalized size = 1.32

$$\frac{\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(f\sqrt{b^2-4ac}-2ah-bf\right)+bh\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(f\sqrt{b^2-4ac}+2ah+bf\right)+bh\left(\sqrt{b^2-4ac}+b\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \sqrt{c}(\cdots}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*h*x + (2*Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*h + c*(-(b
*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
```

```
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt
[2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b*f + Sqrt[b^2 - 4*a*c]*f +
 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(2*c*e + (-b + Sqrt[b^2 -
 4*a*c])*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (Sqr
t[c]*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [B] time = 0.03, size = 1132, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] h*x/c-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g+1/
4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/
2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-1/2*(-
4*a*c+b^2)/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c
*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h+1/2*(-4*a*c+b^2)/(4*a*c-b^
2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*f-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*a*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*h-1/2*(-4
*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctan
h(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4
*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(2*c*x^2+(
-4*a*c+b^2)^(1/2)+b)*g-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*
a*c+b^2)^(1/2)+b)*b*g+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)+1/2*(-4*a*c+b^2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h-1/2*
(-4*a*c+b^2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^2*h-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4
*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
```

```
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.24 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=321

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\tanh^{-1}\left(\frac{b+2c\cdots}{\sqrt{b^2-4\cdots}}\right.\cdots}{\cdots}$$

```
[Out] (h*x)/c + (i*x^2)/(2*c) + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d -
b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
(2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 0.533943, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1676, 1166, 205, 1663, 1657, 634, 618, 206, 628}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\tanh^{-1}\left(\frac{b+2c\cdots}{\sqrt{b^2-4\cdots}}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + (i*x^2)/(2*c) + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d -
b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
(2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4+24x^5}{a+bx^2+cx^4}\,dx &= \int \frac{x\left(e+gx^2+24x^4\right)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+hx^4}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx+24x^2}{a+bx+cx^2}\,dx, x, x^2\right) + \int \left(\frac{h}{c} + \frac{cd-ah+(cf-bh)x^2}{c\left(a+bx^2+cx^4\right)}\right) dx \\
&= \frac{hx}{c} + \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{24}{c} - \frac{24a-ce+(24b-cg)x}{c\left(a+bx+cx^2\right)}\right) dx, x, x^2\right) + \frac{\int \frac{cd-ah+(cf-bh)x^2}{a+bx^2+cx^4}}{c} \\
&= \frac{hx}{c} + \frac{12x^2}{c} - \frac{\operatorname{Subst}\left(\int \frac{24a-ce+(24b-cg)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} + \frac{\left(cf-bh-\frac{2c^2d-bcf+b^2h-2ach}{\sqrt{b^2-4ac}}\right.}{2c} \\
&= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(cf-bh-\frac{2c}{}\right.}{} \\
&= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(cf-bh-\frac{2c}{}\right.}{} \\
&= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(cf-bh-\frac{2c}{}\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.797854, size = 441, normalized size = 1.37

$$\frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(f\sqrt{b^2-4ac}-2ah-bf\right)+bh\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(f\sqrt{b^2-4ac}+2ah+bf\right)+bh\left(\sqrt{b^2-4ac}+b\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*c*h*x + 2*c*i*x^2 + (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*
c])*h + c*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b*f +
 Sqrt[b^2 - 4*a*c]*f + 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^2*e +
b*(b - Sqrt[b^2 - 4*a*c])*i + c*(-(b*g) + Sqrt[b^2 - 4*a*c]*g - 2*a*i))*Log
[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((2*c^2*e + b*(b +
Sqrt[b^2 - 4*a*c])*i - c*(b*g + Sqrt[b^2 - 4*a*c]*g + 2*a*i))*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

Maple [B] time = 0.029, size = 1435, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*i*x^2/c+h*x/c+1/2*(-4*a*c+b^2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+1/2
*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*h-1/2*(-4*a*c+b^2)
/(4*a*c-b^2)/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*b^2*h-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d-1/2*(-4*a
```

```
*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*d-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)*g-1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^
(1/2)+b)*g+1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*b*i-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
*a*i+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*b^2*i+1/4*(-4*a*c+b^2)/(4*a*c-b^2)/c^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*
b*i+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*
a*i-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b
)*b^2*i+1/2*(-4*a*c+b^2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f-1/2*(-4*a*c+b^2)
/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f+1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-
2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/4*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2
*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*g-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^
2+(-4*a*c+b^2)^(1/2)+b)-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
a*h-(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*h
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ix^2 + 2\,hx}{2\,c} - \frac{-\int \frac{\left(cg-bi\right)x^3+\left(cf-bh\right)x^2+cd-ah+(ce-ai)x}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="max
ima")
```

```
[Out] 1/2*(i*x^2 + 2*h*x)/c - integrate(-((c*g - b*i)*x^3 + (c*f - b*h)*x^2 + c*d
 - a*h + (c*e - a*i)*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="gia
c")
```

```
[Out] Exception raised: NotImplementedError
```

3.25 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=545

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\tan$$

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*k
 - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((c^3*f - c^2*(b*h +
 a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m -
b^2*c*(b*k + 4*a*m) + c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k +
 2*a*m) - (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*
(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^2*l -
c*(b*j + a*l))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 4.21328, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1673, 1676, 1166, 205, 1663, 1657, 634, 618, 206, 628}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2\left(2a^2m+3abk+b^2h\right)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\tan$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*k
 - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((c^3*f - c^2*(b*h +
 a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m -
b^2*c*(b*k + 4*a*m) + c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sq
```

```
rt[b - Sqrt[b^2 - 4*a*c]]) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k +
 2*a*m) - (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*
(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^2*l -
c*(b*j + a*l))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \int \frac{x\left(e+gx^2+jx^4+lx^6\right)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+hx^4+kx^6+mx}{a+bx^2+cx^4}$$

$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e+gx+jx^2+lx^3}{a+bx+cx^2}\,dx, x, x^2\right) + \int \Bigg(\frac{c^2h+b^2m-}{c}$$

$$= \frac{\left(c^2h+b^2m-c(bk+am)\right)x}{c^3} + \frac{(ck-bm)x^3}{3c^2} + \frac{mx^5}{5c} + \frac{1}{2}\,\mathrm{Sub}$$

$$= \frac{\left(c^2h+b^2m-c(bk+am)\right)x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \frac{(ck-bm)x^3}{3c^2} + \frac{l}{}$$

$$= \frac{\left(c^2h+b^2m-c(bk+am)\right)x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \frac{(ck-bm)x^3}{3c^2} + \frac{l}{}$$

$$= \frac{\left(c^2h+b^2m-c(bk+am)\right)x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \frac{(ck-bm)x^3}{3c^2} + \frac{l}{}$$

$$= \frac{\left(c^2h+b^2m-c(bk+am)\right)x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \frac{(ck-bm)x^3}{3c^2} + \frac{l}{}$$

Mathematica [A] time = 1.61428, size = 816, normalized size = 1.5

$$\frac{mx^5}{5c} + \frac{lx^4}{4c} + \frac{(ck-bm)x^3}{3c^2} + \frac{(cj-bl)x^2}{2c^2} + \frac{\left(mb^2+c^2h-c(bk+am)\right)x}{c^3} + \frac{\left(2dc^4+\left(-bf+\sqrt{b^2-4ac}f-2ah\right)c^3+\left(2ma\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*k
 - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((2*c^4*d + c^3*(-(b
*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h) + b^3*(b - Sqrt[b^2 - 4*a*c])*m + c^2*(b
^2*h - b*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k - a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m) +
```

```
 b*c*(-(b^2*k) + b*Sqrt[b^2 - 4*a*c]*k - 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m)
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)
*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d - c^3*(b*f + Sq
rt[b^2 - 4*a*c]*f + 2*a*h) + b^3*(b + Sqrt[b^2 - 4*a*c])*m + c^2*(b^2*h + b
*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k + a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m) - b*c*(b^
2*k + b*Sqrt[b^2 - 4*a*c]*k + 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 -
 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^3*e + c^2*(-(b*g) + Sqrt[b^2 -
 4*a*c]*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + c*(b^2*j - b*Sqrt[b^2
 - 4*a*c]*j + 3*a*b*l - a*Sqrt[b^2 - 4*a*c]*l))*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c]) + ((-2*c^3*e + c^2*(b*g + Sqrt[b^2 -
4*a*c]*g + 2*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c])*l - c*(b^2*j + b*Sqrt[b^2 -
 4*a*c]*j + 3*a*b*l + a*Sqrt[b^2 - 4*a*c]*l))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.046, size = 3835, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*l*x^4/c+1/5*m*x^5/c+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*b^2-1/2/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*f*b^2+1/c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a^2*m-1/2/c^2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^3*k+1/2/c^3*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*m+1/c*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a^2*m-1/2/c^2*(-4*a*c+b^2
)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*k-3/c^2/(4*a*c-b^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^3*m*a+5/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*k*b^2+4/c/(4*a*c-b^2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a^2*b*m+3/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*m*a-
```

```
5/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*k*b^2-4/c/(4*a*c-b^2)*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2))*a^2*b*m+1/2/c^3*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
^4*m+3/2/c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*b*k-2/c^2*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*m+3/2/c*(-4*a*c+b^2)
^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*k-2/c^2*(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2))*a*b^2*m+h*x/c+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/
c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^2*h+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2))*b^2*h-2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arc
tanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*a-1/2*(-4*a*c+b^2)^(1/
2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))*d+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*a-1/2*(-4*a*c+b^2)^
(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*d+1/3/c*x^3*k+1/2/c*x^2*j+1/2/c*(-4*a*c+b^2)^(1/2)/(4*a*c
-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*j+1/4/c^3*(-4*a*c+b^2)^(1/2)/(4*a
*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^3*l-1/4/c^2*(-4*a*c+b^2)^(1/2)/
(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*j-1/2/c*(-4*a*c+b^2)^(1/2
)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*j-1/4/c^3*(-4*a*c+b^2)^(1/
2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3*l+1/4/c^2*(-4*a*c+b^2)^
(1/2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*j+5/4/c^2/(4*a*c-b^2
)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*l*a+5/4/c^2/(4*a*c-b^2)*ln(-2*c*x^2+
(-4*a*c+b^2)^(1/2)-b)*b^2*l*a-1/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2
)-b)*b*j*a-1/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*j*a+2/(4*a*c-
b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2))*a^2*k-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*k+1/4*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b*g-1/4*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b*g+1/(4*a
*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g*a+1/(4*a*c-b^2)*ln(2*c*x^2+(-4*
a*c+b^2)^(1/2)+b)*g*a-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
```

```
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h*a+2/(4*a*c-b^2)*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2))*b*h*a-1/2/c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*h-3/4/c^2*(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*a*b*l+3/4/c^2*(-
4*a*c+b^2)^(1/2)/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a*b*l-1/2/c^2
/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*k+1/2/c^2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*b^4*k+1/2/c^3/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*m+1/2/c/(4*a*c-b^2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b^3*h-1/2/c^3/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^5*m-(-4*a*c
+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*a*h-(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*a*h-1/3/c^2*x^3*b*m-1/2/c^2*x^2*b*l-1/c^2*a*m*x+1/c^
3*b^2*m*x-1/c^2*b*k*x+1/4/c^2/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)
*b^3*j+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^3*j-1/4/c^3/(
4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^4*l-1/4/c^3/(4*a*c-b^2)*ln(-2
*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^4*l-1/c/(4*a*c-b^2)*ln(-2*c*x^2+(-4*a*c+b^2)
^(1/2)-b)*a^2*l-1/c/(4*a*c-b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*a^2*l-1/4/
(4*a*c-b^2)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*g*b^2-1/4/(4*a*c-b^2)/c*ln(-
2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*g*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{12\,c^2mx^5+15\,c^2lx^4+20\left(c^2k-bcm\right)x^3+30\left(c^2j-bcl\right)x^2+60\left(c^2h-bck+\left(b^2-ac\right)m\right)x}{60\,c^3}-\frac{-\int\frac{c^3d-ac^2h+abck+\left(c^3g-bc^2j+\left(b^2c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="maxima")
```

```
[Out] 1/60*(12*c^2*m*x^5 + 15*c^2*l*x^4 + 20*(c^2*k - b*c*m)*x^3 + 30*(c^2*j - b*
c*l)*x^2 + 60*(c^2*h - b*c*k + (b^2 - a*c)*m)*x)/c^3 - integrate(-(c^3*d -
a*c^2*h + a*b*c*k + (c^3*g - b*c^2*j + (b^2*c - a*c^2)*l)*x^3 + (c^3*f - b*
c^2*h + (b^2*c - a*c^2)*k - (b^3 - 2*a*b*c)*m)*x^2 - (a*b^2 - a^2*c)*m + (c
^3*e - a*c^2*j + a*b*c*l)*x)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.26 $\int \frac{d+ex}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=94

$$\frac{dx(17-5x^2)}{72(x^4-5x^2+4)} + \frac{19}{432} d \tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54} d \tanh^{-1}(x) + \frac{e(5-2x^2)}{18(x^4-5x^2+4)} + \frac{1}{27} e \log\left(1-x^2\right) - \frac{1}{27} e \log\left(4-x^2\right)$$

```
[Out] (d*x*(17 - 5*x^2))/(72*(4 - 5*x^2 + x^4)) + (e*(5 - 2*x^2))/(18*(4 - 5*x^2
+ x^4)) + (19*d*ArcTanh[x/2])/432 - (d*ArcTanh[x])/54 + (e*Log[1 - x^2])/27
 - (e*Log[4 - x^2])/27
```

Rubi [A] time = 0.0518091, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1673, 12, 1092, 1166, 207, 1107, 614, 616, 31}

$$\frac{dx(17-5x^2)}{72(x^4-5x^2+4)} + \frac{19}{432} d \tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54} d \tanh^{-1}(x) + \frac{e(5-2x^2)}{18(x^4-5x^2+4)} + \frac{1}{27} e \log\left(1-x^2\right) - \frac{1}{27} e \log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(72*(4 - 5*x^2 + x^4)) + (e*(5 - 2*x^2))/(18*(4 - 5*x^2
+ x^4)) + (19*d*ArcTanh[x/2])/432 - (d*ArcTanh[x])/54 + (e*Log[1 - x^2])/27
 - (e*Log[4 - x^2])/27
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
```

```
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{\left(4-5x^2+x^4\right)^2}\,dx &= \int \frac{d}{\left(4-5x^2+x^4\right)^2}\,dx+\int \frac{ex}{\left(4-5x^2+x^4\right)^2}\,dx\\
&= d\int \frac{1}{\left(4-5x^2+x^4\right)^2}\,dx+e\int \frac{x}{\left(4-5x^2+x^4\right)^2}\,dx\\
&= \frac{dx\left(17-5x^2\right)}{72\left(4-5x^2+x^4\right)}-\frac{1}{72}d\int \frac{-1+5x^2}{4-5x^2+x^4}\,dx+\frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{\left(4-5x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{dx\left(17-5x^2\right)}{72\left(4-5x^2+x^4\right)}+\frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)}+\frac{1}{54}d\int \frac{1}{-1+x^2}\,dx-\frac{1}{216}(19d)\int \frac{1}{-4+x^2}\,dx-\frac{1}{9}e\,\mathrm{Sub}\\
&= \frac{dx\left(17-5x^2\right)}{72\left(4-5x^2+x^4\right)}+\frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)-\frac{1}{27}e\,\mathrm{Subst}\left(\int \right.\\
&= \frac{dx\left(17-5x^2\right)}{72\left(4-5x^2+x^4\right)}+\frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0548821, size = 90, normalized size = 0.96

$$
\frac{1}{864}\left(\frac{12\left(dx\left(17-5x^2\right)+e\left(20-8x^2\right)\right)}{x^4-5x^2+4}+8(d+4e)\log(1-x)-(19d+32e)\log(2-x)-8(d-4e)\log(x+1)+(19d-32
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4) + 8*(d + 4*e)*L
og[1 - x] - (19*d + 32*e)*Log[2 - x] - 8*(d - 4*e)*Log[1 + x] + (19*d - 32*
e)*Log[2 + x])/864
```

Maple [A] time = 0.017, size = 122, normalized size = 1.3

$$
-\frac{d}{288+144\,x}+\frac{e}{144+72\,x}+\frac{19\,\ln(2+x)\,d}{864}-\frac{\ln(2+x)\,e}{27}-\frac{\ln(1+x)\,d}{108}+\frac{\ln(1+x)\,e}{27}-\frac{d}{36+36\,x}+\frac{e}{36+36\,x}-\frac{19\,\ln}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/144/(2+x)*d+1/72/(2+x)*e+19/864*ln(2+x)*d-1/27*ln(2+x)*e-1/108*ln(1+x)*d
+1/27*ln(1+x)*e-1/36/(1+x)*d+1/36/(1+x)*e-19/864*ln(x-2)*d-1/27*ln(x-2)*e-1
/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-1)*d-1/36/(x-1)*e+1/108*ln(x-1)*d+1/27*ln
(x-1)*e
```

Maxima [A] time = 0.93689, size = 112, normalized size = 1.19

$$\frac{1}{864}(19d-32e)\log(x+2)-\frac{1}{108}(d-4e)\log(x+1)+\frac{1}{108}(d+4e)\log(x-1)-\frac{1}{864}(19d+32e)\log(x-2)-\frac{5dx^3}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e)*log(x + 2) - 1/108*(d - 4*e)*log(x + 1) + 1/108*(d + 4*
e)*log(x - 1) - 1/864*(19*d + 32*e)*log(x - 2) - 1/72*(5*d*x^3 + 8*e*x^2 -
17*d*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [B] time = 2.49327, size = 446, normalized size = 4.74

$$-\frac{60dx^3+96ex^2-204dx-\left((19d-32e)x^4-5(19d-32e)x^2+76d-128e\right)\log(x+2)+8\left((d-4e)x^4-5(d-4e)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(60*d*x^3 + 96*e*x^2 - 204*d*x - ((19*d - 32*e)*x^4 - 5*(19*d - 32*e
)*x^2 + 76*d - 128*e)*log(x + 2) + 8*((d - 4*e)*x^4 - 5*(d - 4*e)*x^2 + 4*d
 - 16*e)*log(x + 1) - 8*((d + 4*e)*x^4 - 5*(d + 4*e)*x^2 + 4*d + 16*e)*log(
x - 1) + ((19*d + 32*e)*x^4 - 5*(19*d + 32*e)*x^2 + 76*d + 128*e)*log(x - 2
) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [B] time = 2.65074, size = 604, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d - 4*e)*log(x + (-6006260*d**4*e + 2341251*d**4*(d - 4*e) - 18247680*d**
2*e**3 + 24099840*d**2*e**2*(d - 4*e) + 7387904*d**2*e*(d - 4*e)**2 - 66528
0*d**2*(d - 4*e)**3 + 587202560*e**5 - 12582912*e**4*(d - 4*e) - 36700160*e
**3*(d - 4*e)**2 + 786432*e**2*(d - 4*e)**3)/(1675971*d**5 - 66150400*d**3*
e**2 + 318767104*d*e**4))/108 + (d + 4*e)*log(x + (-6006260*d**4*e - 234125
1*d**4*(d + 4*e) - 18247680*d**2*e**3 - 24099840*d**2*e**2*(d + 4*e) + 7387
904*d**2*e*(d + 4*e)**2 + 665280*d**2*(d + 4*e)**3 + 587202560*e**5 + 12582
912*e**4*(d + 4*e) - 36700160*e**3*(d + 4*e)**2 - 786432*e**2*(d + 4*e)**3)
/(1675971*d**5 - 66150400*d**3*e**2 + 318767104*d*e**4))/108 + (19*d - 32*e
)*log(x + (-6006260*d**4*e - 2341251*d**4*(19*d - 32*e)/8 - 18247680*d**2*e
**3 - 3012480*d**2*e**2*(19*d - 32*e) + 115436*d**2*e*(19*d - 32*e)**2 + 10
395*d**2*(19*d - 32*e)**3/8 + 587202560*e**5 + 1572864*e**4*(19*d - 32*e) -
 573440*e**3*(19*d - 32*e)**2 - 1536*e**2*(19*d - 32*e)**3)/(1675971*d**5 -
 66150400*d**3*e**2 + 318767104*d*e**4))/864 - (19*d + 32*e)*log(x + (-6006
260*d**4*e + 2341251*d**4*(19*d + 32*e)/8 - 18247680*d**2*e**3 + 3012480*d*
*2*e**2*(19*d + 32*e) + 115436*d**2*e*(19*d + 32*e)**2 - 10395*d**2*(19*d +
 32*e)**3/8 + 587202560*e**5 - 1572864*e**4*(19*d + 32*e) - 573440*e**3*(19
*d + 32*e)**2 + 1536*e**2*(19*d + 32*e)**3)/(1675971*d**5 - 66150400*d**3*e
**2 + 318767104*d*e**4))/864 - (5*d*x**3 - 17*d*x + 8*e*x**2 - 20*e)/(72*x*
*4 - 360*x**2 + 288)
```

Giac [A] time = 1.10026, size = 126, normalized size = 1.34

$$\frac{1}{864}\,(19\,d-32\,e)\log\left(|x+2|\right)-\frac{1}{108}\,(d-4\,e)\log\left(|x+1|\right)+\frac{1}{108}\,(d+4\,e)\log\left(|x-1|\right)-\frac{1}{864}\,(19\,d+32\,e)\log\left(|x-2|\right)-\frac{5\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d - 32*e)*log(abs(x + 2)) - 1/108*(d - 4*e)*log(abs(x + 1)) + 1/1
08*(d + 4*e)*log(abs(x - 1)) - 1/864*(19*d + 32*e)*log(abs(x - 2)) - 1/72*(
5*d*x^3 + 8*x^2*e - 17*d*x - 20*e)/(x^4 - 5*x^2 + 4)
```

3.27 $\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=115

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}+\frac{1}{27}e\log$$

```
[Out] (e*(5 - 2*x^2))/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f - (5*d + 8*f)*x^2)
)/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 - ((d + 7*f)*Ar
cTanh[x])/54 + (e*Log[1 - x^2])/27 - (e*Log[4 - x^2])/27
```

Rubi [A] time = 0.140115, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1673, 1178, 1166, 207, 12, 1107, 614, 616, 31}

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{e\left(5-2x^2\right)}{18\left(x^4-5x^2+4\right)}+\frac{1}{27}e\log$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (e*(5 - 2*x^2))/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f - (5*d + 8*f)*x^2)
)/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 - ((d + 7*f)*Ar
cTanh[x])/54 + (e*Log[1 - x^2])/27 - (e*Log[4 - x^2])/27
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{ex}{\left(4-5x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} - \frac{1}{72}\int \frac{-d+20f+(5d+8f)x^2}{4-5x^2+x^4}\,dx + e\int \frac{x}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{\left(4-5x+x^2\right)^2}\,dx, x, x^2\right) - \frac{1}{54}(-d-7f)\int \frac{1}{-1+$$

$$= \frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}(d+7f)$$

$$= \frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}(d+7f)$$

$$= \frac{e\left(5-2x^2\right)}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}(d+7f)$$

Mathematica [A] time = 0.0808132, size = 112, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx\right)}{x^4-5x^2+4} + 8\log(1-x)(d+4e+7f) - \log(2-x)(19d+32e+52f) - 8\,\mathrm{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 + x
^4) + 8*(d + 4*e + 7*f)*Log[1 - x] - (19*d + 32*e + 52*f)*Log[2 - x] - 8*(d
 - 4*e + 7*f)*Log[1 + x] + (19*d - 32*e + 52*f)*Log[2 + x])/864
```

Maple [A] time = 0.018, size = 182, normalized size = 1.6

$$-\frac{d}{288+144\,x}+\frac{e}{144+72\,x}-\frac{f}{72+36\,x}+\frac{19\,\ln\left(2+x\right)d}{864}-\frac{\ln\left(2+x\right)e}{27}+\frac{13\,\ln\left(2+x\right)f}{216}-\frac{\ln\left(1+x\right)d}{108}+\frac{\ln\left(1+x\right)e}{27}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/144/(2+x)*d+1/72/(2+x)*e-1/36/(2+x)*f+19/864*ln(2+x)*d-1/27*ln(2+x)*e+13
/216*ln(2+x)*f-1/108*ln(1+x)*d+1/27*ln(1+x)*e-7/108*ln(1+x)*f-1/36/(1+x)*d+
1/36/(1+x)*e-1/36/(1+x)*f-19/864*ln(x-2)*d-1/27*ln(x-2)*e-13/216*ln(x-2)*f-
1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-2)*f-1/36/(x-1)*d-1/36/(x-1)*e-1/36/(x-1
)*f+1/108*ln(x-1)*d+1/27*ln(x-1)*e+7/108*ln(x-1)*f
```

Maxima [A] time = 0.942326, size = 143, normalized size = 1.24

$$\frac{1}{864}\left(19\,d-32\,e+52\,f\right)\log\left(x+2\right)-\frac{1}{108}\left(d-4\,e+7\,f\right)\log\left(x+1\right)+\frac{1}{108}\left(d+4\,e+7\,f\right)\log\left(x-1\right)-\frac{1}{864}\left(19\,d+32\,e+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f)*log(x + 2) - 1/108*(d - 4*e + 7*f)*log(x + 1) +
1/108*(d + 4*e + 7*f)*log(x - 1) - 1/864*(19*d + 32*e + 52*f)*log(x - 2) -
1/72*((5*d + 8*f)*x^3 + 8*e*x^2 - (17*d + 20*f)*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [B] time = 3.0018, size = 585, normalized size = 5.09

$$-\frac{12\left(5\,d+8\,f\right)x^{3}+96\,ex^{2}-12\left(17\,d+20\,f\right)x-\left(\left(19\,d-32\,e+52\,f\right)x^{4}-5\left(19\,d-32\,e+52\,f\right)x^{2}+76\,d-128\,e+208\,f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 96*e*x^2 - 12*(17*d + 20*f)*x - ((19*d - 32*e
+ 52*f)*x^4 - 5*(19*d - 32*e + 52*f)*x^2 + 76*d - 128*e + 208*f)*log(x + 2)
```

```
 + 8*((d - 4*e + 7*f)*x^4 - 5*(d - 4*e + 7*f)*x^2 + 4*d - 16*e + 28*f)*log(
x + 1) - 8*((d + 4*e + 7*f)*x^4 - 5*(d + 4*e + 7*f)*x^2 + 4*d + 16*e + 28*f
)*log(x - 1) + ((19*d + 32*e + 52*f)*x^4 - 5*(19*d + 32*e + 52*f)*x^2 + 76*
d + 128*e + 208*f)*log(x - 2) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [B] time = 38.346, size = 2689, normalized size = 23.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d - 4*e + 7*f)*log(x + (-6006260*d**5*e + 2341251*d**5*(d - 4*e + 7*f) -
246016240*d**4*e*f + 31626180*d**4*f*(d - 4*e + 7*f) - 18247680*d**3*e**3 +
 24099840*d**3*e**2*(d - 4*e + 7*f) - 2758371200*d**3*e*f**2 + 7387904*d**3
*e*(d - 4*e + 7*f)**2 + 171122976*d**3*f**2*(d - 4*e + 7*f) - 665280*d**3*(
d - 4*e + 7*f)**3 + 298598400*d**2*e**3*f + 369487872*d**2*e**2*f*(d - 4*e
+ 7*f) - 13192256000*d**2*e*f**3 + 90885120*d**2*e*f*(d - 4*e + 7*f)**2 + 4
41486720*d**2*f**3*(d - 4*e + 7*f) - 5536512*d**2*f*(d - 4*e + 7*f)**3 + 58
7202560*d*e**5 - 12582912*d*e**4*(d - 4*e + 7*f) + 1353646080*d*e**3*f**2 -
 36700160*d*e**3*(d - 4*e + 7*f)**2 + 1448755200*d*e**2*f**2*(d - 4*e + 7*f
) + 786432*d*e**2*(d - 4*e + 7*f)**3 - 28282393600*d*e*f**4 + 362729472*d*e
*f**2*(d - 4*e + 7*f)**2 + 399575808*d*f**4*(d - 4*e + 7*f) - 10368000*d*f*
*2*(d - 4*e + 7*f)**3 + 2751463424*e**5*f + 251658240*e**4*f*(d - 4*e + 7*f
) - 530841600*e**3*f**3 - 171966464*e**3*f*(d - 4*e + 7*f)**2 + 1935212544*
e**2*f**3*(d - 4*e + 7*f) - 15728640*e**2*f*(d - 4*e + 7*f)**3 - 2188688998
4*e*f**5 + 483737600*e*f**3*(d - 4*e + 7*f)**2 - 212474880*f**5*(d - 4*e +
7*f) + 4534272*f**3*(d - 4*e + 7*f)**3)/(1675971*d**6 + 28507545*d**5*f - 6
6150400*d**4*e**2 + 168075324*d**4*f**2 - 1091117056*d**3*e**2*f + 38409552
0*d**3*f**3 + 318767104*d**2*e**4 - 6528860160*d**2*e**2*f**2 + 162082944*d
**2*f**4 + 3103784960*d*e**4*f - 17414619136*d*e**2*f**3 - 305130240*d*f**5
 + 6106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f**6))/108 + (d
+ 4*e + 7*f)*log(x + (-6006260*d**5*e - 2341251*d**5*(d + 4*e + 7*f) - 2460
16240*d**4*e*f - 31626180*d**4*f*(d + 4*e + 7*f) - 18247680*d**3*e**3 - 240
99840*d**3*e**2*(d + 4*e + 7*f) - 2758371200*d**3*e*f**2 + 7387904*d**3*e*(
d + 4*e + 7*f)**2 - 171122976*d**3*f**2*(d + 4*e + 7*f) + 665280*d**3*(d +
4*e + 7*f)**3 + 298598400*d**2*e**3*f - 369487872*d**2*e**2*f*(d + 4*e + 7*
f) - 13192256000*d**2*e*f**3 + 90885120*d**2*e*f*(d + 4*e + 7*f)**2 - 44148
6720*d**2*f**3*(d + 4*e + 7*f) + 5536512*d**2*f*(d + 4*e + 7*f)**3 + 587202
560*d*e**5 + 12582912*d*e**4*(d + 4*e + 7*f) + 1353646080*d*e**3*f**2 - 367
00160*d*e**3*(d + 4*e + 7*f)**2 - 1448755200*d*e**2*f**2*(d + 4*e + 7*f) -
```

786432*d*e**2*(d + 4*e + 7*f)**3 - 28282393600*d*e*f**4 + 362729472*d*e*f**
2*(d + 4*e + 7*f)**2 - 399575808*d*f**4*(d + 4*e + 7*f) + 10368000*d*f**2*(
d + 4*e + 7*f)**3 + 2751463424*e**5*f - 251658240*e**4*f*(d + 4*e + 7*f) -
530841600*e**3*f**3 - 171966464*e**3*f*(d + 4*e + 7*f)**2 - 1935212544*e**2
*f**3*(d + 4*e + 7*f) + 15728640*e**2*f*(d + 4*e + 7*f)**3 - 21886889984*e*
f**5 + 483737600*e*f**3*(d + 4*e + 7*f)**2 + 212474880*f**5*(d + 4*e + 7*f)
 - 4534272*f**3*(d + 4*e + 7*f)**3)/(1675971*d**6 + 28507545*d**5*f - 66150
400*d**4*e**2 + 168075324*d**4*f**2 - 1091117056*d**3*e**2*f + 384095520*d*
*3*f**3 + 318767104*d**2*e**4 - 6528860160*d**2*e**2*f**2 + 162082944*d**2*
f**4 + 3103784960*d*e**4*f - 17414619136*d*e**2*f**3 - 305130240*d*f**5 + 6
106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f**6))/108 + (19*d -
 32*e + 52*f)*log(x + (-6006260*d**5*e - 2341251*d**5*(19*d - 32*e + 52*f)/
8 - 246016240*d**4*e*f - 7906545*d**4*f*(19*d - 32*e + 52*f)/2 - 18247680*d
3*e3 - 3012480*d**3*e**2*(19*d - 32*e + 52*f) - 2758371200*d**3*e*f**2
+ 115436*d**3*e*(19*d - 32*e + 52*f)**2 - 21390372*d**3*f**2*(19*d - 32*e +
 52*f) + 10395*d**3*(19*d - 32*e + 52*f)**3/8 + 298598400*d**2*e**3*f - 461
85984*d**2*e**2*f*(19*d - 32*e + 52*f) - 13192256000*d**2*e*f**3 + 1420080*
d**2*e*f*(19*d - 32*e + 52*f)**2 - 55185840*d**2*f**3*(19*d - 32*e + 52*f)
+ 21627*d**2*f*(19*d - 32*e + 52*f)**3/2 + 587202560*d*e**5 + 1572864*d*e**
4*(19*d - 32*e + 52*f) + 1353646080*d*e**3*f**2 - 573440*d*e**3*(19*d - 32*
e + 52*f)**2 - 181094400*d*e**2*f**2*(19*d - 32*e + 52*f) - 1536*d*e**2*(19
*d - 32*e + 52*f)**3 - 28282393600*d*e*f**4 + 5667648*d*e*f**2*(19*d - 32*e
 + 52*f)**2 - 49946976*d*f**4*(19*d - 32*e + 52*f) + 20250*d*f**2*(19*d - 3
2*e + 52*f)**3 + 2751463424*e**5*f - 31457280*e**4*f*(19*d - 32*e + 52*f) -
 530841600*e**3*f**3 - 2686976*e**3*f*(19*d - 32*e + 52*f)**2 - 241901568*e
2*f3*(19*d - 32*e + 52*f) + 30720*e**2*f*(19*d - 32*e + 52*f)**3 - 2188
6889984*e*f**5 + 7558400*e*f**3*(19*d - 32*e + 52*f)**2 + 26559360*f**5*(19
*d - 32*e + 52*f) - 8856*f**3*(19*d - 32*e + 52*f)**3)/(1675971*d**6 + 2850
7545*d**5*f - 66150400*d**4*e**2 + 168075324*d**4*f**2 - 1091117056*d**3*e*
*2*f + 384095520*d**3*f**3 + 318767104*d**2*e**4 - 6528860160*d**2*e**2*f**
2 + 162082944*d**2*f**4 + 3103784960*d*e**4*f - 17414619136*d*e**2*f**3 - 3
05130240*d*f**5 + 6106906624*e**4*f**2 - 17414225920*e**2*f**4 + 67931136*f
6))/864 - (19*d + 32*e + 52*f)*log(x + (-6006260*d5*e + 2341251*d**5*(1
9*d + 32*e + 52*f)/8 - 246016240*d**4*e*f + 7906545*d**4*f*(19*d + 32*e + 5
2*f)/2 - 18247680*d**3*e**3 + 3012480*d**3*e**2*(19*d + 32*e + 52*f) - 2758
371200*d**3*e*f**2 + 115436*d**3*e*(19*d + 32*e + 52*f)**2 + 21390372*d**3*
f**2*(19*d + 32*e + 52*f) - 10395*d**3*(19*d + 32*e + 52*f)**3/8 + 29859840
0*d**2*e**3*f + 46185984*d**2*e**2*f*(19*d + 32*e + 52*f) - 13192256000*d**
2*e*f**3 + 1420080*d**2*e*f*(19*d + 32*e + 52*f)**2 + 55185840*d**2*f**3*(1
9*d + 32*e + 52*f) - 21627*d**2*f*(19*d + 32*e + 52*f)**3/2 + 587202560*d*e
5 - 1572864*d*e4*(19*d + 32*e + 52*f) + 1353646080*d*e**3*f**2 - 573440
*d*e**3*(19*d + 32*e + 52*f)**2 + 181094400*d*e**2*f**2*(19*d + 32*e + 52*f
) + 1536*d*e**2*(19*d + 32*e + 52*f)**3 - 28282393600*d*e*f**4 + 5667648*d*
e*f**2*(19*d + 32*e + 52*f)**2 + 49946976*d*f**4*(19*d + 32*e + 52*f) - 202
50*d*f**2*(19*d + 32*e + 52*f)**3 + 2751463424*e**5*f + 31457280*e**4*f*(19

```
*d + 32*e + 52*f) - 530841600*e**3*f**3 - 2686976*e**3*f*(19*d + 32*e + 52*
f)**2 + 241901568*e**2*f**3*(19*d + 32*e + 52*f) - 30720*e**2*f*(19*d + 32*
e + 52*f)**3 - 21886889984*e*f**5 + 7558400*e*f**3*(19*d + 32*e + 52*f)**2
- 26559360*f**5*(19*d + 32*e + 52*f) + 8856*f**3*(19*d + 32*e + 52*f)**3)/(
1675971*d**6 + 28507545*d**5*f - 66150400*d**4*e**2 + 168075324*d**4*f**2 -
 1091117056*d**3*e**2*f + 384095520*d**3*f**3 + 318767104*d**2*e**4 - 65288
60160*d**2*e**2*f**2 + 162082944*d**2*f**4 + 3103784960*d*e**4*f - 17414619
136*d*e**2*f**3 - 305130240*d*f**5 + 6106906624*e**4*f**2 - 17414225920*e**
2*f**4 + 67931136*f**6))/864 - (8*e*x**2 - 20*e + x**3*(5*d + 8*f) + x*(-17
*d - 20*f))/(72*x**4 - 360*x**2 + 288)
```

Giac [A] time = 1.08665, size = 155, normalized size = 1.35

$$\frac{1}{864}\left(19\,d+52\,f-32\,e\right)\log\left(|x+2|\right)-\frac{1}{108}\left(d+7\,f-4\,e\right)\log\left(|x+1|\right)+\frac{1}{108}\left(d+7\,f+4\,e\right)\log\left(|x-1|\right)-\frac{1}{864}\left(19\,d+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f - 4*e)*log(abs(
x + 1)) + 1/108*(d + 7*f + 4*e)*log(abs(x - 1)) - 1/864*(19*d + 52*f + 32*e
)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 8*x^2*e - 17*d*x - 20*f*x - 2
0*e)/(x^4 - 5*x^2 + 4)
```

3.28 $\int \frac{d+ex+fx^2+gx^3}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=138

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}+\frac{1}{54}$$

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (5*e + 8*g - (
2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 -
 ((d + 7*f)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[1 - x^2])/54 - ((2*e + 5*g)*L
og[4 - x^2])/54
```

Rubi [A] time = 0.153883, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1673, 1178, 1166, 207, 1247, 638, 616, 31}

$$\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{x^2(-(2e+5g))+5e+8g}{18\left(x^4-5x^2+4\right)}+\frac{1}{54}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (5*e + 8*g - (
2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 -
 ((d + 7*f)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[1 - x^2])/54 - ((2*e + 5*g)*L
og[4 - x^2])/54
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} - \frac{1}{72}\int \frac{-d+20f+(5d+8f)x^2}{4-5x^2+x^4}\,dx + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e+gx}{\left(4-5x+x\right.}\right.$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} - \frac{1}{54}(-d-7f)\int \frac{1}{-1+x^2}\,dx - \frac{1}{216}(1$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}(d$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}(d$$

Mathematica [A] time = 0.054027, size = 134, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx-4g\left(5x^2-8\right)\right)}{x^4-5x^2+4} + 8\log(1-x)(d+4e+7f+10g) - \log(2-x)(19d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g*(-8 + 5*x^
2)))/(4 - 5*x^2 + x^4) + 8*(d + 4*e + 7*f + 10*g)*Log[1 - x] - (19*d + 32*e
 + 52*f + 80*g)*Log[2 - x] - 8*(d - 4*e + 7*f - 10*g)*Log[1 + x] + (19*d -
32*e + 52*f - 80*g)*Log[2 + x])/864
```

Maple [A] time = 0.018, size = 242, normalized size = 1.8

$$\frac{19\ln(2+x)d}{864} - \frac{\ln(2+x)e}{27} - \frac{\ln(1+x)d}{108} + \frac{\ln(1+x)e}{27} - \frac{19\ln(x-2)d}{864} - \frac{\ln(x-2)e}{27} + \frac{\ln(x-1)d}{108} + \frac{\ln(x-1)e}{27} + \frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 19/864*ln(2+x)*d-1/27*ln(2+x)*e-1/108*ln(1+x)*d+1/27*ln(1+x)*e-19/864*ln(x-
2)*d-1/27*ln(x-2)*e+1/108*ln(x-1)*d+1/27*ln(x-1)*e+1/18/(2+x)*g-1/36/(1+x)*
d+1/36/(1+x)*e-1/18/(x-2)*g-1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-1)*g-1/36/(x
-1)*d-1/36/(x-1)*e-1/144/(2+x)*d+1/72/(2+x)*e+1/36/(1+x)*g-1/36/(1+x)*f-1/3
6/(x-2)*f-1/36/(x-1)*f-1/36/(2+x)*f-5/54*ln(2+x)*g+5/54*ln(1+x)*g-5/54*ln(x
-2)*g+5/54*ln(x-1)*g-13/216*ln(x-2)*f+7/108*ln(x-1)*f+13/216*ln(2+x)*f-7/10
8*ln(1+x)*f
```

Maxima [A] time = 0.941108, size = 171, normalized size = 1.24

$$\frac{1}{864}\left(19\,d-32\,e+52\,f-80\,g\right)\log\left(x+2\right)-\frac{1}{108}\left(d-4\,e+7\,f-10\,g\right)\log\left(x+1\right)+\frac{1}{108}\left(d+4\,e+7\,f+10\,g\right)\log\left(x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g)*log(x + 2) - 1/108*(d - 4*e + 7*f - 10*g)
*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*g)*log(x - 1) - 1/864*(19*d + 32*e
+ 52*f + 80*g)*log(x - 2) - 1/72*((5*d + 8*f)*x^3 + 4*(2*e + 5*g)*x^2 - (17
*d + 20*f)*x - 20*e - 32*g)/(x^4 - 5*x^2 + 4)
```

Fricas [B] time = 4.61709, size = 725, normalized size = 5.25

$$-\frac{12\left(5\,d+8\,f\right)x^3+48\left(2\,e+5\,g\right)x^2-12\left(17\,d+20\,f\right)x-\left(\left(19\,d-32\,e+52\,f-80\,g\right)x^4-5\left(19\,d-32\,e+52\,f-80\,g\right)x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d + 20*f)*x - ((19
*d - 32*e + 52*f - 80*g)*x^4 - 5*(19*d - 32*e + 52*f - 80*g)*x^2 + 76*d - 1
28*e + 208*f - 320*g)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g)*x^4 - 5*(d - 4
*e + 7*f - 10*g)*x^2 + 4*d - 16*e + 28*f - 40*g)*log(x + 1) - 8*((d + 4*e +
 7*f + 10*g)*x^4 - 5*(d + 4*e + 7*f + 10*g)*x^2 + 4*d + 16*e + 28*f + 40*g)
```

```
*log(x - 1) + ((19*d + 32*e + 52*f + 80*g)*x^4 - 5*(19*d + 32*e + 52*f + 80
*g)*x^2 + 76*d + 128*e + 208*f + 320*g)*log(x - 2) - 240*e - 384*g)/(x^4 -
5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08606, size = 184, normalized size = 1.33

$$\frac{1}{864}\left(19\,d + 52\,f - 80\,g - 32\,e\right)\log\left(|x + 2|\right) - \frac{1}{108}\left(d + 7\,f - 10\,g - 4\,e\right)\log\left(|x + 1|\right) + \frac{1}{108}\left(d + 7\,f + 10\,g + 4\,e\right)\log\left(|x -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 80*g - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f - 10*g -
 4*e)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g + 4*e)*log(abs(x - 1)) - 1/86
4*(19*d + 52*f + 80*g + 32*e)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 2
0*g*x^2 + 8*x^2*e - 17*d*x - 20*f*x - 32*g - 20*e)/(x^4 - 5*x^2 + 4)
```

3.29 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+$$

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f + 32
*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f + 112*
h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[
1 - x^2])/54 - ((2*e + 5*g)*Log[4 - x^2])/54
```

Rubi [A] time = 0.214297, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1673, 1678, 1166, 207, 1247, 638, 616, 31}

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f + 32
*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f + 112*
h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[
1 - x^2])/54 - ((2*e + 5*g)*Log[4 - x^2])/54
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)} - \frac{1}{72}\int \frac{-d+20f+32h+(5d+8f+20h}{4-5x^2+x^4}$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{18}(-2e-5g)\,\mathrm{S}$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)} + \frac{1}{432}(19d+52f$$

Mathematica [A] time = 0.0770761, size = 159, normalized size = 1.06

$$\frac{1}{864}\left(-\frac{12\left(x\left(d\left(5x^2-17\right)+4f\left(2x^2-5\right)+4h\left(5x^2-8\right)\right)+4e\left(2x^2-5\right)+4g\left(5x^2-8\right)\right)}{x^4-5x^2+4}+8\log(1-x)(d+4e+7f+1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(4*e*(-5 + 2*x^2) + 4*g*(-8 + 5*x^2) + x*(4*f*(-5 + 2*x^2) + d*(-17 +
 5*x^2) + 4*h*(-8 + 5*x^2))))/(4 - 5*x^2 + x^4) + 8*(d + 4*e + 7*f + 10*g +
 13*h)*Log[1 - x] - (19*d + 32*e + 52*f + 80*g + 112*h)*Log[2 - x] - 8*(d -
 4*e + 7*f - 10*g + 13*h)*Log[1 + x] + (19*d - 32*e + 52*f - 80*g + 112*h)*
Log[2 + x])/864
```

Maple [B] time = 0.019, size = 302, normalized size = 2.

$$\frac{19\,\ln(2+x)\,d}{864} - \frac{\ln(2+x)\,e}{27} - \frac{\ln(1+x)\,d}{108} + \frac{\ln(1+x)\,e}{27} - \frac{19\,\ln(x-2)\,d}{864} - \frac{\ln(x-2)\,e}{27} + \frac{\ln(x-1)\,d}{108} + \frac{\ln(x-1)\,e}{27} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 19/864*ln(2+x)*d-1/27*ln(2+x)*e-1/108*ln(1+x)*d+1/27*ln(1+x)*e-19/864*ln(x-
2)*d-1/27*ln(x-2)*e+1/108*ln(x-1)*d+1/27*ln(x-1)*e-1/9/(x-2)*h-1/36/(x-1)*h
-1/36/(1+x)*h-1/9/(2+x)*h+1/18/(2+x)*g-1/36/(1+x)*d+1/36/(1+x)*e-1/18/(x-2)
*g-1/144/(x-2)*d-1/72/(x-2)*e-1/36/(x-1)*g-1/36/(x-1)*d-1/36/(x-1)*e-1/144/
(2+x)*d+1/72/(2+x)*e+1/36/(1+x)*g-1/36/(1+x)*f-1/36/(x-2)*f-1/36/(x-1)*f-1/
36/(2+x)*f-5/54*ln(2+x)*g+5/54*ln(1+x)*g-5/54*ln(x-2)*g+5/54*ln(x-1)*g+7/54
*ln(2+x)*h-13/108*ln(1+x)*h-7/54*ln(x-2)*h+13/108*ln(x-1)*h-13/216*ln(x-2)*
f+7/108*ln(x-1)*f+13/216*ln(2+x)*f-7/108*ln(1+x)*f
```

Maxima [A] time = 0.972301, size = 196, normalized size = 1.31

$$\frac{1}{864}\left(19\,d-32\,e+52\,f-80\,g+112\,h\right)\log\left(x+2\right)-\frac{1}{108}\left(d-4\,e+7\,f-10\,g+13\,h\right)\log\left(x+1\right)+\frac{1}{108}\left(d+4\,e+7\,f+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g + 112*h)*log(x + 2) - 1/108*(d - 4*e + 7*f
 - 10*g + 13*h)*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*g + 13*h)*log(x - 1)
 - 1/864*(19*d + 32*e + 52*f + 80*g + 112*h)*log(x - 2) - 1/72*((5*d + 8*f
+ 20*h)*x^3 + 4*(2*e + 5*g)*x^2 - (17*d + 20*f + 32*h)*x - 20*e - 32*g)/(x^
4 - 5*x^2 + 4)
```

Fricas [B] time = 13.6627, size = 865, normalized size = 5.77

$$-\frac{12\left(5\,d+8\,f+20\,h\right)x^3+48\left(2\,e+5\,g\right)x^2-12\left(17\,d+20\,f+32\,h\right)x-\left(\left(19\,d-32\,e+52\,f-80\,g+112\,h\right)x^4-5\left(19\,d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d + 20*f +
32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h)*x^4 - 5*(19*d - 32*e + 52*f
```

```
- 80*g + 112*h)*x^2 + 76*d - 128*e + 208*f - 320*g + 448*h)*log(x + 2) + 8*
((d - 4*e + 7*f - 10*g + 13*h)*x^4 - 5*(d - 4*e + 7*f - 10*g + 13*h)*x^2 +
4*d - 16*e + 28*f - 40*g + 52*h)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13
*h)*x^4 - 5*(d + 4*e + 7*f + 10*g + 13*h)*x^2 + 4*d + 16*e + 28*f + 40*g +
52*h)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h)*x^4 - 5*(19*d + 32*
e + 52*f + 80*g + 112*h)*x^2 + 76*d + 128*e + 208*f + 320*g + 448*h)*log(x
- 2) - 240*e - 384*g)/(x^4 - 5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11335, size = 213, normalized size = 1.42

$$\frac{1}{864}\left(19\,d+52\,f-80\,g+112\,h-32\,e\right)\log\left(|x+2|\right)-\frac{1}{108}\left(d+7\,f-10\,g+13\,h-4\,e\right)\log\left(|x+1|\right)+\frac{1}{108}\left(d+7\,f+10\,g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f
- 10*g + 13*h - 4*e)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g + 13*h + 4*e)*
log(abs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 112*h + 32*e)*log(abs(x - 2))
 - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 20*g*x^2 + 8*x^2*e - 17*d*x - 20*f*
x - 32*h*x - 32*g - 20*e)/(x^4 - 5*x^2 + 4)
```

3.30 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=162

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{x^2}{}$$

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
(5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d
 + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2
*e + 5*g + 8*i)*Log[1 - x^2])/54 - ((2*e + 5*g + 8*i)*Log[4 - x^2])/54
```

Rubi [A] time = 0.231837, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1673, 1678, 1166, 207, 1663, 1660, 12, 616, 31}

$$\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
(5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d
 + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2
*e + 5*g + 8*i)*Log[1 - x^2])/54 - ((2*e + 5*g + 8*i)*Log[4 - x^2])/54
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+30x^5}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{x\left(e+gx^2+30x^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)} - \frac{1}{72}\int \frac{-d+20f+32h+(5d+8f}{4-5x^2+x^4}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{72\left(4-5x^2+x^4\right)}$$

Mathematica [A] time = 0.0940909, size = 185, normalized size = 1.14

$$\frac{-5dx^3+17dx-8ex^2+20e-8fx^3+20fx-20gx^2+32g-20hx^3+32hx-68ix^2+80i}{72\left(x^4-5x^2+4\right)} + \frac{1}{108}\log(1-x)(d+4e+7f+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i*
x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(72*(4 - 5*x^2 + x^4)) + ((d + 4*e + 7*
f + 10*g + 13*h + 16*i)*Log[1 - x])/108 + ((-19*d - 32*e - 52*f - 80*g - 11
2*h - 128*i)*Log[2 - x])/864 + ((-d + 4*e - 7*f + 10*g - 13*h + 16*i)*Log[1
 + x])/108 + ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*Log[2 + x])/864
```

Maple [B] time = 0.019, size = 362, normalized size = 2.2

$$\frac{19\ln(2+x)d}{864}-\frac{\ln(2+x)e}{27}-\frac{\ln(1+x)d}{108}+\frac{\ln(1+x)e}{27}-\frac{19\ln(x-2)d}{864}-\frac{\ln(x-2)e}{27}+\frac{\ln(x-1)d}{108}+\frac{\ln(x-1)e}{27}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 19/864*ln(2+x)*d-1/27*ln(2+x)*e-1/108*ln(1+x)*d+1/27*ln(1+x)*e-19/864*ln(x-
2)*d-1/27*ln(x-2)*e+1/108*ln(x-1)*d+1/27*ln(x-1)*e-2/9/(x-2)*i-1/36/(x-1)*i
+1/36/(1+x)*i+2/9/(2+x)*i-1/9/(x-2)*h-1/36/(x-1)*h-1/36/(1+x)*h-1/9/(2+x)*h
+1/18/(2+x)*g-1/36/(1+x)*d+1/36/(1+x)*e-1/18/(x-2)*g-1/144/(x-2)*d-1/72/(x-
2)*e-1/36/(x-1)*g-1/36/(x-1)*d-1/36/(x-1)*e-1/144/(2+x)*d+1/72/(2+x)*e+1/36
/(1+x)*g-1/36/(1+x)*f-1/36/(x-2)*f-1/36/(x-1)*f-1/36/(2+x)*f-4/27*ln(x-2)*i
+4/27*ln(x-1)*i-4/27*ln(2+x)*i+4/27*ln(1+x)*i-5/54*ln(2+x)*g+5/54*ln(1+x)*g
-5/54*ln(x-2)*g+5/54*ln(x-1)*g+7/54*ln(2+x)*h-13/108*ln(1+x)*h-7/54*ln(x-2)
*h+13/108*ln(x-1)*h-13/216*ln(x-2)*f+7/108*ln(x-1)*f+13/216*ln(2+x)*f-7/108
*ln(1+x)*f
```

Maxima [A] time = 1.00125, size = 220, normalized size = 1.36

$$\frac{1}{864}\left(19\,d-32\,e+52\,f-80\,g+112\,h-128\,i\right)\log\left(x+2\right)-\frac{1}{108}\left(d-4\,e+7\,f-10\,g+13\,h-16\,i\right)\log\left(x+1\right)+\frac{1}{108}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/864*(19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*log(x + 2) - 1/108*(d - 4
*e + 7*f - 10*g + 13*h - 16*i)*log(x + 1) + 1/108*(d + 4*e + 7*f + 10*g + 1
3*h + 16*i)*log(x - 1) - 1/864*(19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*
```

```
log(x - 2) - 1/72*((5*d + 8*f + 20*h)*x^3 + 4*(2*e + 5*g + 17*i)*x^2 - (17*
d + 20*f + 32*h)*x - 20*e - 32*g - 80*i)/(x^4 - 5*x^2 + 4)
```

Fricas [B] time = 65.6791, size = 1007, normalized size = 6.22

$$-\frac{12\left(5d+8f+20h\right)x^3+48\left(2e+5g+17i\right)x^2-12\left(17d+20f+32h\right)x-\left(\left(19d-32e+52f-80g+112h-128i\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g + 17*i)*x^2 - 12*(17*d +
20*f + 32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*x^4 - 5*(19*d
 - 32*e + 52*f - 80*g + 112*h - 128*i)*x^2 + 76*d - 128*e + 208*f - 320*g +
 448*h - 512*i)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^4 -
5*(d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^2 + 4*d - 16*e + 28*f - 40*g + 52*
h - 64*i)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13*h + 16*i)*x^4 - 5*(d +
 4*e + 7*f + 10*g + 13*h + 16*i)*x^2 + 4*d + 16*e + 28*f + 40*g + 52*h + 64
*i)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*x^4 - 5*(19*d
 + 32*e + 52*f + 80*g + 112*h + 128*i)*x^2 + 76*d + 128*e + 208*f + 320*g +
 448*h + 512*i)*log(x - 2) - 240*e - 384*g - 960*i)/(x^4 - 5*x^2 + 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08324, size = 242, normalized size = 1.49

$$\frac{1}{864}\left(19d+52f-80g+112h-128i-32e\right)\log\left(|x+2|\right)-\frac{1}{108}\left(d+7f-10g+13h-16i-4e\right)\log\left(|x+1|\right)+\frac{1}{108}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 128*i - 32*e)*log(abs(x + 2)) - 1/108*(
d + 7*f - 10*g + 13*h - 16*i - 4*e)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g
 + 13*h + 16*i + 4*e)*log(abs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 112*h +
 128*i + 32*e)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 20*g*
x^2 + 68*i*x^2 + 8*x^2*e - 17*d*x - 20*f*x - 32*h*x - 32*g - 80*i - 20*e)/(
x^4 - 5*x^2 + 4)
```

3.31 $\int \frac{d+ex}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=140

$$\frac{dx(1-x^2)}{6(x^4+x^2+1)}-\frac{1}{4}d\log(x^2-x+1)+\frac{1}{4}d\log(x^2+x+1)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e(2x^2+1)}{6(x^4+x^2+1)}+\frac{2e\,\mathrm{ta}}{}$$

```
[Out] (d*x*(1 - x^2))/(6*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) -
 (d*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(3*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - (d*Log[1 - x
+ x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rubi [A] time = 0.09779, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1673, 12, 1092, 1169, 634, 618, 204, 628, 1107, 614}

$$\frac{dx(1-x^2)}{6(x^4+x^2+1)}-\frac{1}{4}d\log(x^2-x+1)+\frac{1}{4}d\log(x^2+x+1)-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{e(2x^2+1)}{6(x^4+x^2+1)}+\frac{2e\,\mathrm{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (d*x*(1 - x^2))/(6*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) -
 (d*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(3*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - (d*Log[1 - x
+ x^2])/4 + (d*Log[1 + x + x^2])/4
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{d}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{ex}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= d\int \frac{1}{\left(1+x^2+x^4\right)^2}\,dx + e\int \frac{x}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{dx\left(1-x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}d\int \frac{5-x^2}{1+x^2+x^4}\,dx + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{dx\left(1-x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}d\int \frac{5-6x}{1-x+x^2}\,dx + \frac{1}{12}d\int \frac{5+6x}{1+x+x^2}\,dx + \frac{1}{3}e\,\mathrm{Subst}\Bigg(\int$$

$$= \frac{dx\left(1-x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}d\int \frac{1}{1-x+x^2}\,dx + \frac{1}{6}d\int \frac{1}{1+x+x^2}\,dx - \frac{1}{4}d\int \frac{-1+2x}{1-x+x^2}$$

$$= \frac{dx\left(1-x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{4}d\log\left(1-x+x^2\right) + \frac{1}{4}d\log\left(1+x+x^2\right)$$

$$= \frac{dx\left(1-x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{4}d\log$$

Mathematica [C] time = 0.494046, size = 146, normalized size = 1.04

$$\frac{d\left(x-x^3\right)+2ex^2+e}{6\left(x^4+x^2+1\right)} - \frac{\left(\sqrt{3}-11i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{6\sqrt{6+6i\sqrt{3}}} - \frac{\left(\sqrt{3}+11i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)}{6\sqrt{6-6i\sqrt{3}}} - \frac{2e\tan^{-1}\left(\frac{\sqrt{3}}{2x^2+1}\right)}{3\sqrt{3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e + 2*e*x^2 + d*(x - x^3))/(6*(1 + x^2 + x^4)) - ((-11*I + Sqrt[3])*d*ArcT
an[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt[3]]) - ((11*I + Sqrt[3])*d
*ArcTan[((I + Sqrt[3])*x)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]]) - (2*e*ArcTan[Sqr
t[3]/(1 + 2*x^2)])/(3*Sqrt[3])
```

Maple [A] time = 0.02, size = 146, normalized size = 1.

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}\right)x-\frac{2d}{3}+\frac{e}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{2\sqrt{3}e}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e)*x-2/3*d+1/3*e)/(x^2+x+1)+1/4*d*ln(x^2+x+1)+1/9*d*arctan
(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e-1/4
*((1/3*d-1/3*e)*x-2/3*d-1/3*e)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/9*3^(1/2)*arct
an(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e
```

Maxima [A] time = 1.43411, size = 130, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{9}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1) - 1/6*(d*x^3 - 2*e*x^2 - d*x - e)/(x^4 + x^2 + 1)
```

Fricas [A] time = 1.58244, size = 416, normalized size = 2.97

$$-\frac{6\,dx^3-12\,ex^2-4\,\sqrt{3}\left((d-2\,e)x^4+(d-2\,e)x^2+d-2\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-4\,\sqrt{3}\left((d+2\,e)x^4+(d+2\,e)x^2+d+\right.}{36\left(x^4+x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(6*d*x^3 - 12*e*x^2 - 4*sqrt(3)*((d - 2*e)*x^4 + (d - 2*e)*x^2 + d -
2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*((d + 2*e)*x^4 + (d + 2*e)*x
^2 + d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*d*x - 9*(d*x^4 + d*x^2 + d)
*log(x^2 + x + 1) + 9*(d*x^4 + d*x^2 + d)*log(x^2 - x + 1) - 6*e)/(x^4 + x^
2 + 1)
```

Sympy [C] time = 2.77069, size = 952, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4*e + 1026*d**4*(-d/4 -
sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 - 31536*d**2*e**2*(-d/4 - sqrt(3)*
I*(d + 2*e)/18) + 108432*d**2*e*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**2 + 163296
*d**2*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**3 + 1792*e**5 + 11520*e**4*(-d/4 - s
qrt(3)*I*(d + 2*e)/18) + 48384*e**3*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**2 + 31
1040*e**2*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5 - 11408*d**3*e**2
- 7936*d*e**4)) + (-d/4 + sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4*e +
1026*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 - 31536*d**2*e**
2*(-d/4 + sqrt(3)*I*(d + 2*e)/18) + 108432*d**2*e*(-d/4 + sqrt(3)*I*(d + 2*
e)/18)**2 + 163296*d**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**3 + 1792*e**5 + 11
520*e**4*(-d/4 + sqrt(3)*I*(d + 2*e)/18) + 48384*e**3*(-d/4 + sqrt(3)*I*(d
+ 2*e)/18)**2 + 311040*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5
- 11408*d**3*e**2 - 7936*d*e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/18)*log(x +
(-10309*d**4*e + 1026*d**4*(d/4 - sqrt(3)*I*(d - 2*e)/18) - 7200*d**2*e**3
- 31536*d**2*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18) + 108432*d**2*e*(d/4 - sqr
t(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d/4 - sqrt(3)*I*(d - 2*e)/18)**3 + 1
792*e**5 + 11520*e**4*(d/4 - sqrt(3)*I*(d - 2*e)/18) + 48384*e**3*(d/4 - sq
rt(3)*I*(d - 2*e)/18)**2 + 311040*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18)**3)/(
```

```
3348*d**5 - 11408*d**3*e**2 - 7936*d*e**4)) + (d/4 + sqrt(3)*I*(d - 2*e)/18
)*log(x + (-10309*d**4*e + 1026*d**4*(d/4 + sqrt(3)*I*(d - 2*e)/18) - 7200*
d**2*e**3 - 31536*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/18) + 108432*d**2*e*
(d/4 + sqrt(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d/4 + sqrt(3)*I*(d - 2*e)/
18)**3 + 1792*e**5 + 11520*e**4*(d/4 + sqrt(3)*I*(d - 2*e)/18) + 48384*e**3
*(d/4 + sqrt(3)*I*(d - 2*e)/18)**2 + 311040*e**2*(d/4 + sqrt(3)*I*(d - 2*e)
/18)**3)/(3348*d**5 - 11408*d**3*e**2 - 7936*d*e**4)) - (d*x**3 - d*x - 2*e
*x**2 - e)/(6*x**4 + 6*x**2 + 6)
```

Giac [A] time = 1.09115, size = 135, normalized size = 0.96

$$\frac{1}{9}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{9}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1) - 1/6*(d*x^3 - 2*x^2*e - d*x - e)/(x^4 + x^2 + 1)
```

3.32 $\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2} dx$

Optimal. Leaf size=165

$$\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\,t}{1}$$

```
[Out] (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x
^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f)*Log[1 + x + x^
2])/8
```

Rubi [A] time = 0.129192, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {1673, 1178, 1169, 634, 618, 204, 628, 12, 1107, 614}

$$\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\,t}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x
^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f)*Log[1 + x + x^
2])/8
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{ex}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+(-d+2f)x^2}{1+x^2+x^4}\,dx + e\int \frac{x}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-f-(6d-3f)x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{5d-f+(6d-3f)x}{1+x+x^2}\,dx + \frac{1}{2}e\,\mathrm{Sub}$$

$$= \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{3}e\,\mathrm{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{8}(2d-f)\int \frac{1}{1+}$$

$$= \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{1}{8}(2d-f)\log\left(1-x+x^2\right) + \frac{1}{8}(2d-f)\log\left(1+x+x^2\right.$$

$$= \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{2e\,\mathrm{t}}{}$$

Mathematica [C] time = 0.42446, size = 186, normalized size = 1.13

$$\frac{1}{36}\left(\frac{6\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e\right)}{x^4+x^2+1} - \frac{\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(\sqrt{3}+11i\right)d-2\left(\sqrt{}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4) - (((-11*I
 + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1
 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f)*ArcTan[((I +
 Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 +
2*x^2)])/36
```

Maple [A] time = 0.014, size = 214, normalized size = 1.3

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}+\frac{f}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e+2/3*f)*x-2/3*d+1/3*e+1/3*f)/(x^2+x+1)+1/4*d*ln(x^2+x+1)-
1/8*ln(x^2+x+1)*f+1/9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arc
tan(1/3*(1+2*x)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f-1/4*(
(1/3*d-1/3*e-2/3*f)*x-2/3*d-1/3*e+1/3*f)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln
(x^2-x+1)*f+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/
3*(2*x-1)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f
```

Maxima [A] time = 1.42986, size = 162, normalized size = 0.98

$$\frac{1}{36}\sqrt{3}\left(4d-8e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+8e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\left(2d-f\right)\log\left(x^2+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(
4*d + 8*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x +
1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d - 2*f)*x^3 - 2*e*x^2 - (d + f
```

```
)*x - e)/(x^4 + x^2 + 1)
```

Fricas [A] time = 1.90264, size = 547, normalized size = 3.32

$$-\frac{12\left(d-2f\right)x^3-24ex^2-2\sqrt{3}\left(\left(4d-8e+f\right)x^4+\left(4d-8e+f\right)x^2+4d-8e+f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left(\left(4d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f)*x^3 - 24*e*x^2 - 2*sqrt(3)*((4*d - 8*e + f)*x^4 + (4*d
- 8*e + f)*x^2 + 4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(
(4*d + 8*e + f)*x^4 + (4*d + 8*e + f)*x^2 + 4*d + 8*e + f)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 12*(d + f)*x - 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*
log(x^2 + x + 1) + 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 - x
+ 1) - 12*e)/(x^4 + x^2 + 1)
```

Sympy [C] time = 33.2861, size = 4107, normalized size = 24.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5*e + 16416
*d**5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 336520*d**4*e*f + 20066
4*d**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 115200*d**3*e**3 - 5
04576*d**3*e**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e
*f**2 + 1734912*d**3*e*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229
500*d**3*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(-
d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d
**2*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 119420*d**2*e*f**3
 - 2477952*d**2*e*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 50436*
d**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 2519424*d**2*f*(-d/
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(
-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e*
*3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**2*(-d/
```

```
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/4 + f/8 - sqrt
(3)*I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(-d/4 + f/
8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*d*f**4*(-d/4 + f/8 - sqrt(3)*I
*(4*d + 8*e + f)/72) + 139968*d*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f
)/72)**3 - 20480*e**5*f - 36864*e**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e +
 f)/72) - 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e
 + f)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)
- 995328*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 3956*e*f**
5 - 209088*e*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 3996*f**
5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 233280*f**3*(-d/4 + f/8 - s
qrt(3)*I*(4*d + 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e
**2 + 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e
**4 - 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e*
*2*f**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**6)) + (-d/
4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5*e + 16416*d**
5*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 336520*d**4*e*f + 200664*d*
*4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 115200*d**3*e**3 - 50457
6*d**3*e**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e*f**
2 + 1734912*d**3*e*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229500*
d**3*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(-d/4
+ f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d**2*
e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 119420*d**2*e*f**3 - 2
477952*d**2*e*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 50436*d**2
*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 2519424*d**2*f*(-d/4 +
f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(-d/4
 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(
-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**2*(-d/4 +
f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/4 + f/8 + sqrt(3)*
I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(-d/4 + f/8 +
sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*d*f**4*(-d/4 + f/8 + sqrt(3)*I*(4*
d + 8*e + f)/72) + 139968*d*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72
)**3 - 20480*e**5*f - 36864*e**4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/
72) - 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f
)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 99
5328*e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 3956*e*f**5 -
209088*e*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 3996*f**5*(-
d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 233280*f**3*(-d/4 + f/8 + sqrt(
3)*I*(4*d + 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e**2
+ 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4
- 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e**2*f
**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**6)) + (d/4 - f
/8 - sqrt(3)*I*(4*d - 8*e + f)/72)*log(x + (-164944*d**5*e + 16416*d**5*(d/
4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 336520*d**4*e*f + 200664*d**4*f*(
d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 115200*d**3*e**3 - 504576*d**3*
e**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 272380*d**3*e*f**2 + 1734
```

```
912*d**3*e*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 229500*d**3*f**2
*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 2612736*d**3*(d/4 - f/8 - sqr
t(3)*I*(4*d - 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d**2*e**2*f*(d/4
 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 119420*d**2*e*f**3 - 2477952*d**2*
e*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 50436*d**2*f**3*(d/4 -
f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 2519424*d**2*f*(d/4 - f/8 - sqrt(3)*I
*(4*d - 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(d/4 - f/8 - sqrt(3)
*I*(4*d - 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(d/4 - f/8 - sqrt
(3)*I*(4*d - 8*e + f)/72)**2 - 409536*d*e**2*f**2*(d/4 - f/8 - sqrt(3)*I*(4
*d - 8*e + f)/72) + 4976640*d*e**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/7
2)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e
 + f)/72)**2 + 14040*d*f**4*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 13
9968*d*f**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3 - 20480*e**5*f -
36864*e**4*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 2880*e**3*f**3 -
552960*e**3*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 70848*e**2*f*
*3*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 995328*e**2*f*(d/4 - f/8 -
sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 3956*e*f**5 - 209088*e*f**3*(d/4 - f/8 -
 sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 3996*f**5*(d/4 - f/8 - sqrt(3)*I*(4*d -
 8*e + f)/72) + 233280*f**3*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**3)/
(53568*d**6 - 69984*d**5*f - 182528*d**4*e**2 + 23652*d**4*f**2 + 377344*d*
*3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4 - 278400*d**2*e**2*f**2 - 413
1*d**2*f**4 + 102400*d*e**4*f + 93568*d*e**2*f**3 + 81*d*f**5 - 28672*e**4*
f**2 - 11648*e**2*f**4 + 189*f**6)) + (d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f
)/72)*log(x + (-164944*d**5*e + 16416*d**5*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*
e + f)/72) + 336520*d**4*e*f + 200664*d**4*f*(d/4 - f/8 + sqrt(3)*I*(4*d -
8*e + f)/72) - 115200*d**3*e**3 - 504576*d**3*e**2*(d/4 - f/8 + sqrt(3)*I*(
4*d - 8*e + f)/72) - 272380*d**3*e*f**2 + 1734912*d**3*e*(d/4 - f/8 + sqrt(
3)*I*(4*d - 8*e + f)/72)**2 - 229500*d**3*f**2*(d/4 - f/8 + sqrt(3)*I*(4*d
- 8*e + f)/72) + 2612736*d**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3
 + 51840*d**2*e**3*f + 881280*d**2*e**2*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e
 + f)/72) + 119420*d**2*e*f**3 - 2477952*d**2*e*f*(d/4 - f/8 + sqrt(3)*I*(4
*d - 8*e + f)/72)**2 + 50436*d**2*f**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e +
f)/72) - 2519424*d**2*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3 + 286
72*d*e**5 + 184320*d*e**4*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 8640
*d*e**3*f**2 + 774144*d*e**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**2
- 409536*d*e**2*f**2*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 4976640*d
*e**2*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270
080*d*e*f**2*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 14040*d*f**4*(
d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 139968*d*f**2*(d/4 - f/8 + sqrt
(3)*I*(4*d - 8*e + f)/72)**3 - 20480*e**5*f - 36864*e**4*f*(d/4 - f/8 + sqr
t(3)*I*(4*d - 8*e + f)/72) - 2880*e**3*f**3 - 552960*e**3*f*(d/4 - f/8 + sq
rt(3)*I*(4*d - 8*e + f)/72)**2 + 70848*e**2*f**3*(d/4 - f/8 + sqrt(3)*I*(4*
d - 8*e + f)/72) - 995328*e**2*f*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)
**3 + 3956*e*f**5 - 209088*e*f**3*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72
)**2 - 3996*f**5*(d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72) + 233280*f**3*(
```

```
d/4 - f/8 + sqrt(3)*I*(4*d - 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f -
182528*d**4*e**2 + 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 -
126976*d**2*e**4 - 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f
 + 93568*d*e**2*f**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*
f**6)) - (-2*e*x**2 - e + x**3*(d - 2*f) + x*(-d - f))/(6*x**4 + 6*x**2 + 6
)
```

Giac [A] time = 1.09278, size = 173, normalized size = 1.05

$$\frac{1}{36}\sqrt{3}\left(4\,d+f-8\,e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{36}\sqrt{3}\left(4\,d+f+8\,e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{8}\left(2\,d-f\right)\log\left(x^2+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(
4*d + f + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x +
1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 - 2*x^2*e - d*x
- f*x - e)/(x^4 + x^2 + 1)
```

3.33 $\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^2} dx$

Optimal. Leaf size=179

$$\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)}{1}$$

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + (2*e - g)*x^2)
/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) +
 ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g)*ArcTan[(1
+ 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f
)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.141012, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1673, 1178, 1169, 634, 618, 204, 628, 1247, 638}

$$\frac{x\left(x^2(-(d-2f))+d+f\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + (2*e - g)*x^2)
/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) +
 ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g)*ArcTan[(1
+ 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f
)*Log[1 + x + x^2])/8
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+(-d+2f)x^2}{1+x^2+x^4}\,dx + \frac{1}{2}\,\text{Subst}\left(\int \frac{e+gx}{\left(1+x+x^2\right)^2}\,dx,x,x\right.$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-f-(6d-3f)x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{5d-f}{1}$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{1}{8}(2d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{8}(-2d+f)\int$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{8}(2d-f)\log\left(1-x\right.$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} - \frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right.}{12\sqrt{3}}$$

Mathematica [C] time = 0.434399, size = 200, normalized size = 1.12

$$\frac{1}{36}\left(\frac{6\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e-g\left(x^2+2\right)\right)}{x^4+x^2+1} - \frac{\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}-2i\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(\sqrt{3}+\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 - g*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x
^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*
x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f
)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g
)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Maple [A] time = 0.013, size = 260, normalized size = 1.5

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{f}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{d\sqrt{3}}{9}\arctan\Bigg(\frac{(1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e-1/3*g+2/3*f)*x-2/3*d+1/3*e-2/3*g+1/3*f)/(x^2+x+1)+1/4*d*
ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/9*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9
*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1
/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g-1/4*((1/3*d-1/3*e-1/3*g-2/
3*f)*x-2/3*d-1/3*e+2/3*g+1/3*f)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)
*f+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1
/3*(2*x-1)*3^(1/2))*g
```

Maxima [A] time = 1.45662, size = 182, normalized size = 1.02

$$\frac{1}{36}\sqrt{3}\left(4d-8e+f+4g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+8e+f-4g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\left(2d-f\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqr
t(3)*(4*d + 8*e + f - 4*g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*lo
g(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d - 2*f)*x^3 - (2*e
 - g)*x^2 - (d + f)*x - e + 2*g)/(x^4 + x^2 + 1)
```

Fricas [A] time = 3.15898, size = 616, normalized size = 3.44

$$-\frac{12\left(d-2f\right)x^3-12\left(2e-g\right)x^2-2\sqrt{3}\left(\left(4d-8e+f+4g\right)x^4+\left(4d-8e+f+4g\right)x^2+4d-8e+f+4g\right)\arctan\left(\frac{1}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f)*x^3 - 12*(2*e - g)*x^2 - 2*sqrt(3)*((4*d - 8*e + f + 4*
g)*x^4 + (4*d - 8*e + f + 4*g)*x^2 + 4*d - 8*e + f + 4*g)*arctan(1/3*sqrt(3
)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e + f - 4*g)*x^4 + (4*d + 8*e + f - 4*g)
*x^2 + 4*d + 8*e + f - 4*g)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(d + f)*x -
9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 + x + 1) + 9*((2*d - f)
*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 - x + 1) - 12*e + 24*g)/(x^4 + x^2
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08282, size = 192, normalized size = 1.07

$$\frac{1}{36}\sqrt{3}\left(4d+f+4g-8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+f-4g+8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\left(2d-f\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqr
t(3)*(4*d + f - 4*g + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*lo
g(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 + g*
x^2 - 2*x^2*e - d*x - f*x + 2*g - e)/(x^4 + x^2 + 1)
```

3.34 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(1+x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=187

$$\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d}{12\sqrt{3}}$$

```
[Out] (e - 2*g + (2*e - g)*x^2)/(6*(1 + x^2 + x^4)) + (x*(d + f - 2*h - (d - 2*f
+ h)*x^2))/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/
(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2
*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x
 + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.167095, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {1673, 1678, 1169, 634, 618, 204, 628, 1247, 638}

$$\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (e - 2*g + (2*e - g)*x^2)/(6*(1 + x^2 + x^4)) + (x*(d + f - 2*h - (d - 2*f
+ h)*x^2))/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/
(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2
*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x
 + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+2h+(-d+2f-h)x^2}{1+x^2+x^4}\,dx + \frac{1}{2}\,\mathrm{Subst}\Big($$

$$= \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-f+2h-(6d-3f}{1-x+x^2}$$

$$= \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{3}(-2e+g)\,\mathrm{Subst}\Big(\int \frac{1}{-3-x}$$

$$= \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{8}(2a$$

$$= \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{(4d+f+h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + $$

Mathematica [C] time = 0.628847, size = 234, normalized size = 1.25

$$\frac{1}{36}\left(-\frac{6\left(x\left(d\left(x^2-1\right)-f\left(2x^2+1\right)+h\left(x^2+2\right)\right)-e\left(2x^2+1\right)+g\left(x^2+2\right)\right)}{x^4+x^2+1} - \frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}-11i\right)d-2\left(\sqrt{3}\right.\right.}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((-6*(g*(2 + x^2) - e*(1 + 2*x^2) + x*(d*(-1 + x^2) + h*(2 + x^2) - f*(1 +
2*x^2))))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f +
(-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] -
 (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*ArcTan[((I
 + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g)*ArcTan[Sqr
t[3]/(1 + 2*x^2)])/36
```

Maple [A] time = 0.014, size = 328, normalized size = 1.8

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}+\frac{2f}{3}-\frac{g}{3}-\frac{e}{3}-\frac{h}{3}\right)x-\frac{2d}{3}+\frac{f}{3}-\frac{2g}{3}+\frac{e}{3}+\frac{h}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}+\frac{\ln\left(x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d+2/3*f-1/3*g-1/3*e-1/3*h)*x-2/3*d+1/3*f-2/3*g+1/3*e+1/3*h)/(x^2
+x+1)+1/4*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/8*ln(x^2+x+1)*h+1/9*d*arctan(1/
3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/36*3
^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2)
)*g+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h-1/4*((1/3*d-2/3*f-1/3*g-1/3*
e+1/3*h)*x-2/3*d+1/3*f+2/3*g-1/3*e+1/3*h)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*l
n(x^2-x+1)*f-1/8*ln(x^2-x+1)*h+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/
9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g+1/36*3^(1/2)*arctan(1/3*(
2*x-1)*3^(1/2))*h
```

Maxima [A] time = 1.50396, size = 193, normalized size = 1.03

$$\frac{1}{36}\sqrt{3}\left(4d-8e+f+4g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+8e+f-4g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + 8*e + f - 4*g + h)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1) - 1/6*((d -
 2*f + h)*x^3 - (2*e - g)*x^2 - (d + f - 2*h)*x - e + 2*g)/(x^4 + x^2 + 1)
```

Fricas [A] time = 10.4954, size = 694, normalized size = 3.71

$$-\frac{12\left(d-2f+h\right)x^3-12\left(2e-g\right)x^2-2\sqrt{3}\left(\left(4d-8e+f+4g+h\right)x^4+\left(4d-8e+f+4g+h\right)x^2+4d-8e+f+4g}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f + h)*x^3 - 12*(2*e - g)*x^2 - 2*sqrt(3)*((4*d - 8*e + f
+ 4*g + h)*x^4 + (4*d - 8*e + f + 4*g + h)*x^2 + 4*d - 8*e + f + 4*g + h)*a
rctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e + f - 4*g + h)*x^4 + (
4*d + 8*e + f - 4*g + h)*x^2 + 4*d + 8*e + f - 4*g + h)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 12*(d + f - 2*h)*x - 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2
+ 2*d - f + h)*log(x^2 + x + 1) + 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2
+ 2*d - f + h)*log(x^2 - x + 1) - 12*e + 24*g)/(x^4 + x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07795, size = 209, normalized size = 1.12

$$\frac{1}{36}\sqrt{3}\left(4d+f+4g+h-8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+f-4g+h+8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\left(2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + f - 4*g + h + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1) - 1/6*(d*x^
3 - 2*f*x^3 + h*x^3 + g*x^2 - 2*x^2*e - d*x - f*x + 2*h*x + 2*g - e)/(x^4 +
 x^2 + 1)
```

3.35 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=194

$$\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d}{12\sqrt{3}}$$

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + i +
(2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sq
rt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3
]) + ((2*e - g + 2*i)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f
+ h)*Log[1 - x + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rubi [A] time = 0.197327, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{6\left(x^4+x^2+1\right)}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)(4d}{12\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```


```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + i +
(2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sq
rt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3
]) + ((2*e - g + 2*i)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f
+ h)*Log[1 - x + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+35x^5}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{x\left(e+gx^2+35x^4\right)}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+2h+(-d+2f-h)x^2}{1+x^2+x^4}\,dx + \frac{1}{2}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-}{}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}(70+2e-$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{1}{8}(2d-f+$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\left(1+x^2+x^4\right)} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{(4d+f+h}{12}$$

Mathematica [C] time = 0.66927, size = 243, normalized size = 1.25

$$\frac{1}{36}\left(\frac{6\left(-dx^3+dx+2ex^2+e+2fx^3+fx-g\left(x^2+2\right)-hx^3-2hx-ix^2+i\right)}{x^4+x^2+1}-\frac{\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)\left(\left(\sqrt{3}-11i\right)d-2\left(\right.\right.}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^3
- g*(2 + x^2)))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3]
)*f + (-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3]
)/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*ArcT
an[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g + 2*i
)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Maple [B] time = 0.015, size = 374, normalized size = 1.9

$$\frac{1}{4x^2+4x+4}\left(\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}-\frac{h}{3}+\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{h}{3}+\frac{f}{3}+\frac{i}{3}\right)+\frac{d\ln\left(x^2+x+1\right)}{4}-\frac{\ln\left(x^2+x+1\right)f}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x)
```

```
[Out] 1/4*((-1/3*d-1/3*e-1/3*g-1/3*h+2/3*f+2/3*i)*x-2/3*d+1/3*e-2/3*g+1/3*h+1/3*f
+1/3*i)/(x^2+x+1)+1/4*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+1/8*ln(x^2+x+1)*h+1/9
*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/
2))*e+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+
2*x)*3^(1/2))*g+1/36*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h-2/9*3^(1/2)*arct
an(1/3*(1+2*x)*3^(1/2))*i-1/4*((1/3*d-1/3*e-1/3*g+1/3*h-2/3*f+2/3*i)*x-2/3*
d-1/3*e+2/3*g+1/3*h+1/3*f-1/3*i)/(x^2-x+1)-1/4*d*ln(x^2-x+1)+1/8*ln(x^2-x+1
)*f-1/8*ln(x^2-x+1)*h+1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)
*arctan(1/3*(2*x-1)*3^(1/2))*e+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1
/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g+1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))*h+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*i
```

Maxima [A] time = 1.46652, size = 209, normalized size = 1.08

$$\frac{1}{36}\sqrt{3}\left(4d-8e+f+4g+h-8i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+8e+f-4g+h+8i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxim
a")
```

```
[Out] 1/36*sqrt(3)*(4*d - 8*e + f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + 8*e + f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1)
 - 1/6*((d - 2*f + h)*x^3 - (2*e - g - i)*x^2 - (d + f - 2*h)*x - e + 2*g -
 i)/(x^4 + x^2 + 1)
```

Fricas [A] time = 48.7816, size = 757, normalized size = 3.9

$$-\frac{12\left(d-2f+h\right)x^3-12\left(2e-g-i\right)x^2-2\sqrt{3}\left(\left(4d-8e+f+4g+h-8i\right)x^4+\left(4d-8e+f+4g+h-8i\right)x^2+4d-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="frica
s")
```

```
[Out] -1/72*(12*(d - 2*f + h)*x^3 - 12*(2*e - g - i)*x^2 - 2*sqrt(3)*((4*d - 8*e
+ f + 4*g + h - 8*i)*x^4 + (4*d - 8*e + f + 4*g + h - 8*i)*x^2 + 4*d - 8*e
+ f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e
+ f - 4*g + h + 8*i)*x^4 + (4*d + 8*e + f - 4*g + h + 8*i)*x^2 + 4*d + 8*e
+ f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(d + f - 2*h)*x - 9
*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 + x + 1) + 9
*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 - x + 1) - 1
2*e + 24*g - 12*i)/(x^4 + x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09133, size = 228, normalized size = 1.18

$$\frac{1}{36}\sqrt{3}\left(4d+f+4g+h-8i-8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}\left(4d+f-4g+h+8i+8e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac"
)
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*i - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + f - 4*g + h + 8*i + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1)
 - 1/6*(d*x^3 - 2*f*x^3 + h*x^3 + g*x^2 + i*x^2 - 2*x^2*e - d*x - f*x + 2*h
*x + 2*g - i - e)/(x^4 + x^2 + 1)
```

3.36 $\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=330

$$\frac{dx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}d\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-}}}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (d*x*(b^2 - 2*a*
c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*
a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
(Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
4*a*c)^(3/2)
```

Rubi [A] time = 0.744729, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {1673, 12, 1092, 1166, 205, 1107, 614, 618, 206}

$$\frac{dx\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}d\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (d*x*(b^2 - 2*a*
c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*
a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
(Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{d}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{ex}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= d\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx + e\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{dx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{d\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)} + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, \ldots\right.$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{dx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)d\right.\ldots}{4a\left(b^2-4ac\right.\ldots}$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{dx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\ldots}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b\ldots}}$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{dx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\ldots}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b\ldots}}$$

Mathematica [A] time = 0.875281, size = 341, normalized size = 1.03

$$\frac{1}{4}\left(\frac{2abe+4acx(d+ex)-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}d\left(b\sqrt{b^2-4a\right.}{a\left(b^2-4\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*e + 4*a*c*x*(d + e*x) - 2*b*d*x*(b + c*x^2))/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*d*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqr
t[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*c*e*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e*Log[b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.122, size = 1237, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*d*x*(-4*a*c+b^2)^(1/
2)-1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*(-4*a*c+b
^2)^(1/2)*b^2-c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*d*x*b+
1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*b^3+2*c/(4*a
*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*a-1/2/(4*a*c-b^2)^2/(x^2
+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*b^2+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)
*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
(-4*a*c+b^2)^(1/2)*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^
(1/2)*b^2*d-c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/4*c/(4*a*c-b^2)^2/a*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
```

```
)^(1/2))*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2
)/c)*d*x*(-4*a*c+b^2)^(1/2)+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4
*a*c+b^2)^(1/2)/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1
/2)/c)/a*d*x*b^3+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e
*a-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e*b^2-c/(4*a*c-
b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+3*c^2/(4*a*c-
b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*b*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*e*x^2 - a*b*e + (b^2 - 2*a*c)*d*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b
*c*d*x^2 - 4*a*c*e*x + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*
a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.37 $\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*a*
c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.870422, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*a*
c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
```

```
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{ex}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d+6acd-abf-c(bd-2af)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)} + e\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,d$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + \frac{\left(c\left(bd-2af\right.\right.}{}$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12}{\sqrt{b}}\right.}{2\sqrt{2}a\left(b^2-4\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12}{\sqrt{b}}\right.}{2\sqrt{2}a\left(b^2-4\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12}{\sqrt{b}}\right.}{2\sqrt{2}a\left(b^2-4\right.}$$

Mathematica [A] time = 1.35201, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e+fx)+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4af\right)-2a\left(f\sqrt{b^2-4ac}+6cd\right)+b^2d\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 -
4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]
*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
 (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.118, size = 1813, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*a*x*f+2*c/(4*a*c-b
^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*a*x*f+1/4*c/(4*a*c-b^2)^2/a*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-c/(4
*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-c/(4*a*c-b^2)^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
```

```
a*c+b^2)^(1/2)*b^2*d+c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)
*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b
/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*b^
3-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*(-4*a*c+b^2)^(
1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*b+1/4/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*d*x*b^3-1/4/(4*a*c-b^2)^2
/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-c^2/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*d+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+2*c^2/(4*a*c-b^2
)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-2*c
^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^2*f+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1
/2)+b)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e*a-c/(4*a*
c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+2*c/(4*a*c-
b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/
2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-
4*a*c+b^2)^(1/2)/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(
1/2)/c)*x*b^2*f-1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*x*
b^2*f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.38 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=386

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*
f - (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.489971, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1673, 1178, 1166, 205, 1247, 638, 618, 206}

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*
f - (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
```

```
b^2 - 4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\int \frac{-b^2d+6acd}{a}\cdots}{2a\cdots}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(bd-2af-\frac{b^2d-12\cdots}{\sqrt{b\cdots}}\right.\right.\cdots}{4a\cdots}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2af+\frac{b^2d-1\cdots}{\sqrt{\cdots}}\right.\cdots}{2\sqrt{2}a\left(b^2-\cdots\right.}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bd-2af+\frac{b^2d-1\cdots}{\sqrt{\cdots}}\right.\cdots}{2\sqrt{2}a\left(b^2-\cdots\right.}$$

Mathematica [A] time = 1.55914, size = 421, normalized size = 1.09

$$\frac{1}{4}\left(\frac{-4a^2g+2ab(e+x(f-gx))+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2-4ac}+4af\right)-2a\left(f\sqrt{b^2-4ac}\cdots\right.\right.}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\cdots}}\right.\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*g - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)) + 2*a*b*(e + x
*(f - g*x)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2
*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d +
b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.122, size = 2310, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*a*x*f+2*c/(4*a*c-b
^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*a*x*f+1/4*c/(4*a*c-b^2)^2/a*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-c/(4
*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-c/(4*a*c-b^2)^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*b^2*d+c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)
*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b
/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*b^
3-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*(-4*a*c+b^2)^(
1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*b+1/4/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*d*x*b^3-1/4/(4*a*c-b^2)^2
/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/
```

```
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-c^2/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*d+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/(4*a*c-b^2)^2/
(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(-4*a*c+b^2)^(1/2)*a*g-1/(4*a*c-b^2)
^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*a*b*g+1/2/(4*a*c-b^2)^2*ln(-2*c*x
^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b*g-1/(4*a*c-b^2)^2/(x^2+1/2*(-
4*a*c+b^2)^(1/2)/c+1/2*b/c)*(-4*a*c+b^2)^(1/2)*a*g-1/(4*a*c-b^2)^2/(x^2+1/2
*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*a*b*g-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b
^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b*g+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^
2)^(1/2)/c+1/2*b/c)*b^3*g+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)*b^3*g+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2
)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2*f-2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/
2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1
/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(
-4*a*c+b^2)^(1/2)/c)*e*a-c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(
-4*a*c+b^2)^(1/2)-b)+2*c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/
c)*e*a-1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(-4*a*c+b
^2)^(1/2)*b^2*g+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*
(-4*a*c+b^2)^(1/2)*b^2*g-1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/
2*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e*b^2
-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*x*b^2*f-1/2/(4*a*
c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*x*b^2*f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(bcd-2\,acf\right)x^3-abe+2\,a^2g-\left(2\,ace-abg\right)x^2-\left(abf-\left(b^2-2\,ac\right)d\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}-\frac{-\int\frac{abf+\left(bcd-2\,acf\right)x^2+\left(b^2-6\,ac\right)d-2\left(2\,ace-abg\right)x}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*f)*x^3 - a*b*e + 2*a^2*g - (2*a*c*e - a*b*g)*x^2 - (a*b
*f - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (
a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(a*b*f + (b*c*d - 2*a*c*f)*x^2 + (
b^2 - 6*a*c)*d - 2*(2*a*c*e - a*b*g)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*
a^2*c)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.39 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=439

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f
 + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f +
b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^
2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.89438, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1673, 1678, 1166, 205, 1247, 638, 618, 206}

$$\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f
 + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f +
b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
```

```
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^
2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^2}\,dx\right.$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} -$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

Mathematica [A] time = 2.24061, size = 489, normalized size = 1.11

$$\frac{1}{4}\left(\frac{-4a^2(g+hx)+2ab(e+x(f-x(g+hx)))+4acx(d+x(e+fx))-2bdx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b}\cdots\right.\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*(g + h*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)) + 2*a*
b*(e + x*(f - x*(g + h*x))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*(b^2*(c*d - a*h) - 2*a*c*(6*c*d + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*S
qrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[
b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 -
 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g)*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*
g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.049, size = 1801, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*
b*f-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f+(-1/2/a*(a*b*
h-2*a*c*f+b*c*d)/(4*a*c-b^2)*x^3-1/2*(b*g-2*c*e)/(4*a*c-b^2)*x^2-1/2*(2*a^2
*h-a*b*f-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x-1/2*(2*a*g-b*e)/(4*a*c-b^2))/(c*x^4
+b*x^2+a)-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
```

```
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d
-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d+a/(4*a*
c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h-a/(4*a*c-b^2)^2*c*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b*h+a/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h
+a/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*(-4*a*c+b^2)^(1/2)*d-c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+3*c^2/
(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*b*d+1/2/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(
-4*a*c+b^2)^(1/2)*b*g-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-
4*a*c+b^2)^(1/2)*b*g+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*
c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*f+1/2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+1/4/(4*a*c-b^2)^2*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(
-4*a*c+b^2)^(1/2)*b^2*h+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4
*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c
+b^2)^(1/2)-b)-1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*h+1/4/(4*a*c-b^2)^
2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*h
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.40 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=468

$$\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\cdots}{\sqrt{b\cdots}}\right.\cdots}{\cdots}$$

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e -
b*c*g + b^2*i - 2*a*c*i)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b
*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*
d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e -
 b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 1.11765, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {1673, 1678, 1166, 205, 1663, 1660, 12, 618, 206}

$$\frac{x^2\left(-\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\cdots}{\sqrt{b\cdots}}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e -
b*c*g + b^2*i - 2*a*c*i)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b
*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*
d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/S
```

```
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e -
 b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+40x^5}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x\left(e+gx^2+40x^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+}{\left(a+bx+\right.}\right.$$

$$= -\frac{40ab+bce-2acg+\left(40b^2-2c(40a-ce)-bcg\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd\right.}{2a\left(b^2-4\right.}$$

$$= -\frac{40ab+bce-2acg+\left(40b^2-2c(40a-ce)-bcg\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd\right.}{2a\left(b^2-4\right.}$$

$$= -\frac{40ab+bce-2acg+\left(40b^2-2c(40a-ce)-bcg\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd\right.}{2a\left(b^2-4\right.}$$

$$= -\frac{40ab+bce-2acg+\left(40b^2-2c(40a-ce)-bcg\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(b^2d-abf-2a(cd\right.}{2a\left(b^2-4\right.}$$

<u>Mathematica</u> [A] time = 2.55582, size = 524, normalized size = 1.12

$$\frac{1}{4}\left(\frac{2\left(a^2(bi-2c(g+x(h+ix)))+a\left(b^2ix^2+bc(e+x(f-x(g+hx)))+2c^2x(d+x(e+fx))\right)-bcdx\left(b+cx^2\right)\right)}{ac\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\tan}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x
]
```

```
[Out] ((2*(-(b*c*d*x*(b + c*x^2)) + a^2*(b*i - 2*c*(g + x*(h + i*x))) + a*(b^2*i*
x^2 + 2*c^2*x*(d + x*(e + f*x)) + b*c*(e + x*(f - x*(g + h*x))))))/(a*c*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*h) - 2*a*c*(6*c*d
 + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sq
rt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
```

```
)/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b
^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g - 2*a*i)*Log[-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (2*(2*c*e - b*g + 2*a*i)*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.034, size = 1917, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*
b*f-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f+(-1/2/a*(a*b*
h-2*a*c*f+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a*c*i-b^2*i+b*c*g-2*c^2*e)/(4*a*c-b
^2)/c*x^2-1/2*(2*a^2*h-a*b*f-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x+1/2/c*(a*b*i-2*
a*c*g+b*c*e)/(4*a*c-b^2))/(c*x^4+b*x^2+a)-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
(-4*a*c+b^2)^(1/2)*b^2*d+a/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^
(1/2)*h-a/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+a/(4*a*c-b^2)^2*c*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h+a/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h+
3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-c^2/(4*a*c-b^2)
^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*d+3*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b
^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
```

```
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+1/2/(4*a*c-b^2)^2*ln(
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b*g-1/2/(4*a*c-b^2)^2*ln(
2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b*g+2*c^2/(4*a*c-b^2)^2*a*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-2*c^2/(4*
a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*
b^2*f+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+1/4/(4
*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+c/(4*a*c-b^2)^2*(-4
*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)-c/(4*a*c-b^2)^2*(-4*a*c+
b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+a/(4*a*c-b^2)^2*ln(2*c*x^2+(
-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*i-a/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a
*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*i-1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*b^3*h+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*h
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="m
axima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 22.393, size = 8384, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="g
iac")
```

```
[Out] -1/16*(16*a^2*b^5*c - 128*a^3*b^3*c^2 + 64*a^2*b^4*c^2 + 256*a^4*b*c^3 - 25
6*a^3*b^2*c^3 + 64*a^2*b^3*c^3 + 8*a^2*b^4*c - 64*a^3*b^2*c^2 + 64*a^2*b^3*
c^2 + 128*a^4*c^3 - 256*a^3*b*c^3 + 96*a^2*b^2*c^3 + 16*a^2*b^2*c^2 - 64*a^
3*c^3 + 48*a^2*b*c^3 + 8*a^2*c^3 + (2*b^6*c - 40*a*b^4*c^2 + 8*b^5*c^2 + 22
4*a^2*b^2*c^3 - 128*a*b^3*c^3 + 8*b^4*c^3 - 384*a^3*c^4 + 384*a^2*b*c^4 - 9
6*a*b^2*c^4 - b^5*c + 8*a*b^3*c^2 - 16*a^2*b*c^3 - 48*a*b^2*c^3 + 4*b^3*c^3
 + 192*a^2*c^4 - 96*a*b*c^4 - 2*b^3*c^2 + 8*a*b*c^3 - 2*b^2*c^3 - 24*a*c^4
- b*c^3)*sqrt(2*b*c + c)*d + 2*(4*a*b^5*c - 32*a^2*b^3*c^2 + 16*a*b^4*c^2 +
 64*a^3*b*c^3 - 64*a^2*b^2*c^3 + 16*a*b^3*c^3 + a*b^4*c - 8*a^2*b^2*c^2 + 1
2*a*b^3*c^2 + 16*a^3*c^3 - 48*a^2*b*c^3 + 20*a*b^2*c^3 + 2*a*b^2*c^2 - 8*a^
2*c^3 + 8*a*b*c^3 + a*c^3)*sqrt(2*b*c + c)*f - (2*a*b^6 - 8*a^2*b^4*c + 8*a
*b^5*c - 32*a^3*b^2*c^2 + 8*a*b^4*c^2 + 128*a^4*c^3 - 128*a^3*b*c^3 + 32*a^
2*b^2*c^3 + a*b^5 - 8*a^2*b^3*c + 8*a*b^4*c + 16*a^3*b*c^2 - 16*a^2*b^2*c^2
 + 12*a*b^3*c^2 - 64*a^3*c^3 + 32*a^2*b*c^3 + 2*a*b^3*c - 8*a^2*b*c^2 + 6*a
*b^2*c^2 + 8*a^2*c^3 + a*b*c^2)*sqrt(2*b*c + c)*h + 4*(2*a*b^6*c*i - 16*a^2
*b^4*c^2*i + 8*a*b^5*c^2*i + 32*a^3*b^2*c^3*i - 32*a^2*b^3*c^3*i + 8*a*b^4*
c^3*i + a*b^5*c*i - 8*a^2*b^3*c^2*i + 8*a*b^4*c^2*i + 16*a^3*b*c^3*i - 32*a
^2*b^2*c^3*i + 12*a*b^3*c^3*i + 2*a*b^3*c^2*i - 8*a^2*b*c^3*i + 6*a*b^2*c^3
```

```
*i + a*b*c^3*i)*g - 8*(2*a*b^5*c^2*i - 16*a^2*b^3*c^3*i + 8*a*b^4*c^3*i + 3
2*a^3*b*c^4*i - 32*a^2*b^2*c^4*i + 8*a*b^3*c^4*i + a*b^4*c^2*i - 8*a^2*b^2*
c^3*i + 8*a*b^3*c^3*i + 16*a^3*c^4*i - 32*a^2*b*c^4*i + 12*a*b^2*c^4*i + 2*
a*b^2*c^3*i - 8*a^2*c^4*i + 6*a*b*c^4*i + a*c^4*i)*e)*log(x + 1/2*sqrt(-(2*
a*b^3*i - 8*a^2*b*c*i + sqrt(-4*(a*b^3 - 4*a^2*b*c)^2 + 16*(a^2*b^2 - 4*a^3
*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c*i - 4*a^2*c^2*i)))/(a*b^8*c*i - 16*a^2
*b^6*c^2*i + 4*a*b^7*c^2*i + 96*a^3*b^4*c^3*i - 48*a^2*b^5*c^3*i + 4*a*b^6*
c^3*i - 256*a^4*b^2*c^4*i + 192*a^3*b^3*c^4*i - 32*a^2*b^4*c^4*i + 256*a^5*
c^5*i - 256*a^4*b*c^5*i + 64*a^3*b^2*c^5*i + 2*a*b^6*c^2*i - 24*a^2*b^4*c^3
*i + 4*a*b^5*c^3*i + 96*a^3*b^2*c^4*i - 32*a^2*b^3*c^4*i - 128*a^4*c^5*i +
64*a^3*b*c^5*i + a*b^4*c^3*i - 8*a^2*b^2*c^4*i + 16*a^3*c^5*i - (a*b^7*c -
12*a^2*b^5*c^2 + 4*a*b^6*c^2 + 48*a^3*b^3*c^3 - 32*a^2*b^4*c^3 + 4*a*b^5*c^
3 - 64*a^4*b*c^4 + 64*a^3*b^2*c^4 - 16*a^2*b^3*c^4 + 2*a*b^5*c^2 - 16*a^2*b
^3*c^3 + 4*a*b^4*c^3 + 32*a^3*b*c^4 - 16*a^2*b^2*c^4 + a*b^3*c^3 - 4*a^2*b*
c^4)*sqrt(-b^2 + 4*a*c)) - 1/16*(16*a^2*b^5*c - 128*a^3*b^3*c^2 + 64*a^2*b^
4*c^2 + 256*a^4*b*c^3 - 256*a^3*b^2*c^3 + 64*a^2*b^3*c^3 + 8*a^2*b^4*c - 64
*a^3*b^2*c^2 + 64*a^2*b^3*c^2 + 128*a^4*c^3 - 256*a^3*b*c^3 + 96*a^2*b^2*c^
3 + 16*a^2*b^2*c^2 - 64*a^3*c^3 + 48*a^2*b*c^3 + 8*a^2*c^3 - (2*b^6*c - 40*
a*b^4*c^2 + 8*b^5*c^2 + 224*a^2*b^2*c^3 - 128*a*b^3*c^3 + 8*b^4*c^3 - 384*a
^3*c^4 + 384*a^2*b*c^4 - 96*a*b^2*c^4 - b^5*c + 8*a*b^3*c^2 - 16*a^2*b*c^3
- 48*a*b^2*c^3 + 4*b^3*c^3 + 192*a^2*c^4 - 96*a*b*c^4 - 2*b^3*c^2 + 8*a*b*c
^3 - 2*b^2*c^3 - 24*a*c^4 - b*c^3)*sqrt(2*b*c + c)*d - 2*(4*a*b^5*c - 32*a^
2*b^3*c^2 + 16*a*b^4*c^2 + 64*a^3*b*c^3 - 64*a^2*b^2*c^3 + 16*a*b^3*c^3 + a
*b^4*c - 8*a^2*b^2*c^2 + 12*a*b^3*c^2 + 16*a^3*c^3 - 48*a^2*b*c^3 + 20*a*b^
2*c^3 + 2*a*b^2*c^2 - 8*a^2*c^3 + 8*a*b*c^3 + a*c^3)*sqrt(2*b*c + c)*f + (2
*a*b^6 - 8*a^2*b^4*c + 8*a*b^5*c - 32*a^3*b^2*c^2 + 8*a*b^4*c^2 + 128*a^4*c
^3 - 128*a^3*b*c^3 + 32*a^2*b^2*c^3 + a*b^5 - 8*a^2*b^3*c + 8*a*b^4*c + 16*
a^3*b*c^2 - 16*a^2*b^2*c^2 + 12*a*b^3*c^2 - 64*a^3*c^3 + 32*a^2*b*c^3 + 2*a
*b^3*c - 8*a^2*b*c^2 + 6*a*b^2*c^2 + 8*a^2*c^3 + a*b*c^2)*sqrt(2*b*c + c)*h
 + 4*(2*a*b^6*c*i - 16*a^2*b^4*c^2*i + 8*a*b^5*c^2*i + 32*a^3*b^2*c^3*i - 3
2*a^2*b^3*c^3*i + 8*a*b^4*c^3*i + a*b^5*c*i - 8*a^2*b^3*c^2*i + 8*a*b^4*c^2
*i + 16*a^3*b*c^3*i - 32*a^2*b^2*c^3*i + 12*a*b^3*c^3*i + 2*a*b^3*c^2*i - 8
*a^2*b*c^3*i + 6*a*b^2*c^3*i + a*b*c^3*i)*g - 8*(2*a*b^5*c^2*i - 16*a^2*b^3
*c^3*i + 8*a*b^4*c^3*i + 32*a^3*b*c^4*i - 32*a^2*b^2*c^4*i + 8*a*b^3*c^4*i
+ a*b^4*c^2*i - 8*a^2*b^2*c^3*i + 8*a*b^3*c^3*i + 16*a^3*c^4*i - 32*a^2*b*c
^4*i + 12*a*b^2*c^4*i + 2*a*b^2*c^3*i - 8*a^2*c^4*i + 6*a*b*c^4*i + a*c^4*i
)*e)*log(x - 1/2*sqrt(-(2*a*b^3*i - 8*a^2*b*c*i + sqrt(-4*(a*b^3 - 4*a^2*b*
c)^2 + 16*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c*i - 4*a^2*c^
2*i)))/(a*b^8*c*i - 16*a^2*b^6*c^2*i + 4*a*b^7*c^2*i + 96*a^3*b^4*c^3*i - 4
8*a^2*b^5*c^3*i + 4*a*b^6*c^3*i - 256*a^4*b^2*c^4*i + 192*a^3*b^3*c^4*i - 3
2*a^2*b^4*c^4*i + 256*a^5*c^5*i - 256*a^4*b*c^5*i + 64*a^3*b^2*c^5*i + 2*a*
b^6*c^2*i - 24*a^2*b^4*c^3*i + 4*a*b^5*c^3*i + 96*a^3*b^2*c^4*i - 32*a^2*b^
3*c^4*i - 128*a^4*c^5*i + 64*a^3*b*c^5*i + a*b^4*c^3*i - 8*a^2*b^2*c^4*i +
16*a^3*c^5*i - (a*b^7*c - 12*a^2*b^5*c^2 + 4*a*b^6*c^2 + 48*a^3*b^3*c^3 - 3
2*a^2*b^4*c^3 + 4*a*b^5*c^3 - 64*a^4*b*c^4 + 64*a^3*b^2*c^4 - 16*a^2*b^3*c^
```

```
4 + 2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 4*a*b^4*c^3 + 32*a^3*b*c^4 - 16*a^2*b^2*
c^4 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(-b^2 + 4*a*c)) - 1/16*(16*a^2*b^5*c - 1
28*a^3*b^3*c^2 + 64*a^2*b^4*c^2 + 256*a^4*b*c^3 - 256*a^3*b^2*c^3 + 64*a^2*
b^3*c^3 - 8*a^2*b^4*c + 64*a^3*b^2*c^2 - 64*a^2*b^3*c^2 - 128*a^4*c^3 + 256
*a^3*b*c^3 - 96*a^2*b^2*c^3 + 16*a^2*b^2*c^2 - 64*a^3*c^3 + 48*a^2*b*c^3 -
8*a^2*c^3 - (2*b^6*c - 40*a*b^4*c^2 + 8*b^5*c^2 + 224*a^2*b^2*c^3 - 128*a*b
^3*c^3 + 8*b^4*c^3 - 384*a^3*c^4 + 384*a^2*b*c^4 - 96*a*b^2*c^4 + b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3 + 48*a*b^2*c^3 - 4*b^3*c^3 - 192*a^2*c^4 + 96*a*b
*c^4 - 2*b^3*c^2 + 8*a*b*c^3 - 2*b^2*c^3 - 24*a*c^4 + b*c^3)*sqrt(2*b*c - c
)*d - 2*(4*a*b^5*c - 32*a^2*b^3*c^2 + 16*a*b^4*c^2 + 64*a^3*b*c^3 - 64*a^2*
b^2*c^3 + 16*a*b^3*c^3 - a*b^4*c + 8*a^2*b^2*c^2 - 12*a*b^3*c^2 - 16*a^3*c^
3 + 48*a^2*b*c^3 - 20*a*b^2*c^3 + 2*a*b^2*c^2 - 8*a^2*c^3 + 8*a*b*c^3 - a*c
^3)*sqrt(2*b*c - c)*f + (2*a*b^6 - 8*a^2*b^4*c + 8*a*b^5*c - 32*a^3*b^2*c^2
 + 8*a*b^4*c^2 + 128*a^4*c^3 - 128*a^3*b*c^3 + 32*a^2*b^2*c^3 - a*b^5 + 8*a
^2*b^3*c - 8*a*b^4*c - 16*a^3*b*c^2 + 16*a^2*b^2*c^2 - 12*a*b^3*c^2 + 64*a^
3*c^3 - 32*a^2*b*c^3 + 2*a*b^3*c - 8*a^2*b*c^2 + 6*a*b^2*c^2 + 8*a^2*c^3 -
a*b*c^2)*sqrt(2*b*c - c)*h + 4*(2*a*b^6*c*i - 16*a^2*b^4*c^2*i + 8*a*b^5*c^
2*i + 32*a^3*b^2*c^3*i - 32*a^2*b^3*c^3*i + 8*a*b^4*c^3*i - a*b^5*c*i + 8*a
^2*b^3*c^2*i - 8*a*b^4*c^2*i - 16*a^3*b*c^3*i + 32*a^2*b^2*c^3*i - 12*a*b^3
*c^3*i + 2*a*b^3*c^2*i - 8*a^2*b*c^3*i + 6*a*b^2*c^3*i - a*b*c^3*i)*g - 8*(
2*a*b^5*c^2*i - 16*a^2*b^3*c^3*i + 8*a*b^4*c^3*i + 32*a^3*b*c^4*i - 32*a^2*
b^2*c^4*i + 8*a*b^3*c^4*i - a*b^4*c^2*i + 8*a^2*b^2*c^3*i - 8*a*b^3*c^3*i -
 16*a^3*c^4*i + 32*a^2*b*c^4*i - 12*a*b^2*c^4*i + 2*a*b^2*c^3*i - 8*a^2*c^4
*i + 6*a*b*c^4*i - a*c^4*i)*e)*log(x + 1/2*sqrt(-(2*a*b^3*i - 8*a^2*b*c*i -
 sqrt(-4*(a*b^3 - 4*a^2*b*c)^2 + 16*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^
2)))/(a*b^2*c*i - 4*a^2*c^2*i)))/(a*b^8*c*i - 16*a^2*b^6*c^2*i + 4*a*b^7*c^
2*i + 96*a^3*b^4*c^3*i - 48*a^2*b^5*c^3*i + 4*a*b^6*c^3*i - 256*a^4*b^2*c^4
*i + 192*a^3*b^3*c^4*i - 32*a^2*b^4*c^4*i + 256*a^5*c^5*i - 256*a^4*b*c^5*i
 + 64*a^3*b^2*c^5*i - 2*a*b^6*c^2*i + 24*a^2*b^4*c^3*i - 4*a*b^5*c^3*i - 96
*a^3*b^2*c^4*i + 32*a^2*b^3*c^4*i + 128*a^4*c^5*i - 64*a^3*b*c^5*i + a*b^4*
c^3*i - 8*a^2*b^2*c^4*i + 16*a^3*c^5*i + (a*b^7*c - 12*a^2*b^5*c^2 + 4*a*b^
6*c^2 + 48*a^3*b^3*c^3 - 32*a^2*b^4*c^3 + 4*a*b^5*c^3 - 64*a^4*b*c^4 + 64*a
^3*b^2*c^4 - 16*a^2*b^3*c^4 - 2*a*b^5*c^2 + 16*a^2*b^3*c^3 - 4*a*b^4*c^3 -
32*a^3*b*c^4 + 16*a^2*b^2*c^4 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(-b^2 + 4*a*c)
) - 1/16*(16*a^2*b^5*c - 128*a^3*b^3*c^2 + 64*a^2*b^4*c^2 + 256*a^4*b*c^3 -
 256*a^3*b^2*c^3 + 64*a^2*b^3*c^3 - 8*a^2*b^4*c + 64*a^3*b^2*c^2 - 64*a^2*b
^3*c^2 - 128*a^4*c^3 + 256*a^3*b*c^3 - 96*a^2*b^2*c^3 + 16*a^2*b^2*c^2 - 64
*a^3*c^3 + 48*a^2*b*c^3 - 8*a^2*c^3 + (2*b^6*c - 40*a*b^4*c^2 + 8*b^5*c^2 +
 224*a^2*b^2*c^3 - 128*a*b^3*c^3 + 8*b^4*c^3 - 384*a^3*c^4 + 384*a^2*b*c^4
- 96*a*b^2*c^4 + b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 48*a*b^2*c^3 - 4*b^3*
c^3 - 192*a^2*c^4 + 96*a*b*c^4 - 2*b^3*c^2 + 8*a*b*c^3 - 2*b^2*c^3 - 24*a*c
^4 + b*c^3)*sqrt(2*b*c - c)*d + 2*(4*a*b^5*c - 32*a^2*b^3*c^2 + 16*a*b^4*c^
2 + 64*a^3*b*c^3 - 64*a^2*b^2*c^3 + 16*a*b^3*c^3 - a*b^4*c + 8*a^2*b^2*c^2
- 12*a*b^3*c^2 - 16*a^3*c^3 + 48*a^2*b*c^3 - 20*a*b^2*c^3 + 2*a*b^2*c^2 - 8
*a^2*c^3 + 8*a*b*c^3 - a*c^3)*sqrt(2*b*c - c)*f - (2*a*b^6 - 8*a^2*b^4*c +
```

```
8*a*b^5*c - 32*a^3*b^2*c^2 + 8*a*b^4*c^2 + 128*a^4*c^3 - 128*a^3*b*c^3 + 32
*a^2*b^2*c^3 - a*b^5 + 8*a^2*b^3*c - 8*a*b^4*c - 16*a^3*b*c^2 + 16*a^2*b^2*
c^2 - 12*a*b^3*c^2 + 64*a^3*c^3 - 32*a^2*b*c^3 + 2*a*b^3*c - 8*a^2*b*c^2 +
6*a*b^2*c^2 + 8*a^2*c^3 - a*b*c^2)*sqrt(2*b*c - c)*h + 4*(2*a*b^6*c*i - 16*
a^2*b^4*c^2*i + 8*a*b^5*c^2*i + 32*a^3*b^2*c^3*i - 32*a^2*b^3*c^3*i + 8*a*b
^4*c^3*i - a*b^5*c*i + 8*a^2*b^3*c^2*i - 8*a*b^4*c^2*i - 16*a^3*b*c^3*i + 3
2*a^2*b^2*c^3*i - 12*a*b^3*c^3*i + 2*a*b^3*c^2*i - 8*a^2*b*c^3*i + 6*a*b^2*
c^3*i - a*b*c^3*i)*g - 8*(2*a*b^5*c^2*i - 16*a^2*b^3*c^3*i + 8*a*b^4*c^3*i
+ 32*a^3*b*c^4*i - 32*a^2*b^2*c^4*i + 8*a*b^3*c^4*i - a*b^4*c^2*i + 8*a^2*b
^2*c^3*i - 8*a*b^3*c^3*i - 16*a^3*c^4*i + 32*a^2*b*c^4*i - 12*a*b^2*c^4*i +
 2*a*b^2*c^3*i - 8*a^2*c^4*i + 6*a*b*c^4*i - a*c^4*i)*e)*log(x - 1/2*sqrt(-
(2*a*b^3*i - 8*a^2*b*c*i - sqrt(-4*(a*b^3 - 4*a^2*b*c)^2 + 16*(a^2*b^2 - 4*
a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c*i - 4*a^2*c^2*i)))/(a*b^8*c*i - 16*
a^2*b^6*c^2*i + 4*a*b^7*c^2*i + 96*a^3*b^4*c^3*i - 48*a^2*b^5*c^3*i + 4*a*b
^6*c^3*i - 256*a^4*b^2*c^4*i + 192*a^3*b^3*c^4*i - 32*a^2*b^4*c^4*i + 256*a
^5*c^5*i - 256*a^4*b*c^5*i + 64*a^3*b^2*c^5*i - 2*a*b^6*c^2*i + 24*a^2*b^4*
c^3*i - 4*a*b^5*c^3*i - 96*a^3*b^2*c^4*i + 32*a^2*b^3*c^4*i + 128*a^4*c^5*i
 - 64*a^3*b*c^5*i + a*b^4*c^3*i - 8*a^2*b^2*c^4*i + 16*a^3*c^5*i + (a*b^7*c
 - 12*a^2*b^5*c^2 + 4*a*b^6*c^2 + 48*a^3*b^3*c^3 - 32*a^2*b^4*c^3 + 4*a*b^5
*c^3 - 64*a^4*b*c^4 + 64*a^3*b^2*c^4 - 16*a^2*b^3*c^4 - 2*a*b^5*c^2 + 16*a^
2*b^3*c^3 - 4*a*b^4*c^3 - 32*a^3*b*c^4 + 16*a^2*b^2*c^4 + a*b^3*c^3 - 4*a^2
*b*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*c^2*d*i*x^3 - 2*a*c^2*f*i*x^3 + a*b*c*
h*i*x^3 + a*b*c*g*i*x^2 - 2*a*c^2*i*x^2*e + b^2*c*d*i*x - 2*a*c^2*d*i*x - a
*b*c*f*i*x + 2*a^2*c*h*i*x + 2*a^2*c*g*i + a*b^2*x^2 - 2*a^2*c*x^2 - a*b*c*
i*e + a^2*b)/((a*b^2*c*i - 4*a^2*c^2*i)*(c*x^4 + b*x^2 + a))
```

3.41 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=770

$$-\frac{x\left(x^2\left(-bc\left(-3a^2m+ach+c^2d\right)+ab^2ck-ab^3m+2ac^2(cf-ak)\right)+b^2\left(-\left(a^2m+c^2d\right)\right)+2ac\left(a^2m-ach+c^2d\right)+abc(a\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (m*x)/c^2 - (b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2
*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*
d - a*c*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*
d + a*c*h - 3*a^2*m))*x^2))/(2*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^
2*m) - (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*
c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2
)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*k - 2*a*c^2*(c*f +
 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m) + (a*b^3*c*k - 4*a*b*c
^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(
3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*j) + b^3*l - 6*a*b*c*l)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (l*Lo
g[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 7.83472, antiderivative size = 770, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1673, 1678, 1676, 1166, 205, 1663, 1660, 634, 618, 206, 628}

$$-\frac{x\left(x^2\left(-bc\left(-3a^2m+ach+c^2d\right)+ab^2ck-ab^3m+2ac^2(cf-ak)\right)+b^2\left(-\left(a^2m+c^2d\right)\right)+2ac\left(a^2m-ach+c^2d\right)+abc(a\right.}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4)^2,x]
```

```
[Out] (m*x)/c^2 - (b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2
*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*
d - a*c*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*
d + a*c*h - 3*a^2*m))*x^2))/(2*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^
2*m) - (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*
c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2
)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*k - 2*a*c^2*(c*f +
 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m) + (a*b^3*c*k - 4*a*b*c
^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(
3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*j) + b^3*l - 6*a*b*c*l)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (l*Lo
g[a + b*x^2 + c*x^4])/(4*c^2)
```

<u>Rule 1673</u>

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

<u>Rule 1678</u>

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

<u>Rule 1676</u>

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x\left(e+gx^2+jx^4+lx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4+kx^6+m}{\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(abc(cf+ak)-b^2\left(c^2d+a^2m\right)+2ac\left(c^2d-ach+a^2m\right)}{2ac^2\left(b^2-4a\right.}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 6.56622, size = 935, normalized size = 1.21

$$4\sqrt{c}mx + \frac{2\sqrt{c}\left(2c(l+mx)a^3-\left((l+mx)b^2-c(j+x(k+3x(l+mx)))b+2c^2(g+x(h+x(j+kx)))\right)a^2+\left(-x^2(l+mx)b^3+cx^2(j+kx)b^2+c^2(e+x(f-x(g+hx)))b+2c^3x(d+x(e+fx\right.}{a\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*m*x + (2*Sqrt[c]*(2*a^3*c*(l + m*x) - b*c^2*d*x*(b + c*x^2) + a*
(b^2*c*x^2*(j + k*x) - b^3*x^2*(l + m*x) + 2*c^3*x*(d + x*(e + f*x)) + b*c^
2*(e + x*(f - x*(g + h*x)))) - a^2*(b^2*(l + m*x) + 2*c^2*(g + x*(h + x*(j
+ k*x))) - b*c*(j + x*(k + 3*x*(l + m*x))))))/(a*(-b^2 + 4*a*c)*(a + b*x^2
+ c*x^4)) - (Sqrt[2]*(-3*a*b^4*m + 2*a*c^2*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f
 + 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*k - 10*a^2*m) + a*b^3*(c*k + 3*Sqrt[b^2
- 4*a*c]*m) - b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) + a*c*(Sqrt[b^2 - 4*a*
c]*h + 8*a*k) + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(-(c^2*d) + a*c*h + a*(
-(Sqrt[b^2 - 4*a*c]*k) + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(3*a*b^4*m + 2*a*c^2*(-6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f - 2*a*c*h + 3*a
*Sqrt[b^2 - 4*a*c]*k + 10*a^2*m) + a*b^3*(-(c*k) + 3*Sqrt[b^2 - 4*a*c]*m) -
 b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d - 4*a*f) + a*c*(Sqrt[b^2 - 4*a*c]*h - 8*a*k)
 + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(c^2*d - a*c*h - a*(Sqrt[b^2 - 4*a*c
]*k + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-4*c^3*e + 2*
c^2*(b*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + a*c*(6*b*l - 4*Sqrt[b^
2 - 4*a*c]*l))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) +
 (Sqrt[c]*(4*c^3*e + c^2*(-2*b*g + 4*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c])*l -
 2*a*c*(3*b + 2*Sqrt[b^2 - 4*a*c])*l)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(b^2 - 4*a*c)^(3/2))/(4*c^(5/2))
```

Maple [B] time = 0.07, size = 4570, normalized size = 5.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*
b*f-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f+19/4/c*a/(4*a
```

```
*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*m+19/4/c*a/(4*a*c-b^2)^
2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^
2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*m-1/4*c/(4*a*c-b^2)^2/a*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*(-4*a*c+b^2)^(1/2)*b^2*d+a/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+
b^2)^(1/2)*h-a/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+a/(4*a*c-b^2)^2*c*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h+a/(4*a*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))
*b*h+m*x/c^2+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-c^
2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+3*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d-a/(4*a*c
-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*j+a/(4*a*c-b^2
)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*j+c/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x^3*f+c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*d+1/c/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*a^2*l+c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*e-1/2/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*x^2*b*g-1/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*h-1/2/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*x^3*b*h+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*f-1/(c*x^4+b*x^2+a)*a
/(4*a*c-b^2)*x^3*k-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*j+1/4/c^2/(4*a*c-b^2
)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^4*l+1/4/c^2/(4*a*c-b^2)^2*ln(-2*c*x^
2+(-4*a*c+b^2)^(1/2)-b)*b^4*l+1/2/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)*(-4*a*c+b^2)^(1/2)*b*g-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/
2)+b)*(-4*a*c+b^2)^(1/2)*b*g-1/4/c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*k+
1/4/c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*k+3/2/c/(c*x^4+b*x^2+a)*a/(4*a*c
-b^2)*x^3*b*m-1/2*c/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)*x^3*b*d+3/2/c/(c*x^4+b*x^
2+a)/(4*a*c-b^2)*x^2*a*b*l+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*k-1/2/c^
2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*m+13*a^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b*m+5/2*a/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arct
anh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*k-5*a^2/(4*a*c-b^2)^2
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*m-13*a^2/(4*a*c-b^2)^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b*m-5/2*a/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
```

```
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*k-5*a^2/(4*a*c-b^2)^2
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*m-3/4/c^2/(4*a*c-b^2)^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^5*m-3/2/c*a/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c
+b^2)^(1/2)*b*l+6*c*a^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*k+3/2/c*a/(4*a*c-b
^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b*l-6*c*a^2/(4*a
*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2))*k+3/4/c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b
^5*m+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2)^2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b^2*f-2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-
b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*b^2*h+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2
)*b^2*h+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+
b)-c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-1
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a*g+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*e+4*a^2/
(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*l+4*a^2/(4*a*c-b^2)^2*ln(2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)*l-2*a/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2
)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c
+b^2)^(1/2)*b*k-2*a/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*k
-3/4/c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*m-25/4/c*a
/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*m+25/4/c*a/(4*a*c-b^2)^2*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^3*m+1/4/c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^
3*k-3/4/c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(
c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*m+1/4/
c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*k-1/2/(c*x^4+b*x^
2+a)/a/(4*a*c-b^2)*x*b^2*d+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*m+1/2/c/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*a*b*j+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*k
-2/c*a/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*l-2/c*a/(4*a*c-b
^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*l+1/4/c^2/(4*a*c-b^2)^2*ln(2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b^3*l-1/4/c^2/(4*a*c-b^2)^2*ln(
```

```
-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b^3*l-1/2/c^2/(c*x^4+b*x^
2+a)/(4*a*c-b^2)*x^3*b^3*m-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a*b^2*l-1/2/
c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*l+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*x^2*b^2*j-1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*h+1/4/(4*a*c-b^2)^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*h
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc^2e - 2\,a^2c^2g + a^2bcj - \left(bc^3d - 2\,ac^3f + abc^2h - \left(ab^2c - 2\,a^2c^2\right)k + \left(ab^3 - 3\,a^2bc\right)m\right)x^3 + \left(2\,ac^3e - abc^2g + \left(ab^2c - \right.\right.}{2\left(a^2b^2c^2 - 4\,a^3c^3 + \left(ab^2c^3 - 4\,a^2c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - (b*c^3*d - 2*a*c^3*f + a*b*c^2*
h - (a*b^2*c - 2*a^2*c^2)*k + (a*b^3 - 3*a^2*b*c)*m)*x^3 + (2*a*c^3*e - a*b
*c^2*g + (a*b^2*c - 2*a^2*c^2)*j - (a*b^3 - 3*a^2*b*c)*l)*x^2 - (a^2*b^2 -
2*a^3*c)*l + (a*b*c^2*f - 2*a^2*c^2*h + a^2*b*c*k - (b^2*c^2 - 2*a*c^3)*d -
 (a^2*b^2 - 2*a^3*c)*m)*x)/(a^2*b^2*c^2 - 4*a^3*c^3 + (a*b^2*c^3 - 4*a^2*c^
4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) + m*x/c^2 - 1/2*integrate(-(a*b*c^2
*f - 2*a^2*c^2*h + a^2*b*c*k + 2*(a*b^2*c - 4*a^2*c^2)*l*x^3 + (b*c^3*d - 2
*a*c^3*f + a*b*c^2*h + (a*b^2*c - 6*a^2*c^2)*k - (3*a*b^3 - 13*a^2*b*c)*m)*
x^2 + (b^2*c^2 - 6*a*c^3)*d - (3*a^2*b^2 - 10*a^3*c)*m - 2*(2*a*c^3*e - a*b
*c^2*g + 2*a^2*c^2*j - a^2*b*c*l)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c^2 - 4
*a^2*c^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.42 $\int \frac{d+ex}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{dx(59-35x^2)}{3456(x^4-5x^2+4)}+\frac{dx(17-5x^2)}{144(x^4-5x^2+4)^2}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}d\tanh^{-1}(x)-\frac{e(5-2x^2)}{54(x^4-5x^2+4)}+\frac{e(5-2x}{36(x^4-5x^2}$$

```
[Out] (d*x*(17 - 5*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (e*(5 - 2*x^2))/(36*(4 - 5*x
^2 + x^4)^2) - (d*x*(59 - 35*x^2))/(3456*(4 - 5*x^2 + x^4)) - (e*(5 - 2*x^2
))/(54*(4 - 5*x^2 + x^4)) - (313*d*ArcTanh[x/2])/20736 + (13*d*ArcTanh[x])/
648 - (e*Log[1 - x^2])/81 + (e*Log[4 - x^2])/81
```

Rubi [A] time = 0.0758338, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1673, 12, 1092, 1178, 1166, 207, 1107, 614, 616, 31}

$$-\frac{dx(59-35x^2)}{3456(x^4-5x^2+4)}+\frac{dx(17-5x^2)}{144(x^4-5x^2+4)^2}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}d\tanh^{-1}(x)-\frac{e(5-2x^2)}{54(x^4-5x^2+4)}+\frac{e(5-2x}{36(x^4-5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (e*(5 - 2*x^2))/(36*(4 - 5*x
^2 + x^4)^2) - (d*x*(59 - 35*x^2))/(3456*(4 - 5*x^2 + x^4)) - (e*(5 - 2*x^2
))/(54*(4 - 5*x^2 + x^4)) - (313*d*ArcTanh[x/2])/20736 + (13*d*ArcTanh[x])/
648 - (e*Log[1 - x^2])/81 + (e*Log[4 - x^2])/81
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(4-5x^2+x^4\right)^3}\,dx = \int \frac{d}{\left(4-5x^2+x^4\right)^3}\,dx + \int \frac{ex}{\left(4-5x^2+x^4\right)^3}\,dx$$

$$= d\int \frac{1}{\left(4-5x^2+x^4\right)^3}\,dx + e\int \frac{x}{\left(4-5x^2+x^4\right)^3}\,dx$$

$$= \frac{dx\left(17-5x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{1}{144}d\int \frac{-19+25x^2}{\left(4-5x^2+x^4\right)^2}\,dx + \frac{1}{2}e\,\mathrm{Subst}\left(\int \frac{1}{\left(4-5x+x^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{dx\left(17-5x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2} - \frac{dx\left(59-35x^2\right)}{3456\left(4-5x^2+x^4\right)} + \frac{d\int \frac{519+105x^2}{4-5x^2+x^4}\,dx}{10368} - \frac{1}{6}e\,\mathrm{Subs}$$

$$= \frac{dx\left(17-5x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2} - \frac{dx\left(59-35x^2\right)}{3456\left(4-5x^2+x^4\right)} - \frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)} - \frac{1}{648}(13$$

$$= \frac{dx\left(17-5x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2} - \frac{dx\left(59-35x^2\right)}{3456\left(4-5x^2+x^4\right)} - \frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)} - \frac{313d\,\mathrm{t}}{2}$$

$$= \frac{dx\left(17-5x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2} - \frac{dx\left(59-35x^2\right)}{3456\left(4-5x^2+x^4\right)} - \frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)} - \frac{313d\,\mathrm{t}}{2}$$

Mathematica [A] time = 0.098161, size = 128, normalized size = 0.9

$$\frac{\frac{288\left(dx\left(17-5x^2\right)+e\left(20-8x^2\right)\right)}{\left(x^4-5x^2+4\right)^2}+\frac{12\left(dx\left(35x^2-59\right)+64e\left(2x^2-5\right)\right)}{x^4-5x^2+4}-32(13d+16e)\log(1-x)+(313d+512e)\log(2-x)+32(13d-16e)\log$$
$$41472$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4)^2 + (12*(64*e*
(-5 + 2*x^2) + d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e)*Lo
g[1 - x] + (313*d + 512*e)*Log[2 - x] + 32*(13*d - 16*e)*Log[1 + x] + (-313
*d + 512*e)*Log[2 + x])/41472
```

Maple [A] time = 0.019, size = 186, normalized size = 1.3

$$-\frac{313\ln(2+x)d}{41472}+\frac{\ln(2+x)e}{81}+\frac{19d}{13824+6912x}-\frac{17e}{6912+3456x}+\frac{d}{3456(2+x)^2}-\frac{e}{1728(2+x)^2}+\frac{d}{432+432x}-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+19/6912/(2+x)*d-17/3456/(2+x)*e+1/3456/
(2+x)^2*d-1/1728/(2+x)^2*e+1/432/(1+x)*d-1/144/(1+x)*e-1/432/(1+x)^2*d+1/43
2/(1+x)^2*e+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+19/6912/(x-2)*d+17/3456/(x-2)*
e-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+313/41472*ln(x-2)*d+1/81*ln(x-2)*e-13/1
296*ln(x-1)*d-1/81*ln(x-1)*e+1/432/(x-1)*d+1/144/(x-1)*e+1/432/(x-1)^2*d+1/
432/(x-1)^2*e
```

Maxima [A] time = 0.940788, size = 163, normalized size = 1.14

$$-\frac{1}{41472}(313d-512e)\log(x+2)+\frac{1}{1296}(13d-16e)\log(x+1)-\frac{1}{1296}(13d+16e)\log(x-1)+\frac{1}{41472}(313d+512e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e)*log(x + 2) + 1/1296*(13*d - 16*e)*log(x + 1) - 1/1
296*(13*d + 16*e)*log(x - 1) + 1/41472*(313*d + 512*e)*log(x - 2) + 1/3456*
(35*d*x^7 + 128*e*x^6 - 234*d*x^5 - 960*e*x^4 + 315*d*x^3 + 1920*e*x^2 + 17
2*d*x - 800*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] time = 2.07694, size = 869, normalized size = 6.08

$$\frac{420\,dx^7 + 1536\,ex^6 - 2808\,dx^5 - 11520\,ex^4 + 3780\,dx^3 + 23040\,ex^2 + 2064\,dx - \left((313\,d - 512\,e)x^8 - 10\,(313\,d - 512\,e)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*d*x^7 + 1536*e*x^6 - 2808*d*x^5 - 11520*e*x^4 + 3780*d*x^3 + 2
3040*e*x^2 + 2064*d*x - ((313*d - 512*e)*x^8 - 10*(313*d - 512*e)*x^6 + 33*
(313*d - 512*e)*x^4 - 40*(313*d - 512*e)*x^2 + 5008*d - 8192*e)*log(x + 2)
+ 32*((13*d - 16*e)*x^8 - 10*(13*d - 16*e)*x^6 + 33*(13*d - 16*e)*x^4 - 40*
(13*d - 16*e)*x^2 + 208*d - 256*e)*log(x + 1) - 32*((13*d + 16*e)*x^8 - 10*
(13*d + 16*e)*x^6 + 33*(13*d + 16*e)*x^4 - 40*(13*d + 16*e)*x^2 + 208*d + 2
56*e)*log(x - 1) + ((313*d + 512*e)*x^8 - 10*(313*d + 512*e)*x^6 + 33*(313*
d + 512*e)*x^4 - 40*(313*d + 512*e)*x^2 + 5008*d + 8192*e)*log(x - 2) - 960
0*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [B] time = 2.79382, size = 668, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] (13*d - 16*e)*log(x + (-1106258459719280*d**4*e - 13113710954343*d**4*(13*d
 - 16*e) - 817263343042560*d**2*e**3 + 153628968222720*d**2*e**2*(13*d - 16
*e) + 9530197557248*d**2*e*(13*d - 16*e)**2 + 88038005760*d**2*(13*d - 16*e
)**3 + 5035763255214080*e**5 + 142661633703936*e**4*(13*d - 16*e) - 1967095
0215680*e**3*(13*d - 16*e)**2 - 557272006656*e**2*(13*d - 16*e)**3)/(229412
56248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1296
 - (13*d + 16*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d**4*(1
```

```
3*d + 16*e) - 817263343042560*d**2*e**3 - 153628968222720*d**2*e**2*(13*d +
 16*e) + 9530197557248*d**2*e*(13*d + 16*e)**2 - 88038005760*d**2*(13*d + 1
6*e)**3 + 5035763255214080*e**5 - 142661633703936*e**4*(13*d + 16*e) - 1967
0950215680*e**3*(13*d + 16*e)**2 + 557272006656*e**2*(13*d + 16*e)**3)/(229
41256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1
296 - (313*d - 512*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d*
*4*(313*d - 512*e)/32 - 817263343042560*d**2*e**3 - 4800905256960*d**2*e**2
*(313*d - 512*e) + 9306833552*d**2*e*(313*d - 512*e)**2 - 85974615*d**2*(31
3*d - 512*e)**3/32 + 5035763255214080*e**5 - 4458176053248*e**4*(313*d - 51
2*e) - 19209912320*e**3*(313*d - 512*e)**2 + 17006592*e**2*(313*d - 512*e)*
*3)/(22941256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*
e**4))/41472 + (313*d + 512*e)*log(x + (-1106258459719280*d**4*e - 13113710
954343*d**4*(313*d + 512*e)/32 - 817263343042560*d**2*e**3 + 4800905256960*
d**2*e**2*(313*d + 512*e) + 9306833552*d**2*e*(313*d + 512*e)**2 + 85974615
*d**2*(313*d + 512*e)**3/32 + 5035763255214080*e**5 + 4458176053248*e**4*(3
13*d + 512*e) - 19209912320*e**3*(313*d + 512*e)**2 - 17006592*e**2*(313*d
+ 512*e)**3)/(22941256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813
420544*d*e**4))/41472 + (35*d*x**7 - 234*d*x**5 + 315*d*x**3 + 172*d*x + 12
8*e*x**6 - 960*e*x**4 + 1920*e*x**2 - 800*e)/(3456*x**8 - 34560*x**6 + 1140
48*x**4 - 138240*x**2 + 55296)
```

Giac [A] time = 1.13233, size = 166, normalized size = 1.16

$$-\frac{1}{41472}\,(313\,d-512\,e)\log\left(|x+2|\right)+\frac{1}{1296}\,(13\,d-16\,e)\log\left(|x+1|\right)-\frac{1}{1296}\,(13\,d+16\,e)\log\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+51$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d - 512*e)*log(abs(x + 2)) + 1/1296*(13*d - 16*e)*log(abs(x +
 1)) - 1/1296*(13*d + 16*e)*log(abs(x - 1)) + 1/41472*(313*d + 512*e)*log(a
bs(x - 2)) + 1/3456*(35*d*x^7 + 128*x^6*e - 234*d*x^5 - 960*x^4*e + 315*d*x
^3 + 1920*x^2*e + 172*d*x - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.43 $\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=175

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\,\mathrm{t}$$

```
[Out] (e*(5 - 2*x^2))/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f - (5*d + 8*f)*x^
2))/(144*(4 - 5*x^2 + x^4)^2) - (e*(5 - 2*x^2))/(54*(4 - 5*x^2 + x^4)) - (x
*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 82
0*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[x])/648 - (e*Log[1 - x^2]
)/81 + (e*Log[4 - x^2])/81
```

Rubi [A] time = 0.223885, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1673, 1178, 1166, 207, 12, 1107, 614, 616, 31}

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\,\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (e*(5 - 2*x^2))/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f - (5*d + 8*f)*x^
2))/(144*(4 - 5*x^2 + x^4)^2) - (e*(5 - 2*x^2))/(54*(4 - 5*x^2 + x^4)) - (x
*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 82
0*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[x])/648 - (e*Log[1 - x^2]
)/81 + (e*Log[4 - x^2])/81
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^3}\,dx &= \int \frac{ex}{\left(4-5x^2+x^4\right)^3}\,dx+\int \frac{d+fx^2}{\left(4-5x^2+x^4\right)^3}\,dx \\
&= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{1}{144}\int \frac{-19d+20f+5(5d+8f)x^2}{\left(4-5x^2+x^4\right)^2}\,dx+e\int \frac{x}{\left(4-5x^2+x^4\right)^3}\,d \\
&= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{x\left(59d+380f-35(d+4f)x^2\right)}{3456\left(4-5x^2+x^4\right)}+\frac{\int \frac{3(173d+260f)+105(d+4f)x^2}{4-5x^2+x^4}\,dx}{10368} \\
&= \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2}+\frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{x\left(59d+380f-35(d+4f)x^2\right)}{3456\left(4-5x^2+x^4\right)}-\frac{1}{6}e\,\mathrm{Sub} \\
&= \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2}+\frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)}-\frac{x\left(59d+380f-35(\right.}{3456\left(4-5x^2\right.} \\
&= \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2}+\frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)}-\frac{x\left(59d+380f-35(\right.}{3456\left(4-5x^2\right.} \\
&= \frac{e\left(5-2x^2\right)}{36\left(4-5x^2+x^4\right)^2}+\frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2}-\frac{e\left(5-2x^2\right)}{54\left(4-5x^2+x^4\right)}-\frac{x\left(59d+380f-35(\right.}{3456\left(4-5x^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.1285, size = 161, normalized size = 0.92

$$\frac{\frac{288\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx\right)}{\left(x^4-5x^2+4\right)^2}+\frac{12\left(dx\left(35x^2-59\right)+64e\left(2x^2-5\right)+20fx\left(7x^2-19\right)\right)}{x^4-5x^2+4}-32\log(1-x)(13d+16e+25f)+\log(2-x)(3}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 +
x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 20*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^2)
))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e + 25*f)*Log[1 - x] + (313*d + 512*e
+ 820*f)*Log[2 - x] + 32*(13*d - 16*e + 25*f)*Log[1 + x] + (-313*d + 512*e
- 820*f)*Log[2 + x])/41472
```

Maple [A] time = 0.018, size = 278, normalized size = 1.6

$$-\frac{313\ln(2+x)d}{41472}+\frac{\ln(2+x)e}{81}+\frac{13\ln(1+x)d}{1296}-\frac{\ln(1+x)e}{81}+\frac{313\ln(x-2)d}{41472}+\frac{\ln(x-2)e}{81}-\frac{13\ln(x-1)d}{1296}-\frac{\ln(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+313/41
472*ln(x-2)*d+1/81*ln(x-2)*e-13/1296*ln(x-1)*d-1/81*ln(x-1)*e-1/432/(1+x)^2
*f+1/864/(2+x)^2*f+1/432/(x-1)^2*d+1/432/(x-1)^2*e+1/3456/(2+x)^2*d-1/1728/
(2+x)^2*e-1/864/(x-2)^2*f-1/432/(1+x)^2*d+1/432/(1+x)^2*e+1/432/(x-1)^2*f-1
/3456/(x-2)^2*d-1/1728/(x-2)^2*e+1/432/(1+x)*d-1/144/(1+x)*e+19/6912/(x-2)*
d+17/3456/(x-2)*e+1/432/(x-1)*d+1/144/(x-1)*e+19/6912/(2+x)*d-17/3456/(2+x)
*e+5/432/(1+x)*f+5/576/(x-2)*f+5/432/(x-1)*f+5/576/(2+x)*f+205/10368*ln(x-2
)*f-25/1296*ln(x-1)*f-205/10368*ln(2+x)*f+25/1296*ln(1+x)*f
```

Maxima [A] time = 0.953977, size = 209, normalized size = 1.19

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f\right)\log\left(x+2\right)+\frac{1}{1296}\left(13\,d-16\,e+25\,f\right)\log\left(x+1\right)-\frac{1}{1296}\left(13\,d+16\,e+25\,f\right)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f)*log(x + 2) + 1/1296*(13*d - 16*e + 25*f)*l
og(x + 1) - 1/1296*(13*d + 16*e + 25*f)*log(x - 1) + 1/41472*(313*d + 512*e
 + 820*f)*log(x - 2) + 1/3456*(35*(d + 4*f)*x^7 + 128*e*x^6 - 18*(13*d + 60
*f)*x^5 - 960*e*x^4 + 63*(5*d + 36*f)*x^3 + 1920*e*x^2 + 4*(43*d - 260*f)*x
 - 800*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] time = 2.3227, size = 1134, normalized size = 6.48

$$\frac{420\left(d+4f\right)x^{7}+1536\,ex^{6}-216\left(13\,d+60\,f\right)x^{5}-11520\,ex^{4}+756\left(5\,d+36\,f\right)x^{3}+23040\,ex^{2}+48\left(43\,d-260\,f\right)x-\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 1536*e*x^6 - 216*(13*d + 60*f)*x^5 - 11520*e*x
^4 + 756*(5*d + 36*f)*x^3 + 23040*e*x^2 + 48*(43*d - 260*f)*x - ((313*d - 5
12*e + 820*f)*x^8 - 10*(313*d - 512*e + 820*f)*x^6 + 33*(313*d - 512*e + 82
0*f)*x^4 - 40*(313*d - 512*e + 820*f)*x^2 + 5008*d - 8192*e + 13120*f)*log(
x + 2) + 32*((13*d - 16*e + 25*f)*x^8 - 10*(13*d - 16*e + 25*f)*x^6 + 33*(1
3*d - 16*e + 25*f)*x^4 - 40*(13*d - 16*e + 25*f)*x^2 + 208*d - 256*e + 400*
f)*log(x + 1) - 32*((13*d + 16*e + 25*f)*x^8 - 10*(13*d + 16*e + 25*f)*x^6
+ 33*(13*d + 16*e + 25*f)*x^4 - 40*(13*d + 16*e + 25*f)*x^2 + 208*d + 256*e
 + 400*f)*log(x - 1) + ((313*d + 512*e + 820*f)*x^8 - 10*(313*d + 512*e + 8
20*f)*x^6 + 33*(313*d + 512*e + 820*f)*x^4 - 40*(313*d + 512*e + 820*f)*x^2
 + 5008*d + 8192*e + 13120*f)*log(x - 2) - 9600*e)/(x^8 - 10*x^6 + 33*x^4 -
 40*x^2 + 16)
```

Sympy [B] time = 43.3403, size = 2822, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] (13*d - 16*e + 25*f)*log(x + (-1106258459719280*d**5*e - 13113710954343*d**
5*(13*d - 16*e + 25*f) - 12929482401572800*d**4*e*f - 107063904267900*d**4*
f*(13*d - 16*e + 25*f) - 817263343042560*d**3*e**3 + 153628968222720*d**3*e
**2*(13*d - 16*e + 25*f) - 59478343838144000*d**3*e*f**2 + 9530197557248*d*
*3*e*(13*d - 16*e + 25*f)**2 - 324891412840800*d**3*f**2*(13*d - 16*e + 25*
f) + 88038005760*d**3*(13*d - 16*e + 25*f)**3 - 2885705898393600*d**2*e**3*
f + 1014848673546240*d**2*e**2*f*(13*d - 16*e + 25*f) - 134905286808320000*
d**2*e*f**3 + 63469758382080*d**2*e*f*(13*d - 16*e + 25*f)**2 - 42297272452
8000*d**2*f**3*(13*d - 16*e + 25*f) + 364616847360*d**2*f*(13*d - 16*e + 25
*f)**3 + 5035763255214080*d*e**5 + 142661633703936*d*e**4*(13*d - 16*e + 25
```

```
*f) - 2138314899456000*d*e**3*f**2 - 19670950215680*d*e**3*(13*d - 16*e + 2
5*f)**2 + 2257033730457600*d*e**2*f**2*(13*d - 16*e + 25*f) - 557272006656*
d*e**2*(13*d - 16*e + 25*f)**3 - 15108264559360000*d*e*f**4 + 141056507904
000*d*e*f**2*(13*d - 16*e + 25*f)**2 - 167683154400000*d*f**4*(13*d - 16*e
+ 25*f) + 339373670400*d*f**2*(13*d - 16*e + 25*f)**3 + 10643272556871680*e
**5*f + 214404767416320*e**4*f*(13*d - 16*e + 25*f) + 529992253440000*e**3*
f**3 - 41575283425280*e**3*f*(13*d - 16*e + 25*f)**2 + 1671759396864000*e**
2*f**3*(13*d - 16*e + 25*f) - 837518622720*e**2*f*(13*d - 16*e + 25*f)**3 -
 66895452108800000*e*f**5 + 104485486592000*e*f**3*(13*d - 16*e + 25*f)**2
+ 51041923200000*f**5*(13*d - 16*e + 25*f) - 80289792000*f**3*(13*d - 16*e
+ 25*f)**3)/(22941256248261*d**6 + 197271407316645*d**5*f - 231274074603520
0*d**4*e**2 + 612862910928900*d**4*f**2 - 20566607354920960*d**3*e**2*f + 7
67363353812000*d**3*f**3 + 4473912813420544*d**2*e**4 - 68552762169753600*d
**2*e**2*f**2 + 197499222000000*d**2*f**4 + 20324472439439360*d*e**4*f - 10
1559983669248000*d*e**2*f**3 - 182883938400000*d*f**5 + 22539988369408000*e
**4*f**2 - 56422196838400000*e**2*f**4 + 21520080000000*f**6))/1296 - (13*d
 + 16*e + 25*f)*log(x + (-1106258459719280*d**5*e + 13113710954343*d**5*(13
*d + 16*e + 25*f) - 12929482401572800*d**4*e*f + 107063904267900*d**4*f*(13
*d + 16*e + 25*f) - 817263343042560*d**3*e**3 - 153628968222720*d**3*e**2*(
13*d + 16*e + 25*f) - 59478343838144000*d**3*e*f**2 + 9530197557248*d**3*e*
(13*d + 16*e + 25*f)**2 + 324891412840800*d**3*f**2*(13*d + 16*e + 25*f) -
88038005760*d**3*(13*d + 16*e + 25*f)**3 - 2885705898393600*d**2*e**3*f - 1
014848673546240*d**2*e**2*f*(13*d + 16*e + 25*f) - 134905286808320000*d**2*
e*f**3 + 63469758382080*d**2*e*f*(13*d + 16*e + 25*f)**2 + 422972724528000*
d**2*f**3*(13*d + 16*e + 25*f) - 364616847360*d**2*f*(13*d + 16*e + 25*f)**
3 + 5035763255214080*d*e**5 - 142661633703936*d*e**4*(13*d + 16*e + 25*f) -
 2138314899456000*d*e**3*f**2 - 19670950215680*d*e**3*(13*d + 16*e + 25*f)*
*2 - 2257033730457600*d*e**2*f**2*(13*d + 16*e + 25*f) + 557272006656*d*e**
2*(13*d + 16*e + 25*f)**3 - 15108264559360000*d*e*f**4 + 141056507904000*d
*e*f**2*(13*d + 16*e + 25*f)**2 + 167683154400000*d*f**4*(13*d + 16*e + 25*
f) - 339373670400*d*f**2*(13*d + 16*e + 25*f)**3 + 10643272556871680*e**5*f
 - 214404767416320*e**4*f*(13*d + 16*e + 25*f) + 529992253440000*e**3*f**3
- 41575283425280*e**3*f*(13*d + 16*e + 25*f)**2 - 1671759396864000*e**2*f**
3*(13*d + 16*e + 25*f) + 837518622720*e**2*f*(13*d + 16*e + 25*f)**3 - 6689
5452108800000*e*f**5 + 104485486592000*e*f**3*(13*d + 16*e + 25*f)**2 - 510
41923200000*f**5*(13*d + 16*e + 25*f) + 80289792000*f**3*(13*d + 16*e + 25*
f)**3)/(22941256248261*d**6 + 197271407316645*d**5*f - 2312740746035200*d**
4*e**2 + 612862910928900*d**4*f**2 - 20566607354920960*d**3*e**2*f + 767363
353812000*d**3*f**3 + 4473912813420544*d**2*e**4 - 68552762169753600*d**2*e
**2*f**2 + 197499222000000*d**2*f**4 + 20324472439439360*d*e**4*f - 1015599
83669248000*d*e**2*f**3 - 182883938400000*d*f**5 + 22539988369408000*e**4*f
**2 - 56422196838400000*e**2*f**4 + 21520080000000*f**6))/1296 - (313*d - 5
12*e + 820*f)*log(x + (-1106258459719280*d**5*e + 13113710954343*d**5*(313*
d - 512*e + 820*f)/32 - 12929482401572800*d**4*e*f + 26765976066975*d**4*f*
(313*d - 512*e + 820*f)/8 - 817263343042560*d**3*e**3 - 4800905256960*d**3*
```

```
e**2*(313*d - 512*e + 820*f) - 59478343838144000*d**3*e*f**2 + 9306833552*d
**3*e*(313*d - 512*e + 820*f)**2 + 10152856651275*d**3*f**2*(313*d - 512*e
+ 820*f) - 85974615*d**3*(313*d - 512*e + 820*f)**3/32 - 2885705898393600*d
**2*e**3*f - 31714021048320*d**2*e**2*f*(313*d - 512*e + 820*f) - 134905286
808320000*d**2*e*f**3 + 61982185920*d**2*e*f*(313*d - 512*e + 820*f)**2 + 1
3217897641500*d**2*f**3*(313*d - 512*e + 820*f) - 89017785*d**2*f*(313*d -
512*e + 820*f)**3/8 + 5035763255214080*d*e**5 - 4458176053248*d*e**4*(313*d
 - 512*e + 820*f) - 2138314899456000*d*e**3*f**2 - 19209912320*d*e**3*(313*
d - 512*e + 820*f)**2 - 70532304076800*d*e**2*f**2*(313*d - 512*e + 820*f)
+ 17006592*d*e**2*(313*d - 512*e + 820*f)**3 - 151082645593600000*d*e*f**4
+ 137750496000*d*e*f**2*(313*d - 512*e + 820*f)**2 + 5240098575000*d*f**4*(
313*d - 512*e + 820*f) - 20713725*d*f**2*(313*d - 512*e + 820*f)**3/2 + 106
43272556871680*e**5*f - 6700148981760*e**4*f*(313*d - 512*e + 820*f) + 5299
92253440000*e**3*f**3 - 40600862720*e**3*f*(313*d - 512*e + 820*f)**2 - 522
42481152000*e**2*f**3*(313*d - 512*e + 820*f) + 25559040*e**2*f*(313*d - 51
2*e + 820*f)**3 - 66895452108800000*e*f**5 + 102036608000*e*f**3*(313*d - 5
12*e + 820*f)**2 - 1595060100000*f**5*(313*d - 512*e + 820*f) + 2450250*f**
3*(313*d - 512*e + 820*f)**3)/(22941256248261*d**6 + 197271407316645*d**5*f
 - 2312740746035200*d**4*e**2 + 612862910928900*d**4*f**2 - 205666073549209
60*d**3*e**2*f + 767363353812000*d**3*f**3 + 4473912813420544*d**2*e**4 - 6
8552762169753600*d**2*e**2*f**2 + 197499222000000*d**2*f**4 + 2032447243943
9360*d*e**4*f - 101559983669248000*d*e**2*f**3 - 182883938400000*d*f**5 + 2
2539988369408000*e**4*f**2 - 56422196838400000*e**2*f**4 + 21520080000000*f
**6))/41472 + (313*d + 512*e + 820*f)*log(x + (-1106258459719280*d**5*e - 1
3113710954343*d**5*(313*d + 512*e + 820*f)/32 - 12929482401572800*d**4*e*f
- 26765976066975*d**4*f*(313*d + 512*e + 820*f)/8 - 817263343042560*d**3*e*
*3 + 4800905256960*d**3*e**2*(313*d + 512*e + 820*f) - 59478343838144000*d*
*3*e*f**2 + 9306833552*d**3*e*(313*d + 512*e + 820*f)**2 - 10152856651275*d
**3*f**2*(313*d + 512*e + 820*f) + 85974615*d**3*(313*d + 512*e + 820*f)**3
/32 - 2885705898393600*d**2*e**3*f + 31714021048320*d**2*e**2*f*(313*d + 51
2*e + 820*f) - 134905286808320000*d**2*e*f**3 + 61982185920*d**2*e*f*(313*d
 + 512*e + 820*f)**2 - 13217897641500*d**2*f**3*(313*d + 512*e + 820*f) + 8
9017785*d**2*f*(313*d + 512*e + 820*f)**3/8 + 5035763255214080*d*e**5 + 445
8176053248*d*e**4*(313*d + 512*e + 820*f) - 2138314899456000*d*e**3*f**2 -
19209912320*d*e**3*(313*d + 512*e + 820*f)**2 + 70532304076800*d*e**2*f**2*
(313*d + 512*e + 820*f) - 17006592*d*e**2*(313*d + 512*e + 820*f)**3 - 1510
82645593600000*d*e*f**4 + 137750496000*d*e*f**2*(313*d + 512*e + 820*f)**2
- 5240098575000*d*f**4*(313*d + 512*e + 820*f) + 20713725*d*f**2*(313*d + 5
12*e + 820*f)**3/2 + 10643272556871680*e**5*f + 6700148981760*e**4*f*(313*d
 + 512*e + 820*f) + 529992253440000*e**3*f**3 - 40600862720*e**3*f*(313*d +
 512*e + 820*f)**2 + 52242481152000*e**2*f**3*(313*d + 512*e + 820*f) - 255
59040*e**2*f*(313*d + 512*e + 820*f)**3 - 66895452108800000*e*f**5 + 102036
608000*e*f**3*(313*d + 512*e + 820*f)**2 + 1595060100000*f**5*(313*d + 512*
e + 820*f) - 2450250*f**3*(313*d + 512*e + 820*f)**3)/(22941256248261*d**6
+ 197271407316645*d**5*f - 2312740746035200*d**4*e**2 + 612862910928900*d**
```

```
4*f**2 - 20566607354920960*d**3*e**2*f + 767363353812000*d**3*f**3 + 447391
2813420544*d**2*e**4 - 68552762169753600*d**2*e**2*f**2 + 197499222000000*d
**2*f**4 + 20324472439439360*d*e**4*f - 101559983669248000*d*e**2*f**3 - 18
2883938400000*d*f**5 + 22539988369408000*e**4*f**2 - 56422196838400000*e**2
*f**4 + 21520080000000*f**6))/41472 + (128*e*x**6 - 960*e*x**4 + 1920*e*x**
2 - 800*e + x**7*(35*d + 140*f) + x**5*(-234*d - 1080*f) + x**3*(315*d + 22
68*f) + x*(172*d - 1040*f))/(3456*x**8 - 34560*x**6 + 114048*x**4 - 138240*
x**2 + 55296)
```

Giac [A] time = 1.0935, size = 212, normalized size = 1.21

$$-\frac{1}{41472}\left(313\,d + 820\,f - 512\,e\right)\log\left(|x+2|\right) + \frac{1}{1296}\left(13\,d + 25\,f - 16\,e\right)\log\left(|x+1|\right) - \frac{1}{1296}\left(13\,d + 25\,f + 16\,e\right)\log\left(|x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 512*e)*log(abs(x + 2)) + 1/1296*(13*d + 25*f - 16
*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f + 16*e)*log(abs(x - 1)) + 1/41472
*(313*d + 820*f + 512*e)*log(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 1
28*x^6*e - 234*d*x^5 - 1080*f*x^5 - 960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 19
20*x^2*e + 172*d*x - 1040*f*x - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.44 $\int \frac{d+ex+fx^2+gx^3}{\left(4-5x^2+x^4\right)^3} dx$

Optimal. Leaf size=204

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\mathrm{t}$$

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (5*e + 8*g
- (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5 - 2*x^2))/(10
8*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x
^2 + x^4)) - ((313*d + 820*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[
x])/648 - ((2*e + 5*g)*Log[1 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi [A] time = 0.251946, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1673, 1178, 1166, 207, 1247, 638, 614, 616, 31}

$$-\frac{x\left(-35x^2(d+4f)+59d+380f\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f))+17d+20f\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\mathrm{t}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (5*e + 8*g
- (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5 - 2*x^2))/(10
8*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x
^2 + x^4)) - ((313*d + 820*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[
x])/648 - ((2*e + 5*g)*Log[1 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(4-5x^2+x^4\right)^3}\,dx = \int \frac{d+fx^2}{\left(4-5x^2+x^4\right)^3}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(4-5x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{1}{144}\int \frac{-19d+20f+5(5d+8f)x^2}{\left(4-5x^2+x^4\right)^2}\,dx + \frac{1}{2}\,\text{Subst}\Bigg(\int \frac{e}{(4-$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} - \frac{x\left(59d+380f-35(d+4f)x^2\right)}{3456\left(4-5x^2+x^4\right)} +$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)} - \frac{x\left(59d+380\right.}{3456\left(\right.}$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)} - \frac{x\left(59d+380\right.}{3456\left(\right.}$$

$$= \frac{x\left(17d+20f-(5d+8f)x^2\right)}{144\left(4-5x^2+x^4\right)^2} + \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)} - \frac{x\left(59d+380\right.}{3456\left(\right.}$$

Mathematica [A] time = 0.0914973, size = 193, normalized size = 0.95

$$\frac{\frac{288\left(-5dx^3+17dx+e\left(20-8x^2\right)-8fx^3+20fx-4g\left(5x^2-8\right)\right)}{\left(x^4-5x^2+4\right)^2} + \frac{12\left(dx\left(35x^2-59\right)+64e\left(2x^2-5\right)+20fx\left(7x^2-19\right)+160g\left(2x^2-5\right)\right)}{x^4-5x^2+4} - 32\log(1-x)(13d+16e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g*(-8 + 5*x
^2)))/(4 - 5*x^2 + x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 160*g*(-5 + 2*x^2) + 2
0*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 1
6*e + 25*f + 40*g)*Log[1 - x] + (313*d + 512*e + 820*f + 1280*g)*Log[2 - x]
 + 32*(13*d - 16*e + 25*f - 40*g)*Log[1 + x] + (-313*d + 512*e - 820*f + 12
80*g)*Log[2 + x])/41472
```

Maple [A] time = 0.022, size = 370, normalized size = 1.8

$$-\frac{313\ \ln(2+x)\,d}{41472}+\frac{\ln(2+x)\,e}{81}+\frac{13\ \ln(1+x)\,d}{1296}-\frac{\ln(1+x)\,e}{81}+\frac{313\ \ln(x-2)\,d}{41472}+\frac{\ln(x-2)\,e}{81}-\frac{13\ \ln(x-1)\,d}{1296}-\frac{\ln(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+313/41
472*ln(x-2)*d+1/81*ln(x-2)*e-13/1296*ln(x-1)*d-1/81*ln(x-1)*e-1/432/(2+x)^2
*g+1/432/(1+x)^2*g-1/432/(x-2)^2*g+1/432/(x-1)^2*g-1/432/(1+x)^2*f+1/864/(2
+x)^2*f+1/432/(x-1)^2*d+1/432/(x-1)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)^2*e-1
/864/(x-2)^2*f-1/432/(1+x)^2*d+1/432/(1+x)^2*e+1/432/(x-1)^2*f-1/3456/(x-2)
^2*d-1/1728/(x-2)^2*e-13/864/(2+x)*g+1/432/(1+x)*d-1/144/(1+x)*e+13/864/(x-
2)*g+19/6912/(x-2)*d+17/3456/(x-2)*e+7/432/(x-1)*g+1/432/(x-1)*d+1/144/(x-1
)*e+19/6912/(2+x)*d-17/3456/(2+x)*e-7/432/(1+x)*g+5/432/(1+x)*f+5/576/(x-2)
*f+5/432/(x-1)*f+5/576/(2+x)*f+5/162*ln(2+x)*g-5/162*ln(1+x)*g+5/162*ln(x-2
)*g-5/162*ln(x-1)*g+205/10368*ln(x-2)*f-25/1296*ln(x-1)*f-205/10368*ln(2+x)
*f+25/1296*ln(1+x)*f
```

Maxima [A] time = 0.946445, size = 254, normalized size = 1.25

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g\right)\log\left(x+2\right)+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g\right)\log\left(x+1\right)-\frac{1}{1296}\left(13\,d+16\,e+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g)*log(x + 2) + 1/1296*(13*d - 16*e
+ 25*f - 40*g)*log(x + 1) - 1/1296*(13*d + 16*e + 25*f + 40*g)*log(x - 1) +
```

```
 1/41472*(313*d + 512*e + 820*f + 1280*g)*log(x - 2) + 1/3456*(35*(d + 4*f)
*x^7 + 64*(2*e + 5*g)*x^6 - 18*(13*d + 60*f)*x^5 - 480*(2*e + 5*g)*x^4 + 63
*(5*d + 36*f)*x^3 + 960*(2*e + 5*g)*x^2 + 4*(43*d - 260*f)*x - 800*e - 2432
*g)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] time = 4.31871, size = 1407, normalized size = 6.9

$$420\left(d+4f\right)x^{7}+768\left(2e+5g\right)x^{6}-216\left(13d+60f\right)x^{5}-5760\left(2e+5g\right)x^{4}+756\left(5d+36f\right)x^{3}+11520\left(2e+5g\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(13*d + 60*f)*x^5 -
5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f)*x^3 + 11520*(2*e + 5*g)*x^2 + 48*(4
3*d - 260*f)*x - ((313*d - 512*e + 820*f - 1280*g)*x^8 - 10*(313*d - 512*e
+ 820*f - 1280*g)*x^6 + 33*(313*d - 512*e + 820*f - 1280*g)*x^4 - 40*(313*d
 - 512*e + 820*f - 1280*g)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g)*log(x
 + 2) + 32*((13*d - 16*e + 25*f - 40*g)*x^8 - 10*(13*d - 16*e + 25*f - 40*g
)*x^6 + 33*(13*d - 16*e + 25*f - 40*g)*x^4 - 40*(13*d - 16*e + 25*f - 40*g)
*x^2 + 208*d - 256*e + 400*f - 640*g)*log(x + 1) - 32*((13*d + 16*e + 25*f
+ 40*g)*x^8 - 10*(13*d + 16*e + 25*f + 40*g)*x^6 + 33*(13*d + 16*e + 25*f +
 40*g)*x^4 - 40*(13*d + 16*e + 25*f + 40*g)*x^2 + 208*d + 256*e + 400*f + 6
40*g)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g)*x^8 - 10*(313*d + 512*
e + 820*f + 1280*g)*x^6 + 33*(313*d + 512*e + 820*f + 1280*g)*x^4 - 40*(313
*d + 512*e + 820*f + 1280*g)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g)*log
(x - 2) - 9600*e - 29184*g)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12401, size = 257, normalized size = 1.26

$$-\frac{1}{41472}\left(313\,d+820\,f-1280\,g-512\,e\right)\log\left(|x+2|\right)+\frac{1}{1296}\left(13\,d+25\,f-40\,g-16\,e\right)\log\left(|x+1|\right)-\frac{1}{1296}\left(13\,d+25\,f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g - 512*e)*log(abs(x + 2)) + 1/1296*(13*d +
25*f - 40*g - 16*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f + 40*g + 16*e)*lo
g(abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g + 512*e)*log(abs(x - 2)) +
1/3456*(35*d*x^7 + 140*f*x^7 + 320*g*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*x
^5 - 2400*g*x^4 - 960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 4800*g*x^2 + 1920*x^
2*e + 172*d*x - 1040*f*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.45 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^3}\,dx$

Optimal. Leaf size=224

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d}{207}$$

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f +
32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5
 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f + 848*h - 5*(7*d + 28
*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f + 1936*h)*ArcTa
nh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g)*Log[1
 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rubi [A] time = 0.306831, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {1673, 1678, 1178, 1166, 207, 1247, 638, 614, 616, 31}

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d}{207}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```


```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f +
32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5
 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f + 848*h - 5*(7*d + 28
*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f + 1936*h)*ArcTa
nh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g)*Log[1
 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(4-5x^2+x^4\right)^3}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(4-5x^2+x^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(4-5x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{1}{144}\int \frac{-19d+20f+32h+5(5d+8f+}{\left(4-5x^2+x^4\right)^2}$$

$$= \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{x\left(59d+380f+\right.}{345}$$

$$= \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x\right.}{108\left(4-5x^2+x\right.}$$

$$= \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x\right.}{108\left(4-5x^2+x\right.}$$

$$= \frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{(2e+5g)\left(5-2x\right.}{108\left(4-5x^2+x\right.}$$

Mathematica [A] time = 0.125163, size = 231, normalized size = 1.03

$$\frac{-5dx^3+17dx-8ex^2+20e-8fx^3+20fx-20gx^2+32g-20hx^3+32hx}{144\left(x^4-5x^2+4\right)^2} + \frac{35dx^3-59dx+128ex^2-320e+140fx^3-}{3456\left(x^4-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 5*d*x^3 - 8*
f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e - 800*g - 59*d*x - 38
0*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*x^3)
/(3456*(4 - 5*x^2 + x^4)) + ((-13*d - 16*e - 25*f - 40*g - 61*h)*Log[1 - x]
)/1296 + ((313*d + 512*e + 820*f + 1280*g + 1936*h)*Log[2 - x])/41472 + ((1
3*d - 16*e + 25*f - 40*g + 61*h)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*
f + 1280*g - 1936*h)*Log[2 + x])/41472
```

Maple [B] time = 0.02, size = 462, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+313/41
472*ln(x-2)*d+1/81*ln(x-2)*e-13/1296*ln(x-1)*d-1/81*ln(x-1)*e+1/432/(x-1)^2
*h+1/216/(2+x)^2*h-1/432/(1+x)^2*h-1/216/(x-2)^2*h-1/432/(2+x)^2*g+1/432/(1
+x)^2*g-1/432/(x-2)^2*g+1/432/(x-1)^2*g-1/432/(1+x)^2*f+1/864/(2+x)^2*f+1/4
32/(x-1)^2*d+1/432/(x-1)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)^2*e-1/864/(x-2)^
2*f-1/432/(1+x)^2*d+1/432/(1+x)^2*e+1/432/(x-1)^2*f-1/3456/(x-2)^2*d-1/1728
/(x-2)^2*e+11/432/(x-2)*h+1/48/(x-1)*h+1/48/(1+x)*h+11/432/(2+x)*h-13/864/(
2+x)*g+1/432/(1+x)*d-1/144/(1+x)*e+13/864/(x-2)*g+19/6912/(x-2)*d+17/3456/(
x-2)*e+7/432/(x-1)*g+1/432/(x-1)*d+1/144/(x-1)*e+19/6912/(2+x)*d-17/3456/(2
+x)*e-7/432/(1+x)*g+5/432/(1+x)*f+5/576/(x-2)*f+5/432/(x-1)*f+5/576/(2+x)*f
+5/162*ln(2+x)*g-5/162*ln(1+x)*g+5/162*ln(x-2)*g-5/162*ln(x-1)*g-121/2592*l
n(2+x)*h+61/1296*ln(1+x)*h+121/2592*ln(x-2)*h-61/1296*ln(x-1)*h+205/10368*l
n(x-2)*f-25/1296*ln(x-1)*f-205/10368*ln(2+x)*f+25/1296*ln(1+x)*f
```

Maxima [A] time = 0.950256, size = 289, normalized size = 1.29

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g+1936\,h\right)\log\left(x+2\right)+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g+61\,h\right)\log\left(x+1\right)-\frac{1}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h)*log(x + 2) + 1/1296*(13*
d - 16*e + 25*f - 40*g + 61*h)*log(x + 1) - 1/1296*(13*d + 16*e + 25*f + 40
*g + 61*h)*log(x - 1) + 1/41472*(313*d + 512*e + 820*f + 1280*g + 1936*h)*l
og(x - 2) + 1/3456*(5*(7*d + 28*f + 64*h)*x^7 + 64*(2*e + 5*g)*x^6 - 18*(13
*d + 60*f + 136*h)*x^5 - 480*(2*e + 5*g)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 +
 960*(2*e + 5*g)*x^2 + 4*(43*d - 260*f - 656*h)*x - 800*e - 2432*g)/(x^8 -
10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] time = 12.9746, size = 1671, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(13*d + 60*
f + 136*h)*x^5 - 5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f + 80*h)*x^3 + 11520
*(2*e + 5*g)*x^2 + 48*(43*d - 260*f - 656*h)*x - ((313*d - 512*e + 820*f -
1280*g + 1936*h)*x^8 - 10*(313*d - 512*e + 820*f - 1280*g + 1936*h)*x^6 + 3
3*(313*d - 512*e + 820*f - 1280*g + 1936*h)*x^4 - 40*(313*d - 512*e + 820*f
 - 1280*g + 1936*h)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h)*lo
g(x + 2) + 32*((13*d - 16*e + 25*f - 40*g + 61*h)*x^8 - 10*(13*d - 16*e + 2
5*f - 40*g + 61*h)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61*h)*x^4 - 40*(13
*d - 16*e + 25*f - 40*g + 61*h)*x^2 + 208*d - 256*e + 400*f - 640*g + 976*h
)*log(x + 1) - 32*((13*d + 16*e + 25*f + 40*g + 61*h)*x^8 - 10*(13*d + 16*e
 + 25*f + 40*g + 61*h)*x^6 + 33*(13*d + 16*e + 25*f + 40*g + 61*h)*x^4 - 40
*(13*d + 16*e + 25*f + 40*g + 61*h)*x^2 + 208*d + 256*e + 400*f + 640*g + 9
76*h)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h)*x^8 - 10*(313
*d + 512*e + 820*f + 1280*g + 1936*h)*x^6 + 33*(313*d + 512*e + 820*f + 128
0*g + 1936*h)*x^4 - 40*(313*d + 512*e + 820*f + 1280*g + 1936*h)*x^2 + 5008
*d + 8192*e + 13120*f + 20480*g + 30976*h)*log(x - 2) - 9600*e - 29184*g)/(
x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13631, size = 302, normalized size = 1.35

$$-\frac{1}{41472}\left(313\,d+820\,f-1280\,g+1936\,h-512\,e\right)\log\left(|x+2|\right)+\frac{1}{1296}\left(13\,d+25\,f-40\,g+61\,h-16\,e\right)\log\left(|x+1|\right)-\frac{1}{12\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e)*log(abs(x + 2)) + 1/1296
*(13*d + 25*f - 40*g + 61*h - 16*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f +
 40*g + 61*h + 16*e)*log(abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g + 19
36*h + 512*e)*log(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 320*h*x^7 +
320*g*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*x^5 - 2448*h*x^5 - 2400*g*x^4 -
960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 5040*h*x^3 + 4800*g*x^2 + 1920*x^2*e +
 172*d*x - 1040*f*x - 2624*h*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.46 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(4-5x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+}{2073}$$

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2)
 + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - (
(2*e + 5*g + 11*i)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f
+ 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d +
820*f + 1936*h)*ArcTanh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648
 - ((2*e + 5*g + 11*i)*Log[1 - x^2])/162 + ((2*e + 5*g + 11*i)*Log[4 - x^2]
)/162
```

Rubi [A] time = 0.344993, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {1673, 1678, 1178, 1166, 207, 1663, 1660, 12, 614, 616, 31}

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(x^2(-(5d+8f+20h))+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+}{2073}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2)
 + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - (
(2*e + 5*g + 11*i)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f
+ 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d +
820*f + 1936*h)*ArcTanh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648
 - ((2*e + 5*g + 11*i)*Log[1 - x^2])/162 + ((2*e + 5*g + 11*i)*Log[4 - x^2]
)/162
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
```

```
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+46x^5}{\left(4-5x^2+x^4\right)^3}\,dx = \int \frac{x\left(e+gx^2+46x^4\right)}{\left(4-5x^2+x^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(4-5x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(17d+20f+32h-(5d+8f+20h)x^2\right)}{144\left(4-5x^2+x^4\right)^2} - \frac{1}{144}\int \frac{-19d+20f+32h+5(5d}{\left(4-5x^2+x\right.}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x\right.}{144\left(4-5x^2+x^4\right)^2}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x\right.}{144\left(4-5x^2+x^4\right)^2}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x\right.}{144\left(4-5x^2+x^4\right)^2}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x\right.}{144\left(4-5x^2+x^4\right)^2}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2} + \frac{x\left(17d+20f+32h-(5d+8f+20h)x\right.}{144\left(4-5x^2+x^4\right)^2}$$

Mathematica [A] time = 0.141789, size = 261, normalized size = 1.09

$$\frac{-5dx^3+17dx-8ex^2+20e-8fx^3+20fx-20gx^2+32g-20hx^3+32hx-68ix^2+80i}{144\left(x^4-5x^2+4\right)^2} + \frac{35dx^3-59dx+128ex^2-32}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i*
x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e - 8
00*g - 1760*i - 59*d*x - 380*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 704*i*
x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*x^3)/(3456*(4 - 5*x^2 + x^4)) + ((-13*d
- 16*e - 25*f - 40*g - 61*h - 88*i)*Log[1 - x])/1296 + ((313*d + 512*e + 82
0*f + 1280*g + 1936*h + 2816*i)*Log[2 - x])/41472 + ((13*d - 16*e + 25*f -
40*g + 61*h - 88*i)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*f + 1280*g -
1936*h + 2816*i)*Log[2 + x])/41472
```

Maple [B] time = 0.021, size = 554, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x)
```

```
[Out] -313/41472*ln(2+x)*d+1/81*ln(2+x)*e+13/1296*ln(1+x)*d-1/81*ln(1+x)*e+313/41
472*ln(x-2)*d+1/81*ln(x-2)*e-13/1296*ln(x-1)*d-1/81*ln(x-1)*e+1/432/(x-1)^2
*i+1/432/(1+x)^2*i-1/108/(x-2)^2*i-1/108/(2+x)^2*i+1/432/(x-1)^2*h+1/216/(2
+x)^2*h-1/432/(1+x)^2*h-1/216/(x-2)^2*h-1/432/(2+x)^2*g+1/432/(1+x)^2*g-1/4
32/(x-2)^2*g+1/432/(x-1)^2*g-1/432/(1+x)^2*f+1/864/(2+x)^2*f+1/432/(x-1)^2*
d+1/432/(x-1)^2*e+1/3456/(2+x)^2*d-1/1728/(2+x)^2*e-1/864/(x-2)^2*f-1/432/(
1+x)^2*d+1/432/(1+x)^2*e+1/432/(x-1)^2*f-1/3456/(x-2)^2*d-1/1728/(x-2)^2*e+
1/24/(x-2)*i+11/432/(x-1)*i-11/432/(1+x)*i-1/24/(2+x)*i+11/432/(x-2)*h+1/48
/(x-1)*h+1/48/(1+x)*h+11/432/(2+x)*h-13/864/(2+x)*g+1/432/(1+x)*d-1/144/(1+
x)*e+13/864/(x-2)*g+19/6912/(x-2)*d+17/3456/(x-2)*e+7/432/(x-1)*g+1/432/(x-
1)*d+1/144/(x-1)*e+19/6912/(2+x)*d-17/3456/(2+x)*e-7/432/(1+x)*g+5/432/(1+x
)*f+5/576/(x-2)*f+5/432/(x-1)*f+5/576/(2+x)*f+11/162*ln(x-2)*i-11/162*ln(x-
1)*i+11/162*ln(2+x)*i-11/162*ln(1+x)*i+5/162*ln(2+x)*g-5/162*ln(1+x)*g+5/16
2*ln(x-2)*g-5/162*ln(x-1)*g-121/2592*ln(2+x)*h+61/1296*ln(1+x)*h+121/2592*l
n(x-2)*h-61/1296*ln(x-1)*h+205/10368*ln(x-2)*f-25/1296*ln(x-1)*f-205/10368*
ln(2+x)*f+25/1296*ln(1+x)*f
```

Maxima [A] time = 0.987893, size = 321, normalized size = 1.34

$$-\frac{1}{41472}\left(313\,d-512\,e+820\,f-1280\,g+1936\,h-2816\,i\right)\log\left(x+2\right)+\frac{1}{1296}\left(13\,d-16\,e+25\,f-40\,g+61\,h-88\,i\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="max
ima")
```

```
[Out] -1/41472*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*log(x + 2) + 1/
1296*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*log(x + 1) - 1/1296*(13*d +
16*e + 25*f + 40*g + 61*h + 88*i)*log(x - 1) + 1/41472*(313*d + 512*e + 820
*f + 1280*g + 1936*h + 2816*i)*log(x - 2) + 1/3456*(5*(7*d + 28*f + 64*h)*x
^7 + 64*(2*e + 5*g + 11*i)*x^6 - 18*(13*d + 60*f + 136*h)*x^5 - 480*(2*e +
```

```
5*g + 11*i)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 + 192*(10*e + 25*g + 52*i)*x^2
 + 4*(43*d - 260*f - 656*h)*x - 800*e - 2432*g - 5120*i)/(x^8 - 10*x^6 + 33
*x^4 - 40*x^2 + 16)
```

Fricas [B] time = 59.8284, size = 1939, normalized size = 8.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fri
cas")
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g + 11*i)*x^6 - 216*(13*
d + 60*f + 136*h)*x^5 - 5760*(2*e + 5*g + 11*i)*x^4 + 756*(5*d + 36*f + 80*
h)*x^3 + 2304*(10*e + 25*g + 52*i)*x^2 + 48*(43*d - 260*f - 656*h)*x - ((31
3*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*x^8 - 10*(313*d - 512*e + 8
20*f - 1280*g + 1936*h - 2816*i)*x^6 + 33*(313*d - 512*e + 820*f - 1280*g +
 1936*h - 2816*i)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*
i)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h - 45056*i)*log(x + 2
) + 32*((13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^8 - 10*(13*d - 16*e + 2
5*f - 40*g + 61*h - 88*i)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i
)*x^4 - 40*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^2 + 208*d - 256*e +
400*f - 640*g + 976*h - 1408*i)*log(x + 1) - 32*((13*d + 16*e + 25*f + 40*g
 + 61*h + 88*i)*x^8 - 10*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^6 + 33
*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^4 - 40*(13*d + 16*e + 25*f + 4
0*g + 61*h + 88*i)*x^2 + 208*d + 256*e + 400*f + 640*g + 976*h + 1408*i)*lo
g(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^8 - 10*(31
3*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^6 + 33*(313*d + 512*e + 8
20*f + 1280*g + 1936*h + 2816*i)*x^4 - 40*(313*d + 512*e + 820*f + 1280*g +
 1936*h + 2816*i)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g + 30976*h + 450
56*i)*log(x - 2) - 9600*e - 29184*g - 61440*i)/(x^8 - 10*x^6 + 33*x^4 - 40*
x^2 + 16)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12672, size = 347, normalized size = 1.45

$$-\frac{1}{41472}\left(313\,d+820\,f-1280\,g+1936\,h-2816\,i-512\,e\right)\log\left(|x+2|\right)+\frac{1}{1296}\left(13\,d+25\,f-40\,g+61\,h-88\,i-16\,e\right)\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="gia
c")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 2816*i - 512*e)*log(abs(x + 2))
 + 1/1296*(13*d + 25*f - 40*g + 61*h - 88*i - 16*e)*log(abs(x + 1)) - 1/129
6*(13*d + 25*f + 40*g + 61*h + 88*i + 16*e)*log(abs(x - 1)) + 1/41472*(313*
d + 820*f + 1280*g + 1936*h + 2816*i + 512*e)*log(abs(x - 2)) + 1/3456*(35*
d*x^7 + 140*f*x^7 + 320*h*x^7 + 320*g*x^6 + 704*i*x^6 + 128*x^6*e - 234*d*x
^5 - 1080*f*x^5 - 2448*h*x^5 - 2400*g*x^4 - 5280*i*x^4 - 960*x^4*e + 315*d*
x^3 + 2268*f*x^3 + 5040*h*x^3 + 4800*g*x^2 + 9984*i*x^2 + 1920*x^2*e + 172*
d*x - 1040*f*x - 2624*h*x - 2432*g - 5120*i - 800*e)/(x^4 - 5*x^2 + 4)^2
```

3.47 $\int \frac{d+ex}{\left(1+x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=185

$$\frac{dx\left(2-7x^2\right)}{24\left(x^4+x^2+1\right)}+\frac{dx\left(1-x^2\right)}{12\left(x^4+x^2+1\right)^2}-\frac{9}{32}d\log\left(x^2-x+1\right)+\frac{9}{32}d\log\left(x^2+x+1\right)-\frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\tan^{-1}\left(\frac{2x}{\ldots}\right.}{48\sqrt{3}}$$

```
[Out] (d*x*(1 - x^2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4
)^2) + (d*x*(2 - 7*x^2))/(24*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2
 + x^4)) - (13*d*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (13*d*ArcTan[(1
+ 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3
]) - (9*d*Log[1 - x + x^2])/32 + (9*d*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.117298, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {1673, 12, 1092, 1178, 1169, 634, 618, 204, 628, 1107, 614}

$$\frac{dx\left(2-7x^2\right)}{24\left(x^4+x^2+1\right)}+\frac{dx\left(1-x^2\right)}{12\left(x^4+x^2+1\right)^2}-\frac{9}{32}d\log\left(x^2-x+1\right)+\frac{9}{32}d\log\left(x^2+x+1\right)-\frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\tan^{-1}\left(\frac{2x}{\ldots}\right.}{48\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (d*x*(1 - x^2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4
)^2) + (d*x*(2 - 7*x^2))/(24*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2
 + x^4)) - (13*d*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (13*d*ArcTan[(1
+ 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3
]) - (9*d*Log[1 - x + x^2])/32 + (9*d*Log[1 + x + x^2])/32
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex}{\left(1+x^2+x^4\right)^3}\,dx = \int \frac{d}{\left(1+x^2+x^4\right)^3}\,dx + \int \frac{ex}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= d\int \frac{1}{\left(1+x^2+x^4\right)^3}\,dx + e\int \frac{x}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}d\int \frac{11-5x^2}{\left(1+x^2+x^4\right)^2}\,dx + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^3}\,dx, x, x^2\right)$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{dx\left(2-7x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{1}{72}d\int \frac{60-21x^2}{1+x^2+x^4}\,dx + \frac{1}{2}e\,\text{Subst}\Bigg(\int$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{dx\left(2-7x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{144}d\int \frac{60-81x}{1-x+x^2}$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{dx\left(2-7x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{1}{96}(13d)\int \frac{1}{1-x+}$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{dx\left(2-7x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} -$$

$$= \frac{dx\left(1-x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{dx\left(2-7x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} - \frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}} +$$

Mathematica [C] time = 0.753721, size = 186, normalized size = 1.01

$$\frac{1}{144}\left(\frac{6\left(dx\left(2-7x^2\right)+e\left(8x^2+4\right)\right)}{x^4+x^2+1} + \frac{12\left(d\left(x-x^3\right)+2ex^2+e\right)}{\left(x^4+x^2+1\right)^2} - \frac{\left(7\sqrt{3}-47i\right)d\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(7\sqrt{3}+47i\right)d\,\text{t}}{\sqrt{\frac{1}{6}\left(\right.}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(d*x*(2 - 7*x^2) + e*(4 + 8*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 +
 d*(x - x^3)))/(1 + x^2 + x^4)^2 - ((-47*I + 7*Sqrt[3])*d*ArcTan[((-I + Sqr
t[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - ((47*I + 7*Sqrt[3])*d*ArcTan[((I + S
qrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2
*x^2)])/144
```

Maple [A] time = 0.017, size = 180, normalized size = 1.

$$\frac{1}{16\left(x^{2}+x+1\right)^{2}}\left(\left(-\frac{7d}{3}-\frac{4e}{3}\right)x^{3}-6x^{2}d+\left(-\frac{20d}{3}+\frac{e}{3}\right)x-4d+2e\right)+\frac{9d\ln\left(x^{2}+x+1\right)}{32}+\frac{13d\sqrt{3}}{144}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d-4/3*e)*x^3-6*x^2*d+(-20/3*d+1/3*e)*x-4*d+2*e)/(x^2+x+1)^2+9/3
2*d*ln(x^2+x+1)+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*ar
ctan(1/3*(1+2*x)*3^(1/2))*e-1/16*((7/3*d-4/3*e)*x^3-6*x^2*d+(20/3*d+1/3*e)*
x-4*d-2*e)/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+13/144*3^(1/2)*arctan(1/3*(2*x-1)
*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e
```

Maxima [A] time = 1.45449, size = 185, normalized size = 1.

$$\frac{1}{144}\sqrt{3}(13d-32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}(13d+32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{9}{32}d\log\left(x^{2}+x+1\right)-\frac{9}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqrt(3)*(
13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*log(x^2 + x + 1) - 9/32
*d*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*e*x^6 + 5*d*x^5 - 12*e*x^4 + 7*d*x^
3 - 16*e*x^2 - 4*d*x - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 1.67298, size = 726, normalized size = 3.92

$$-\frac{84dx^{7}-96ex^{6}+60dx^{5}-144ex^{4}+84dx^{3}-192ex^{2}-2\sqrt{3}\left((13d-32e)x^{8}+2(13d-32e)x^{6}+3(13d-32e)x^{4}+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*d*x^7 - 96*e*x^6 + 60*d*x^5 - 144*e*x^4 + 84*d*x^3 - 192*e*x^2 -
 2*sqrt(3)*((13*d - 32*e)*x^8 + 2*(13*d - 32*e)*x^6 + 3*(13*d - 32*e)*x^4 +
 2*(13*d - 32*e)*x^2 + 13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(
3)*((13*d + 32*e)*x^8 + 2*(13*d + 32*e)*x^6 + 3*(13*d + 32*e)*x^4 + 2*(13*d
 + 32*e)*x^2 + 13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) - 48*d*x - 81*(d*
x^8 + 2*d*x^6 + 3*d*x^4 + 2*d*x^2 + d)*log(x^2 + x + 1) + 81*(d*x^8 + 2*d*x
^6 + 3*d*x^4 + 2*d*x^2 + d)*log(x^2 - x + 1) - 72*e)/(x^8 + 2*x^6 + 3*x^4 +
 2*x^2 + 1)
```

Sympy [C] time = 2.63088, size = 1103, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-1025428432*d**4*e - 33475
2912*d**4*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288) - 431308800*d**2*e**3 - 3
143688192*d**2*e**2*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288) + 9917005824*d*
*2*e*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288)**2 + 11878244352*d**2*(-9*d/32
 - sqrt(3)*I*(13*d + 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/
32 - sqrt(3)*I*(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 - sqrt(3)*I*(1
3*d + 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/2
88)**3)/(217696167*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (-9*d
/32 + sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*
d**4*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288) - 431308800*d**2*e**3 - 314368
8192*d**2*e**2*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288) + 9917005824*d**2*e*
(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288)**2 + 11878244352*d**2*(-9*d/32 + sq
rt(3)*I*(13*d + 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/32 +
sqrt(3)*I*(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 + sqrt(3)*I*(13*d +
 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288)**
3)/(217696167*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (9*d/32 -
sqrt(3)*I*(13*d - 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*d**4*(
9*d/32 - sqrt(3)*I*(13*d - 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d*
*2*e**2*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32
- sqrt(3)*I*(13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 - sqrt(3)*I*(1
3*d - 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(9*d/32 - sqrt(3)*I*(
13*d - 32*e)/288) + 3850371072*e**3*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288)*
*2 + 20384317440*e**2*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288)**3)/(217696167
```

```
*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (9*d/32 + sqrt(3)*I*(13
*d - 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*d**4*(9*d/32 + sqrt
(3)*I*(13*d - 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d**2*e**2*(9*d/
32 + sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32 + sqrt(3)*I*(
13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/2
88)**3 + 142606336*e**5 + 754974720*e**4*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/
288) + 3850371072*e**3*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/288)**2 + 20384317
440*e**2*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/288)**3)/(217696167*d**5 - 12171
28448*d**3*e**2 - 617611264*d*e**4)) - (7*d*x**7 + 5*d*x**5 + 7*d*x**3 - 4*
d*x - 8*e*x**6 - 12*e*x**4 - 16*e*x**2 - 6*e)/(24*x**8 + 48*x**6 + 72*x**4
+ 48*x**2 + 24)
```

Giac [A] time = 1.09851, size = 177, normalized size = 0.96

$$\frac{1}{144}\sqrt{3}(13\,d-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}(13\,d+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{9}{32}\,d\log\left(x^2+x+1\right)-\frac{9}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqrt(3)*(
13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*log(x^2 + x + 1) - 9/32
*d*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*x^6*e + 5*d*x^5 - 12*x^4*e + 7*d*x^
3 - 16*x^2*e - 4*d*x - 6*e)/(x^4 + x^2 + 1)^2
```

3.48 $\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=223

$$\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)-$$

```
[Out] (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4)^2) + (x*(d + f - (d - 2*f)*x^2))/(12*(1
 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(2*d + 3*f - 7*
(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (
2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1 - x + x^2
])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.214941, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {1673, 1178, 1169, 634, 618, 204, 628, 12, 1107, 614}

$$\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4)^2) + (x*(d + f - (d - 2*f)*x^2))/(12*(1
 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(2*d + 3*f - 7*
(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (
2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1 - x + x^2
])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^3}\,dx = \int \frac{ex}{\left(1+x^2+x^4\right)^3}\,dx + \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}\int \frac{11d-f-5(d-2f)x^2}{\left(1+x^2+x^4\right)^2}\,dx + e\int \frac{x}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{1}{72}\int \frac{15(4d-f)-21(d-f)x^2}{1+x^2+x^4}\,dx + \frac{1}{2}e\,\mathrm{Sul}$$

$$= \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{1}{144}\int \frac{15(4d-f)-(21}{1}$$

$$= \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{1}{3}e\,\mathrm{Su}$$

$$= \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} - \frac{1}{32}(9a$$

$$= \frac{e\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e\left(1+2x^2\right)}{6\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} - \frac{(13d}{}$$

Mathematica [C] time = 0.589907, size = 235, normalized size = 1.05

$$\frac{1}{144}\left(\frac{6\left(-7dx^3+2dx+e\left(8x^2+4\right)+7fx^3+3fx\right)}{x^4+x^2+1}+\frac{12\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e\right)}{\left(x^4+x^2+1\right)^2}-\frac{\left(\left(7\sqrt{3}-47i\right)d+\left(-7\right.\right.}{\sqrt{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 + e*(4 + 8*x^2)))/(1 + x^2 + x^4) +
(12*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4)^2 - (((-47
*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqr
t[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f)*ArcTa
n[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[
3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.018, size = 264, normalized size = 1.2

$$\frac{1}{16\left(x^2+x+1\right)^2}\left(\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}\right)x^3+\left(-6d+4f\right)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}\right)x-4d+\frac{4f}{3}+2e\right)+\frac{9d\ln\left(x^2+x+\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e)*x^3+(-6*d+4*f)*x^2+(-20/3*d+13/3*f+1/3*e)*x-4*d+
4/3*f+2*e)/(x^2+x+1)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+13/144*d*arctan
(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/7
2*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f-1/16*((7/3*d-7/3*f-4/3*e)*x^3+(-6*d
+4*f)*x^2+(20/3*d-13/3*f+1/3*e)*x-4*d+4/3*f-2*e)/(x^2-x+1)^2-9/32*d*ln(x^2-
x+1)+1/8*ln(x^2-x+1)*f+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(
1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
*f
```

Maxima [A] time = 1.43095, size = 234, normalized size = 1.05

$$\frac{1}{144}\sqrt{3}\left(13d-32e+2f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}\left(13d+32e+2f\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{32}\left(9d-4f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqr
t(3)*(13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)*l
og(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*(d - f)*x^7 -
 8*e*x^6 + 5*(d - 2*f)*x^5 - 12*e*x^4 + 7*(d - 2*f)*x^3 - 16*e*x^2 - (4*d +
 5*f)*x - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A] time = 2.21264, size = 977, normalized size = 4.38

$$-\frac{84\left(d-f\right)x^7-96\,ex^6+60\left(d-2\,f\right)x^5-144\,ex^4+84\left(d-2\,f\right)x^3-192\,ex^2-2\,\sqrt{3}\left(\left(13\,d-32\,e+2\,f\right)x^8+2\left(13\,d-32\,e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*(d - f)*x^7 - 96*e*x^6 + 60*(d - 2*f)*x^5 - 144*e*x^4 + 84*(d -
2*f)*x^3 - 192*e*x^2 - 2*sqrt(3)*((13*d - 32*e + 2*f)*x^8 + 2*(13*d - 32*e
+ 2*f)*x^6 + 3*(13*d - 32*e + 2*f)*x^4 + 2*(13*d - 32*e + 2*f)*x^2 + 13*d -
 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f)
*x^8 + 2*(13*d + 32*e + 2*f)*x^6 + 3*(13*d + 32*e + 2*f)*x^4 + 2*(13*d + 32
*e + 2*f)*x^2 + 13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(4*d
+ 5*f)*x - 9*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(
9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1) + 9*((9*d - 4*f)*x^8 + 2*(9*d
- 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 - x
 + 1) - 72*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [C] time = 36.7534, size = 4498, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-1025428432*d*
*5*e - 334752912*d**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) +
 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d +
 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(-9*d/32 + f
/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 99170058
24*d**3*e*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 94430016
0*d**3*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 118782443
52*d**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 233164800*
d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*
e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(-9*d/32 + f
/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*e
**4*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1843200*d*e**3*f*
*2 + 3850371072*d*e**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*
*2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/
288) + 20384317440*d*e**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/28
8)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 - sqrt(3)*I
*(13*d + 32*e + 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 - sqrt(3)*I*(
13*d + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*
d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e**3*f**3 - 2151677952*e**3
*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 287096832*e**2*
f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 5096079360*e**2*
f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e*f**5
 - 859521024*e*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 -
 7648128*f**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 4538695
68*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3)/(217696167*
d**6 - 301346487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2 + 2181
281792*d**3*e**2*f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 1450149888*d
**2*e**2*f**2 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**2*f
**3 + 783648*d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 188352*f**
6)) + (-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-1025428
432*d**5*e - 334752912*d**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/
288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*d/32 + f/8 + sqrt(3)*I*(
13*d + 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(-9*d/
32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 99
17005824*d**3*e*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 94
4300160*d**3*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 118
78244352*d**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 2331
64800*d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d
 + 32*e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(-9*d/
32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 7549747
```

```
20*d*e**4*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1843200*d*e
**3*f**2 + 3850371072*d*e**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)
/288)**2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e +
 2*f)/288) + 20384317440*d*e**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2
*f)/288)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 + sqr
t(3)*I*(13*d + 32*e + 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 + sqrt(
3)*I*(13*d + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 + sqrt(3)*
I*(13*d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e**3*f**3 - 215167795
2*e**3*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 287096832
*e**2*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 5096079360
*e**2*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e
*f**5 - 859521024*e*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288
)**2 - 7648128*f**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 4
53869568*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3)/(2176
96167*d**6 - 301346487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2
+ 2181281792*d**3*e**2*f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 145014
9888*d**2*e**2*f**2 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*
e**2*f**3 + 783648*d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 1883
52*f**6)) + (9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)*log(x + (-10
25428432*d**5*e - 334752912*d**5*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2
*f)/288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(9*d/32 - f/8 - sqrt(3)*
I*(13*d - 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(9*
d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 +
9917005824*d**3*e*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 9
44300160*d**3*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 118
78244352*d**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 23316
4800*d**2*e**3*f + 4409634816*d**2*e**2*f*(9*d/32 - f/8 - sqrt(3)*I*(13*d -
 32*e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(9*d/32 -
f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(9*d/32 -
 f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 10089639936*d**2*f*(9*d/32 - f/
8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*
e**4*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 1843200*d*e**3*f*
*2 + 3850371072*d*e**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**
2 - 1926291456*d*e**2*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/28
8) + 20384317440*d*e**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)*
*3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13
*d - 32*e + 2*f)/288)**2 + 12679200*d*f**4*(9*d/32 - f/8 - sqrt(3)*I*(13*d
- 32*e + 2*f)/288) + 1116758016*d*f**2*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32
*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(9*d/32 - f/8 - sqrt
(3)*I*(13*d - 32*e + 2*f)/288) - 7372800*e**3*f**3 - 2151677952*e**3*f*(9*d
/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 + 287096832*e**2*f**3*(9*
d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 5096079360*e**2*f*(9*d/32
 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 14093632*e*f**5 - 85952102
4*e*f**3*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 7648128*f*
```

```
*5*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 453869568*f**3*(9*d
/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3)/(217696167*d**6 - 301346
487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2 + 2181281792*d**3*e
**2*f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 1450149888*d**2*e**2*f**2
 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**2*f**3 + 783648*
d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 188352*f**6)) + (9*d/32
 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)*log(x + (-1025428432*d**5*e - 3
34752912*d**5*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 20089613
60*d**4*e*f + 1151575920*d**4*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*
f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(9*d/32 - f/8 + sqrt(3
)*I*(13*d - 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 9917005824*d**3*e*(
9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 944300160*d**3*f**2*
(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 11878244352*d**3*(9*d/
32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 + 233164800*d**2*e**3*f +
4409634816*d**2*e**2*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) +
 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(9*d/32 - f/8 + sqrt(3)*I*(13
*d - 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(9*d/32 - f/8 + sqrt(3)*I*(1
3*d - 32*e + 2*f)/288) - 10089639936*d**2*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d
 - 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*e**4*(9*d/32 - f/8
+ sqrt(3)*I*(13*d - 32*e + 2*f)/288) - 1843200*d*e**3*f**2 + 3850371072*d*e
**3*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 1926291456*d*e*
*2*f**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 20384317440*d*
e**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 - 146756960*d*e*
f**4 + 5813379072*d*e*f**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/28
8)**2 + 12679200*d*f**4*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)
+ 1116758016*d*f**2*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**3 -
 79691776*e**5*f - 188743680*e**4*f*(9*d/32 - f/8 + sqrt(3)*I*(13*d - 32*e
+ 2*f)/288) - 7372800*e**3*f**3 - 2151677952*e**3*f*(9*d/32 - f/8 + sqrt(3)
*I*(13*d - 32*e + 2*f)/288)**2 + 287096832*e**2*f**3*(9*d/32 - f/8 + sqrt(3
)*I*(13*d - 32*e + 2*f)/288) - 5096079360*e**2*f*(9*d/32 - f/8 + sqrt(3)*I*
(13*d - 32*e + 2*f)/288)**3 + 14093632*e*f**5 - 859521024*e*f**3*(9*d/32 -
f/8 + sqrt(3)*I*(13*d - 32*e + 2*f)/288)**2 - 7648128*f**5*(9*d/32 - f/8 +
sqrt(3)*I*(13*d - 32*e + 2*f)/288) + 453869568*f**3*(9*d/32 - f/8 + sqrt(3)
*I*(13*d - 32*e + 2*f)/288)**3)/(217696167*d**6 - 301346487*d**5*f - 121712
8448*d**4*e**2 + 130506255*d**4*f**2 + 2181281792*d**3*e**2*f - 5619240*d**
3*f**3 - 617611264*d**2*e**4 - 1450149888*d**2*e**2*f**2 - 8036820*d**2*f**
4 + 495976448*d*e**4*f + 430088192*d*e**2*f**3 + 783648*d*f**5 - 114294784*
e**4*f**2 - 47771648*e**2*f**4 + 188352*f**6)) - (-8*e*x**6 - 12*e*x**4 - 1
6*e*x**2 - 6*e + x**7*(7*d - 7*f) + x**5*(5*d - 10*f) + x**3*(7*d - 14*f) +
 x*(-4*d - 5*f))/(24*x**8 + 48*x**6 + 72*x**4 + 48*x**2 + 24)
```

Giac [A] time = 1.11395, size = 231, normalized size = 1.04

$$\frac{1}{144}\sqrt{3}\left(13\,d+2\,f-32\,e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\left(13\,d+2\,f+32\,e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)+\frac{1}{32}\left(9\,d-4\,f\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqr
t(3)*(13*d + 2*f + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)*l
og(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x
^7 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 - 16*x^2*
e - 4*d*x - 5*f*x - 6*e)/(x^4 + x^2 + 1)^2
```

3.49 $\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=243

$$\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)$$

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + (2*e - g)*x
^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2 + x^4)) +
 (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTa
n[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9
*d - 4*f)*Log[1 - x + x^2])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.227032, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1673, 1178, 1169, 634, 618, 204, 628, 1247, 638, 614}

$$\frac{x\left(-7x^2(d-f)+2d+3f\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f))+d+f\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + (2*e - g)*x
^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2 + x^4)) +
 (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTa
n[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9
*d - 4*f)*Log[1 - x + x^2])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^3}\,dx = \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^3}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}\int \frac{11d-f-5(d-2f)x^2}{\left(1+x^2+x^4\right)^2}\,dx + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e+gx}{\left(1+x+x^2\right)^3}\,dx, x,\right.$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-7(d-f)x^2\right)}{24\left(1+x^2+x^4\right)} + \frac{1}{72}\int \frac{15(4d-f)}{1+}$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right.}{24\left(1+x^2+x^4\right)}$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right.}{24\left(1+x^2+x^4\right)}$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right.}{24\left(1+x^2+x^4\right)}$$

$$= \frac{x\left(d+f-(d-2f)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+3f-7(d-f)x^2\right.}{24\left(1+x^2+x^4\right)}$$

Mathematica [C] time = 0.749252, size = 259, normalized size = 1.07

$$\frac{1}{144}\left(\frac{6\left(-7dx^3+2dx+e\left(8x^2+4\right)+7fx^3+3fx-2g\left(2x^2+1\right)\right)}{x^4+x^2+1} + \frac{12\left(x\left(-dx^2+d+2fx^2+f\right)+2ex^2+e-g\left(x^2+2\right)\right)}{\left(x^4+x^2+1\right)^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 - 2*g*(1 + 2*x^2) + e*(4 + 8*x^2)))/
(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x
^2)))/(1 + x^2 + x^4)^2 - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*A
rcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])
*d - (17*I + 7*Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3]
)/6] - 16*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.016, size = 322, normalized size = 1.3

$$\frac{1}{16\left(x^2+x+1\right)^2}\left(\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^3+\left(-6d+4f-2g\right)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4d+\frac{4f}{3}+2e-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*g)*x^2+(-20/3*d+13/3*f+1/3
*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+
1)*f+13/144*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1
+2*x)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arc
tan(1/3*(1+2*x)*3^(1/2))*g-1/16*((7/3*d-7/3*f-4/3*e-1/3*g)*x^3+(-6*d+4*f+2*
g)*x^2+(20/3*d-13/3*f+1/3*e-8/3*g)*x-4*d+4/3*f-2*e+2*g)/(x^2-x+1)^2-9/32*d*
ln(x^2-x+1)+1/8*ln(x^2-x+1)*f+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+
2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*g
```

Maxima [A] time = 1.42711, size = 270, normalized size = 1.11

$$\frac{1}{144}\sqrt{3}\left(13d-32e+2f+16g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}\left(13d+32e+2f-16g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/
144*sqrt(3)*(13*d + 32*e + 2*f - 16*g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32
*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7
*(d - f)*x^7 - 4*(2*e - g)*x^6 + 5*(d - 2*f)*x^5 - 6*(2*e - g)*x^4 + 7*(d -
 2*f)*x^3 - 8*(2*e - g)*x^2 - (4*d + 5*f)*x - 6*e + 6*g)/(x^8 + 2*x^6 + 3*x
^4 + 2*x^2 + 1)
```

Fricas [A] time = 4.00098, size = 1111, normalized size = 4.57

$$-\frac{84\left(d-f\right)x^7-48\left(2e-g\right)x^6+60\left(d-2f\right)x^5-72\left(2e-g\right)x^4+84\left(d-2f\right)x^3-96\left(2e-g\right)x^2-2\sqrt{3}\left(\left(13d-32e+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*(d - f)*x^7 - 48*(2*e - g)*x^6 + 60*(d - 2*f)*x^5 - 72*(2*e - g)
*x^4 + 84*(d - 2*f)*x^3 - 96*(2*e - g)*x^2 - 2*sqrt(3)*((13*d - 32*e + 2*f
+ 16*g)*x^8 + 2*(13*d - 32*e + 2*f + 16*g)*x^6 + 3*(13*d - 32*e + 2*f + 16*
g)*x^4 + 2*(13*d - 32*e + 2*f + 16*g)*x^2 + 13*d - 32*e + 2*f + 16*g)*arcta
n(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f - 16*g)*x^8 + 2*(1
3*d + 32*e + 2*f - 16*g)*x^6 + 3*(13*d + 32*e + 2*f - 16*g)*x^4 + 2*(13*d +
 32*e + 2*f - 16*g)*x^2 + 13*d + 32*e + 2*f - 16*g)*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 12*(4*d + 5*f)*x - 9*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d
 - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1) + 9*((9*d - 4
*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d -
 4*f)*log(x^2 - x + 1) - 72*e + 72*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09717, size = 267, normalized size = 1.1

$$\frac{1}{144}\sqrt{3}\left(13\,d+2\,f+16\,g-32\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\,\sqrt{3}\left(13\,d+2\,f-16\,g+32\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/
144*sqrt(3)*(13*d + 2*f - 16*g + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32
*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7
*d*x^7 - 7*f*x^7 + 4*g*x^6 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 + 6*g*x^4 - 12*x^
```

```
4*e + 7*d*x^3 - 14*f*x^3 + 8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 6*g - 6*e)/
(x^4 + x^2 + 1)^2
```

3.50 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(1+x^2+x^4\right)^3} dx$

Optimal. Leaf size=263

$$\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\text{l}$$

```
[Out] (e - 2*g + (2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + (x*(d + f - 2*h - (d - 2
*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2
 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 + x^2 + x^4))
 - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f
 + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^
2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/32 + ((9*d
- 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.262926, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {1673, 1678, 1178, 1169, 634, 618, 204, 628, 1247, 638, 614}

$$\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (e - 2*g + (2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + (x*(d + f - 2*h - (d - 2
*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2
 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 + x^2 + x^4))
 - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f
 + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^
2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/32 + ((9*d
- 4*f + 3*h)*Log[1 + x + x^2])/32
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(1+x^2+x^4\right)^3}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(1+x^2+x^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}\int \frac{11d-f+2h-5(d-2f+h)x^2}{\left(1+x^2+x^4\right)^2}\,dx + \frac{1}{2}\,\mathrm{Sub}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-h-(7d-7f+\right.}{24\left(1+x^2+x^4\right)}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+\right.}{}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+\right.}{}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+\right.}{}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)} + \frac{x\left(2d+\right.}{}$$

Mathematica [C] time = 0.942372, size = 303, normalized size = 1.15

$$\frac{1}{144}\left(-\frac{6\left(x\left(7dx^2-2d-7fx^2-3f+4hx^2+h\right)-4e\left(2x^2+1\right)+g\left(4x^2+2\right)\right)}{x^4+x^2+1} + \frac{12\left(x\left(-dx^2+d+2fx^2+f-h\left(x^2+2\right.\right.\right.}{\left(x^4+x^2+1\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((-6*(-4*e*(1 + 2*x^2) + g*(2 + 4*x^2) + x*(-2*d - 3*f + h + 7*d*x^2 - 7*f*
x^2 + 4*h*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g*(2 + x^2) + x*(d +
f - d*x^2 + 2*f*x^2 - h*(2 + x^2))))/(1 + x^2 + x^4)^2 - (((-47*I + 7*Sqrt[
3])*d + (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2*Sqrt[3])*h)*ArcTan[((-I + Sqrt[3
])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt
[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sq
rt[3])/6] - 16*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.017, size = 396, normalized size = 1.5

$$\frac{1}{16\left(x^{2}+x+1\right)^{2}}\left(\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^{3}+\left(-6d+4f-2h-2g\right)x^{2}+\left(-\frac{20d}{3}+\frac{13f}{3}-\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*h-2*g)*x^2+(-20/3*d+
13/3*f-5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1)^2+9/32*d*ln(x^2+x+
1)-1/8*ln(x^2+x+1)*f+3/32*ln(x^2+x+1)*h+13/144*d*arctan(1/3*(1+2*x)*3^(1/2)
)*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3
*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*g+1/144*3^(1/2)
*arctan(1/3*(1+2*x)*3^(1/2))*h-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g)*x^3+(-
6*d+4*f-2*h+2*g)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f-2*e+2*g)
/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)*f-3/32*ln(x^2-x+1)*h+13/144
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arctan(1/3*(2*
x-1)*3^(1/2))*g+1/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h
```

Maxima [A] time = 1.44078, size = 293, normalized size = 1.11

$$\frac{1}{144}\sqrt{3}\left(13d-32e+2f+16g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}\left(13d+32e+2f-16g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*log(x^
2 - x + 1) - 1/24*((7*d - 7*f + 4*h)*x^7 - 4*(2*e - g)*x^6 + 5*(d - 2*f + h
)*x^5 - 6*(2*e - g)*x^4 + 7*(d - 2*f + h)*x^3 - 8*(2*e - g)*x^2 - (4*d + 5*
f - 5*h)*x - 6*e + 6*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [B] time = 13.0623, size = 1278, normalized size = 4.86

$$-\frac{12\left(7\,d-7\,f+4\,h\right)x^{7}-48\left(2\,e-g\right)x^{6}+60\left(d-2\,f+h\right)x^{5}-72\left(2\,e-g\right)x^{4}+84\left(d-2\,f+h\right)x^{3}-96\left(2\,e-g\right)x^{2}-2\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(12*(7*d - 7*f + 4*h)*x^7 - 48*(2*e - g)*x^6 + 60*(d - 2*f + h)*x^5
- 72*(2*e - g)*x^4 + 84*(d - 2*f + h)*x^3 - 96*(2*e - g)*x^2 - 2*sqrt(3)*((
13*d - 32*e + 2*f + 16*g + h)*x^8 + 2*(13*d - 32*e + 2*f + 16*g + h)*x^6 +
3*(13*d - 32*e + 2*f + 16*g + h)*x^4 + 2*(13*d - 32*e + 2*f + 16*g + h)*x^2
 + 13*d - 32*e + 2*f + 16*g + h)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*
((13*d + 32*e + 2*f - 16*g + h)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h)*x^6
+ 3*(13*d + 32*e + 2*f - 16*g + h)*x^4 + 2*(13*d + 32*e + 2*f - 16*g + h)*x
^2 + 13*d + 32*e + 2*f - 16*g + h)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(4*d
+ 5*f - 5*h)*x - 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*
d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 + x
 + 1) + 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*d - 4*f +
 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x + 1) - 7
2*e + 72*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12826, size = 308, normalized size = 1.17

$$\frac{1}{144}\,\sqrt{3}\left(13\,d+2\,f+16\,g+h-32\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\,\sqrt{3}\left(13\,d+2\,f-16\,g+h+32\,e\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*log(x^
2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*h*x^7 + 4*g*x^6 - 8*x^6*e + 5*d*x^
5 - 10*f*x^5 + 5*h*x^5 + 6*g*x^4 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 + 7*h*x^3
+ 8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 5*h*x + 6*g - 6*e)/(x^4 + x^2 + 1)^2
```

3.51 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(1+x^2+x^4\right)^3} dx$

Optimal. Leaf size=269

$$\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}$$

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + i
 + (2*e - g - i)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g + i)*(1 + 2*x^2))/
(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 +
 x^2 + x^4)) - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) +
((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g + i)*
ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x
^2])/32 + ((9*d - 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rubi [A] time = 0.286473, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1673, 1678, 1178, 1169, 634, 618, 204, 628, 1663, 1660, 12, 614}

$$\frac{x\left(x^2(-(7d-7f+4h))+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(x^2(-(d-2f+h))+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```


```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + i
 + (2*e - g - i)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g + i)*(1 + 2*x^2))/
(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 +
 x^2 + x^4)) - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) +
((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g + i)*
ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x
^2])/32 + ((9*d - 4*f + 3*h)*Log[1 + x + x^2])/32
```


Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+51x^5}{\left(1+x^2+x^4\right)^3}\,dx = \int \frac{x\left(e+gx^2+51x^4\right)}{\left(1+x^2+x^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(1+x^2+x^4\right)^3}\,dx$$

$$= \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}\int \frac{11d-f+2h-5(d-2f+h)x^2}{\left(1+x^2+x^4\right)^2}\,dx + \frac{1}{2}$$

$$= \frac{51+e-2g-(51-2e+g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-\right.}{24}$$

$$= \frac{51+e-2g-(51-2e+g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(2d+3f-\right.}{24}$$

$$= \frac{51+e-2g-(51-2e+g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(51+2e-g}{12\left(1+x\right.}$$

$$= \frac{51+e-2g-(51-2e+g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(51+2e-g}{12\left(1+x\right.}$$

$$= \frac{51+e-2g-(51-2e+g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2} + \frac{(51+2e-g}{12\left(1+x\right.}$$

Mathematica [C] time = 1.09887, size = 325, normalized size = 1.21

$$\frac{1}{144}\left(\frac{12\left(-dx^3+dx+2ex^2+e+2fx^3+fx-g\left(x^2+2\right)-hx^3-2hx-ix^2+i\right)}{\left(x^4+x^2+1\right)^2} + \frac{6\left(-7dx^3+2dx+e\left(8x^2+4\right)+7fx^3+\right.}{x^4}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((12*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^3
 - g*(2 + x^2)))/(1 + x^2 + x^4)^2 + (6*(2*i + 2*d*x + 3*f*x - h*x + 4*i*x^
2 - 7*d*x^3 + 7*f*x^3 - 4*h*x^3 - 2*g*(1 + 2*x^2) + e*(4 + 8*x^2)))/(1 + x^
2 + x^4) - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2*Sqr
t[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I +
7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[((I + S
qrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 16*Sqrt[3]*(2*e - g + i)*ArcTan[Sq
```

```
rt[3]/(1 + 2*x^2)])/144
```

Maple [A] time = 0.017, size = 454, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-6*d+4*f-2*h-2*g+2*i)*x^2
+(-20/3*d+13/3*f-5/3*h+1/3*e-8/3*g+7/3*i)*x-4*d+4/3*f+2*e-2*g+4/3*i)/(x^2+x
+1)^2+9/32*d*ln(x^2+x+1)-1/8*ln(x^2+x+1)*f+3/32*ln(x^2+x+1)*h+13/144*d*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/9*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*e+1
/72*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*f+1/9*3^(1/2)*arctan(1/3*(1+2*x)*3^
(1/2))*g+1/144*3^(1/2)*arctan(1/3*(1+2*x)*3^(1/2))*h-1/9*3^(1/2)*arctan(1/3
*(1+2*x)*3^(1/2))*i-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-6*d+4
*f-2*h+2*g-2*i)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g+7/3*i)*x-4*d+4/3*f-2*e
+2*g-4/3*i)/(x^2-x+1)^2-9/32*d*ln(x^2-x+1)+1/8*ln(x^2-x+1)*f-3/32*ln(x^2-x+
1)*h+13/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*d+2/9*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))*e+1/72*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*f-1/9*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))*g+1/144*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*h+1/9*
3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))*i
```

Maxima [A] time = 1.47001, size = 309, normalized size = 1.15

$$\frac{1}{144}\sqrt{3}\left(13\,d-32\,e+2\,f+16\,g+h-16\,i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\left(13\,d+32\,e+2\,f-16\,g+h+16\,i\right)\text{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxim
a")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e + 2*f + 16*g + h - 16*i)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/144*sqrt(3)*(13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*sqr
t(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f
 + 3*h)*log(x^2 - x + 1) - 1/24*((7*d - 7*f + 4*h)*x^7 - 4*(2*e - g + i)*x^
6 + 5*(d - 2*f + h)*x^5 - 6*(2*e - g + i)*x^4 + 7*(d - 2*f + h)*x^3 - 4*(4*
```

```
e - 2*g + i)*x^2 - (4*d + 5*f - 5*h)*x - 6*e + 6*g - 4*i)/(x^8 + 2*x^6 + 3*
x^4 + 2*x^2 + 1)
```

Fricas [B] time = 59.0039, size = 1401, normalized size = 5.21

$$-\frac{12\left(7d-7f+4h\right)x^{7}-48\left(2e-g+i\right)x^{6}+60\left(d-2f+h\right)x^{5}-72\left(2e-g+i\right)x^{4}+84\left(d-2f+h\right)x^{3}-48\left(4e-2g+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="frica
s")
```

```
[Out] -1/288*(12*(7*d - 7*f + 4*h)*x^7 - 48*(2*e - g + i)*x^6 + 60*(d - 2*f + h)*
x^5 - 72*(2*e - g + i)*x^4 + 84*(d - 2*f + h)*x^3 - 48*(4*e - 2*g + i)*x^2
- 2*sqrt(3)*((13*d - 32*e + 2*f + 16*g + h - 16*i)*x^8 + 2*(13*d - 32*e + 2
*f + 16*g + h - 16*i)*x^6 + 3*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^4 + 2
*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^2 + 13*d - 32*e + 2*f + 16*g + h -
 16*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f - 16*g
 + h + 16*i)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h + 16*i)*x^6 + 3*(13*d +
32*e + 2*f - 16*g + h + 16*i)*x^4 + 2*(13*d + 32*e + 2*f - 16*g + h + 16*i)
*x^2 + 13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*sqrt(3)*(2*x - 1)) -
 12*(4*d + 5*f - 5*h)*x - 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^
6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*lo
g(x^2 + x + 1) + 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*
d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x
 + 1) - 72*e + 72*g - 48*i)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0845, size = 344, normalized size = 1.28

$$\frac{1}{144}\sqrt{3}\left(13\,d+2\,f+16\,g+h-16\,i-32\,e\right)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\left(13\,d+2\,f-16\,g+h+16\,i+32\,e\right)\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac"
)
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 16*i - 32*e)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 16*i + 32*e)*arctan(1/3*sqr
t(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f
 + 3*h)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*h*x^7 + 4*g*x^6 - 4*
i*x^6 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 + 5*h*x^5 + 6*g*x^4 - 6*i*x^4 - 12*x^4
*e + 7*d*x^3 - 14*f*x^3 + 7*h*x^3 + 8*g*x^2 - 4*i*x^2 - 16*x^2*e - 4*d*x -
5*f*x + 5*h*x + 6*g - 4*i - 6*e)/(x^4 + x^2 + 1)^2
```

3.52 $\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=474

$$\frac{3\sqrt{c}d\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}d\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (d*x*(b^2 - 2*
a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c
*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (d*x*((b^2 - 7*a*c)*(3*b^2
 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b
^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(
b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*d*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 2.19345, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1673, 12, 1092, 1178, 1166, 205, 1107, 614, 618, 206}

$$\frac{3\sqrt{c}d\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}d\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\right.}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (d*x*(b^2 - 2*
a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2*c
*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (d*x*((b^2 - 7*a*c)*(3*b^2
 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b
^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
```

```
Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]*(
b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*d*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{d}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{ex}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= d\int \frac{1}{\left(a+bx^2+cx^4\right)^3}\,dx + e\int \frac{x}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{dx\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{d\int \frac{b^2-2ac-4\left(b^2-4ac\right)-5bcx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^3}\right.$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{dx\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{dx\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)\right.}{8a^2\left(b^2-4ac\right)^2\left(a\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{dx\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{dx\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{dx\left(b^2-2ac+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx\right.}$$

Mathematica [A] time = 2.30072, size = 488, normalized size = 1.03

$$\frac{1}{16}\left(\frac{3\sqrt{2}\sqrt{c}d\left(56a^2c^2+b^3\sqrt{b^2-4ac}-10ab^2c-8abc\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{3\sqrt{2}\sqrt{c}d\left(56a^2c^2-b^3\sqrt{b^2-\right.}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*a*b*e + 8*a*c*x*(d + e*x) - 4*b*d*x*(b + c*x^2))/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) - 2*a*b*c*d*x*(25*b + 24*c*x^2)
 + 8*a^2*c*(3*b*e + c*x*(7*d + 6*e*x)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c
```

```
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 -
 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c]
 + 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (48*c
^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48*c^2*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] time = 0.24, size = 3725, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/a^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*
c+b^2)^(1/2)*b^4*d-15/8*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d+3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)/a^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d-15/8*c^2/(16*a^2
*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2
*d-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)^2*e*b^3-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a
*c+b^2)^(1/2)/c+1/2*b/c)^2*e*b^3+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^
2)*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)-3*c^2/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^
2)^(1/2)/c)^2*e*(-4*a*c+b^2)^(1/2)*b^2-1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-
b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*e*(-4*a*c+b^2)^(1/2)*b^2-11*c
^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2
*b/c)^2*d*a*x+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b
^2)^(1/2)/c+1/2*b/c)^2*d*x*b^2+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(
x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*e*(-4*a*c+b^2)^(1/2)*a+3*c/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*e*a
*b+9/2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/
2)/c+1/2*b/c)^2*d*x^3*(-4*a*c+b^2)^(1/2)-6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*d*x^3*b-9/2*c^2/(16*a^2
```

*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*d*
x^3*(-4*a*c+b^2)^(1/2)-6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/
2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*d*x^3*b+6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/
(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*e*x^2*a-3/2*c/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*e*x
^2*b^2-11*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a
*c+b^2)^(1/2)/c)^2*d*a*x+4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/
2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*d*x*b^2-4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2*e*(-4*a*c+b^2)^(1/2)*a-3
/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/
2*b/c)^2/a^2*d*x^3*b^5-5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2
*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*d/a*x*b^4-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*d*x^3*b^5-5/16/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^
2*d/a*x*b^4+3*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4
*a*c+b^2)^(1/2)/c)^2*e*a*b+6*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^
2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*e*x^2*a-3/2*c/(16*a^2*c^2-8*a*b^2*c+b
^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2*e*x^2*b^2+3/16*c/(
16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/a^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^5*d-15/8*c/
(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/
c)^2/a*d*x^3*(-4*a*c+b^2)^(1/2)*b^2+9/4*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-3/16*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+15/8*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*d*x^3*(-4*a*c+b^2)^(1/2)*b
^2-9/4*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/a*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3
*d+5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/
c+1/2*b/c)^2*d/a*x*(-4*a*c+b^2)^(1/2)*b^3-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(
4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a^2*d*x^3*(-4*a*c+b^2)^
(1/2)*b^4-5/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*
a*c+b^2)^(1/2)/c)^2*d/a*x*(-4*a*c+b^2)^(1/2)*b^3+21/2*c^3/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d+6*c^3/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+21/2*c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-6*c^3
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+9/4*c/(16*a^
2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2/a
*d*x^3*b^3-5/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*(-4*a*c+b^
2)^(1/2)/c+1/2*b/c)^2*d*x*(-4*a*c+b^2)^(1/2)*b+9/4*c/(16*a^2*c^2-8*a*b^2*c+

```
b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)^2/a*d*x^3*b^3+5/4*c
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)
/c)^2*d*x*(-4*a*c+b^2)^(1/2)*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)/
(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)^2/a^2*d*x^3*(-4*a*c+b^2)^(1/2)*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{24\,a^2c^3ex^6 + 36\,a^2bc^2ex^4 + 3\left(b^3c^2 - 8\,abc^3\right)dx^7 + \left(6\,b^4c - 49\,ab^2c^2 + 28\,a^2c^3\right)dx^5 + \left(3\,b^5 - 20\,ab^3c - 4\,a^2bc^2\right)dx^3 + 8\left(a^2 \cdots\right)}{8\left(\left(a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4\right)x^8 + a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2 + 2\left(a^2b^5c - 8\,a^3b^3c^2 + 16\,a^4bc^3\right)x^6 + \left(a^2b^6 - \cdots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*e*x^6 + 36*a^2*b*c^2*e*x^4 + 3*(b^3*c^2 - 8*a*b*c^3)*d*x^7
+ (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d*x^5 + (3*b^5 - 20*a*b^3*c - 4*a^2
*b*c^2)*d*x^3 + 8*(a^2*b^2*c + 5*a^3*c^2)*e*x^2 + (5*a*b^4 - 37*a^2*b^2*c +
 44*a^3*c^2)*d*x - 2*(a^2*b^3 - 10*a^3*b*c)*e)/((a^2*b^4*c^2 - 8*a^3*b^2*c^
3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8
*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4
 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - 3/8*integrate(-(16*a^2*c
^2*e*x + (b^3*c - 8*a*b*c^2)*d*x^2 + (b^4 - 9*a*b^2*c + 28*a^2*c^2)*d)/(c*x
^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.53 $\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=621

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2cd+}{\sqrt{b^2-4ac}}\right.}{8}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(
3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b
*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f)
+ 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3
*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 4.51193, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2cd+}{\sqrt{b^2-4ac}}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(
```

```
3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b
*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f)
+ 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3
*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{ex}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-3b^2d+14acd-abf-5c(bd-2af)x^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)} + e\int \frac{x}{\left(a+bx^2+cx^4\right.}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(3b^4d-25ab^2cd+28a^2c^2d+ab^3f+8a^2bcf+c\left(3\right.\right.}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(3b^4d-25ab^2cd\right.}{}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx\right.}{2\left(b^2-4ac\right)^2\left(a+b\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx\right.}{2\left(b^2-4ac\right)^2\left(a+b\right.}$$

$$= -\frac{e\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3ce\left(b+2cx\right.}{2\left(b^2-4ac\right)^2\left(a+b\right.}$$

Mathematica [A] time = 4.4632, size = 625, normalized size = 1.01

$$\frac{1}{16}\left(\frac{8a^2c\left(b(3e+2fx)+cx\left(7d+6ex+5fx^2\right)\right)+2abx\left(b^2f-25bcd+bcfx^2-24c^2dx^2\right)+6b^3dx\left(b+cx^2\right)}{a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(4a\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-25*
```

```
b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c*x*
(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[
b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*
(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*S
qrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2
*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48
*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] time = 0.28, size = 7858, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.54 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=646

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2cd+}{\sqrt{b^2-4ac}}\right.}{8}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f +
8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2
- 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*
d - Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d +
a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f -
52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c
)^(5/2)
```

Rubi [A] time = 3.29948, antiderivative size = 646, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1673, 1178, 1166, 205, 1247, 638, 614, 618, 206}

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2cd+}{\sqrt{b^2-4ac}}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a
```

```
+ b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f +
8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2
- 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*
d - Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d +
a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f -
52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c
)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right) - \frac{\int \frac{-3b^2d+1}{}}{}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(3b^4d-25ab^2cd\right.}{}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(2ce-bg)\left(b\right.}{4\left(b^2-4ac\right)^2\left(a+\right.}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(2ce-bg)\left(b\right.}{4\left(b^2-4ac\right)^2\left(a+\right.}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(2ce-bg)\left(b\right.}{4\left(b^2-4ac\right)^2\left(a+\right.}$$

Mathematica [A] time = 5.19493, size = 661, normalized size = 1.02

$$\frac{1}{16}\left(\frac{2\left(a^2\left(-6b^2g+4bc\left(3e+2fx-3gx^2\right)+4c^2x\left(7d+6ex+5fx^2\right)\right)+abx\left(b^2f-25bcd+bcfx^2-24c^2dx^2\right)+3b^3dx\left(\right.\right.}{a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-8*a^2*g - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)) + 4*a*b*(e + x
*(f - g*x)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*(3*b^3*d*x*(b +
c*x^2) + a*b*x*(-25*b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + a^2*(-6*b^2
*g + 4*c^2*x*(7*d + 6*e*x + 5*f*x^2) + 4*b*c*(3*e + 2*f*x - 3*g*x^2))))/(a^
2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3
```

```
*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*
b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4
*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-3*b^4*d + b^3
*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*Sqrt[b^2 - 4*a*c]*d + 13*a*f)
+ a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(-42*c*d + 5*Sqrt[b^2 - 4*
a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2
 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (24*c*(-2*c*e + b*g)*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (24*c*(-2*c*e + b*g)*
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] time = 0.285, size = 10222, normalized size = 15.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.55 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=679

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)-ab^2(7ah+25cd)+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*
a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/(4
*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f
 + 4*a^2*c*(7*c*d + a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f +
20*a^2*c*f - 12*a*b*(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h)
 + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(
7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*
d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*
h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (3*c*(2*
c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 4.18216, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1673, 1678, 1178, 1166, 205, 1247, 638, 614, 618, 206}

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)-ab^2(7ah+25cd)+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```



```
[Out] -(b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*
```

```
a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/(4
*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f
 + 4*a^2*c*(7*c*d + a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f +
20*a^2*c*f - 12*a*b*(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h)
 + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(
7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*
d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*
h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (3*c*(2*
c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^3}\right.$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A] time = 6.64503, size = 845, normalized size = 1.24

$$-\frac{bcdx^3-2acfx^3+abhx^3-2acex^2+abgx^2+b^2dx-2acdx-abfx+2a^2hx-abe+2a^2g}{4a\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)^2} + \frac{\sqrt{c}\left(3db^4+3\sqrt{b^2-4ac}db^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(-(a*b*e) + 2*a^2*g + b^2*d*x - 2*a*c*d*x - a*b*f*x + 2*a^2*h*x - 2*a*c*e*
x^2 + a*b*g*x^2 + b*c*d*x^3 - 2*a*c*f*x^3 + a*b*h*x^3)/(4*a*(-b^2 + 4*a*c)*
```

```
(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c*e - 6*a^2*b^2*g + 3*b^4*d*x - 25*a*b^2
*c*d*x + 28*a^2*c^2*d*x + a*b^3*f*x + 8*a^2*b*c*f*x - 7*a^2*b^2*h*x + 4*a^3
*c*h*x + 24*a^2*c^2*e*x^2 - 12*a^2*b*c*g*x^2 + 3*b^3*c*d*x^3 - 24*a*b*c^2*d
*x^3 + a*b^2*c*f*x^3 + 20*a^2*c^2*f*x^3 - 12*a^2*b*c*h*x^3)/(8*a^2*(-b^2 +
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168*a^2*
c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d + a*b^3*f
- 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c]*f +
 18*a^2*b^2*h + 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a^2
*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b^3*f
 + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c]*f
- 18*a^2*b^2*h - 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (3*c*(2*c*e - b*g)*Log[-b + Sqrt[b^2 - 4*a
*c] - 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2)) - (3*c*(2*c*e - b*g)*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.067, size = 3492, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -12*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+
9/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+
12*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h-
9/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d
+6*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*h-1/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b^4*f-13/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-13/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c
-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
```

```
*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f+9/2/(16*a^2*c^2-8*a*b^2*c
+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+9/2/(16*
a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^
(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1
/2)*b^2*h+1/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a
*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1
/2))*b^4*f+3/4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-
4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2))*b^5*d-3/4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*b^5*d+6*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*(-4*a*c+b^2)^(1/2)*h+1/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4
*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*f+1/4/a/(16*a^2*c^2-8*a*b^2
*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c
*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*f+3/4/a
^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b
^2)^(1/2)*b^4*d+3/4/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-
b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d-15/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2
/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-15/2/a/(16*a^2
*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2
)*b^2*d+(-1/8*c^2*(12*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^7-3/2*c^2*(b*g-2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^6+1/8/a^2*c*(4*a^3*c*h-19*a^2*b^2*h+28*a^2*b*c*f+28*a^2*c^2*d+2*a*b^3*f-
49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*(b*g-2*c*e)/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*(16*a^3*b*c*h-36*a^3*c^2*f+5*a^2*b^3*h-5*a
^2*b^2*c*f+4*a^2*b*c^2*d-a*b^4*f+20*a*b^3*c*d-3*b^5*d)/a^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3-1/2*(5*a*c+b^2)*(b*g-2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1
/8*(12*a^3*c*h+3*a^2*b^2*h-16*a^2*b*c*f-44*a^2*c^2*d+a*b^3*f+37*a*b^2*c*d-5
*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x-1/4*(8*a^2*c*g+a*b^2*g-10*a*b*c*e+b^
3*e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+6/(16*a^2*c^2-8*a*b^2*c+
b^4)*c/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*
b*g-6/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(
1/2)+b)*(-4*a*c+b^2)^(1/2)*b*g+42/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-4/(16*a^2*c^2-8*a*b^2*c+b^4)*c
^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-24/(16*a^2*c^2-8*a*b^2*c+b^4)*c^
3/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
```

```
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(1
6*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*f+24/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(1
6*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+42/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*
a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d+3/(16*a^2*c^2-8*a*b^2*c
+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*h-3/(16*a^2*c^2-8*a*b^2*c+b^4
)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*h+20*a/(16*a^2*c^2-8*a*b^2*c+b^4
)*c^3/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-20*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c
^3/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f-12/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(
16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*e+12/(16
*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
*(-4*a*c+b^2)^(1/2)*e
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima"
)
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.56 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=728

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)-ab^2(7ah+25cd)+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e
- b*c*g + b^2*i - 2*a*c*i)*x^2)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 ((6*c*e - 3*b*g + 2*a*i + (b^2*i)/c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d +
a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*
(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]
*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) + (3*b^4*d + a*b^3*
f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d
+ a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2
*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*e - 3*b*c*g + b^2*i
+ 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 2.73276, antiderivative size = 728, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.275, Rules used = {1673, 1678, 1178, 1166, 205, 1663, 1660, 12, 614, 618, 206}

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)-ab^2(7ah+25cd)+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e
```

```
- b*c*g + b^2*i - 2*a*c*i)*x^2)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 ((6*c*e - 3*b*g + 2*a*i + (b^2*i)/c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d +
a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*
(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]
*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) + (3*b^4*d + a*b^3*
f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d
+ a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2
*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*e - 3*b*c*g + b^2*i
+ 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+56x^5}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{x\left(e+gx^2+56x^4\right)}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= \frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{e+gx+}{(a+bx+}\right.$$

$$= -\frac{56ab+bce-2acg+\left(56b^2-2c(56a-ce)-bcg\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd\right.}{4a\left(b^2-4\right.}$$

$$= -\frac{56ab+bce-2acg+\left(56b^2-2c(56a-ce)-bcg\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd\right.}{4a\left(b^2-4\right.}$$

$$= -\frac{56ab+bce-2acg+\left(56b^2-2c(56a-ce)-bcg\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd\right.}{4a\left(b^2-4\right.}$$

$$= -\frac{56ab+bce-2acg+\left(56b^2-2c(56a-ce)-bcg\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd\right.}{4a\left(b^2-4\right.}$$

$$= -\frac{56ab+bce-2acg+\left(56b^2-2c(56a-ce)-bcg\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(b^2d-abf-2a(cd\right.}{4a\left(b^2-4\right.}$$

Mathematica [A] time = 6.85054, size = 980, normalized size = 1.35

$$\frac{-bc^2dx^3+2ac^2fx^3-abchx^3+2ac^2ex^2-abcgx^2+ab^2ix^2-2a^2cix^2+2ac^2dx-b^2cdx+abcfx-2a^2chx+abce-2a^2cg+}{4ac\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x
]
```

```
[Out] (a*b*c*e - 2*a^2*c*g + a^2*b*i - b^2*c*d*x + 2*a*c^2*d*x + a*b*c*f*x - 2*a^
2*c*h*x + 2*a*c^2*e*x^2 - a*b*c*g*x^2 + a*b^2*i*x^2 - 2*a^2*c*i*x^2 - b*c^2
*d*x^3 + 2*a*c^2*f*x^3 - a*b*c*h*x^3)/(4*a*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (12*a^2*b*c^2*e - 6*a^2*b^2*c*g + 2*a^2*b^3*i + 4*a^3*b*c*i + 3*b
^4*c*d*x - 25*a*b^2*c^2*d*x + 28*a^2*c^3*d*x + a*b^3*c*f*x + 8*a^2*b*c^2*f*
x - 7*a^2*b^2*c*h*x + 4*a^3*c^2*h*x + 24*a^2*c^3*e*x^2 - 12*a^2*b*c^2*g*x^2
 + 4*a^2*b^2*c*i*x^2 + 8*a^3*c^2*i*x^2 + 3*b^3*c^2*d*x^3 - 24*a*b*c^3*d*x^3
 + a*b^2*c^2*f*x^3 + 20*a^2*c^3*f*x^3 - 12*a^2*b*c^2*h*x^3)/(8*a^2*c*(-b^2
+ 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d - 30*a*b^2*c*d + 168*a^
2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d + a*b^3*
f - 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c]*f
 + 18*a^2*b^2*h + 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-3*b^4*d + 30*a*b^2*c*d - 168*a
^2*c^2*d + 3*b^3*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c*Sqrt[b^2 - 4*a*c]*d - a*b^3
*f + 52*a^2*b*c*f + a*b^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c*Sqrt[b^2 - 4*a*c]*
f - 18*a^2*b^2*h - 24*a^3*c*h - 12*a^2*b*Sqrt[b^2 - 4*a*c]*h)*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5
/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g + b^2*i + 2*a*c*i)*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2)) + ((-6*c^2*e
+ 3*b*c*g - b^2*i - 2*a*c*i)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*(b^2
- 4*a*c)^(5/2))
```

Maple [B] time = 0.046, size = 3824, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*c^2*(12*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^7+1/2*c*(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^6+1/8/a^2*c*(4*a^3*c*h-19*a^2*b^2*h+28*a^2*b*c*f+28*a^2*c^2*d+
2*a*b^3*f-49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/4*b*(2*a*c
*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*(16*a^3*b*c*h-
36*a^3*c^2*f+5*a^2*b^3*h-5*a^2*b^2*c*f+4*a^2*b*c^2*d-a*b^4*f+20*a*b^3*c*d-3
*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(2*a^2*c*i-5*a*b^2*i+5*a*b*c
*g-10*a*c^2*e+b^3*g-2*b^2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(12*a^3*c
*h+3*a^2*b^2*h-16*a^2*b*c*f-44*a^2*c^2*d+a*b^3*f+37*a*b^2*c*d-5*b^4*d)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/4*(6*a^2*b*i-8*a^2*c*g-a*b^2*g+10*a*b*c*e-b^3*
e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-12*a/(16*a^2*c^2-8*a*b^2*c
+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+9/a/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d+12*a/(16*a^2*c^2-8*a*b^2*c+
b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*h-9/a/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x
*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d+6*a/(16*a^2*c^2-8*a*b^2*c+
b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*
x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h-1/4/a/(16*
a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*f-13/(16*a^2
*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/
2)*b*f-13/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
(-4*a*c+b^2)^(1/2)*b*f+9/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+9/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(1
6*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*h+1/4/a/(16*a^2*c^2
-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*a
rctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*f+3/4/a^2/(16*a^2*
c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^5*d-3/4/a^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5*d+6*a/(16*a^2
*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2
)*h+1/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-
4*a*c+b^2)^(1/2)*b^3*f+1/4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^
(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*f+3/4/a^2/(16*a^2*c^2-8*a*b^2*c+b^
```

```
4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^4*d+3/4/a^2/(16
*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)
^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(
1/2)*b^4*d-15/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((
-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c
)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-15/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16
*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d+2/(16*a^2*c^2-8*a*b
^2*c+b^4)/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2
)*b^2*i-2/(16*a^2*c^2-8*a*b^2*c+b^4)/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2
)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b^2*i+6/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-
4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b*g-6/(16*a^2*c
^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c
+b^2)^(1/2)*b*g+4*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*ln(2*c*x^2+
(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*i-4*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c
/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*i+42/(
16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2
)^(1/2)*d-4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*f-24/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b*d+4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^
2*f+24/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*
d+42/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*d+3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^3*h-3/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*
c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/
2))*b^3*h+20*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*f-20*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^3/(16*a*c-4*b^2)*2^(1/2)/(((-4*a*c
+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
))*f-12/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c-4*b^2)*ln(-2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*e+12/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(16*a*c
-4*b^2)*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*e
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="m
axima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 29.7546, size = 16458, normalized size = 22.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="g
iac")

[Out] 1/64*(32*a^2*b^7 - 192*a^3*b^5*c + 128*a^2*b^6*c - 256*a^3*b^4*c^2 + 128*a^
2*b^5*c^2 + 1024*a^5*b*c^3 - 1024*a^4*b^2*c^3 + 256*a^3*b^3*c^3 - 16*a^2*b^
6 + 96*a^3*b^4*c - 128*a^2*b^5*c + 256*a^3*b^3*c^2 - 192*a^2*b^4*c^2 - 512*
a^5*c^3 + 1024*a^4*b*c^3 - 384*a^3*b^2*c^3 + 32*a^2*b^4*c - 64*a^3*b^2*c^2
+ 96*a^2*b^3*c^2 - 256*a^4*c^3 + 192*a^3*b*c^3 - 16*a^2*b^2*c^2 - 32*a^3*c^
3 + 3*(2*b^8 - 36*a*b^6*c + 8*b^7*c + 304*a^2*b^4*c^2 - 112*a*b^5*c^2 + 8*b
^6*c^2 - 1216*a^3*b^2*c^3 + 768*a^2*b^3*c^3 - 80*a*b^4*c^3 + 1792*a^4*c^4 -
 1792*a^3*b*c^4 + 448*a^2*b^2*c^4 + b^7 - 16*a*b^5*c + 80*a^2*b^3*c^2 + 8*a
*b^4*c^2 - 4*b^5*c^2 - 128*a^3*b*c^3 - 256*a^2*b^2*c^3 + 48*a*b^3*c^3 + 896
*a^3*c^4 - 448*a^2*b*c^4 - 2*b^5*c + 24*a*b^3*c^2 - 2*b^4*c^2 - 64*a^2*b*c^
3 + 12*a*b^2*c^3 + 112*a^2*c^4 + b^3*c^2 - 8*a*b*c^3)*sqrt(2*b*c - c)*d + (
2*a*b^7 - 120*a^2*b^5*c + 8*a*b^6*c + 864*a^3*b^3*c^2 - 448*a^2*b^4*c^2 + 8
*a*b^5*c^2 - 1664*a^4*b*c^3 + 1664*a^3*b^2*c^3 - 416*a^2*b^3*c^3 + a*b^6 +
12*a^2*b^4*c - 144*a^3*b^2*c^2 + 288*a^2*b^3*c^2 - 4*a*b^4*c^2 + 320*a^4*c^
3 - 1152*a^3*b*c^3 + 496*a^2*b^2*c^3 - 2*a*b^4*c - 32*a^2*b^2*c^2 - 2*a*b^3
*c^2 + 160*a^3*c^3 - 184*a^2*b*c^3 + a*b^2*c^2 + 20*a^2*c^3)*sqrt(2*b*c - c
)*f + 12*(3*a^2*b^6 - 20*a^3*b^4*c + 12*a^2*b^5*c + 16*a^4*b^2*c^2 - 32*a^3
*b^3*c^2 + 12*a^2*b^4*c^2 + 64*a^5*c^3 - 64*a^4*b*c^3 + 16*a^3*b^2*c^3 - a^
2*b^5 + 8*a^3*b^3*c - 10*a^2*b^4*c - 16*a^4*b*c^2 + 32*a^3*b^2*c^2 - 16*a^2
*b^3*c^2 + 32*a^4*c^3 - 16*a^3*b*c^3 + 2*a^2*b^3*c - 8*a^3*b*c^2 + 7*a^2*b^
2*c^2 + 4*a^3*c^3 - a^2*b*c^2)*sqrt(2*b*c - c)*h + 48*(2*a^2*b^6*c*i - 16*a
^3*b^4*c^2*i + 8*a^2*b^5*c^2*i + 32*a^4*b^2*c^3*i - 32*a^3*b^3*c^3*i + 8*a^
2*b^4*c^3*i - a^2*b^5*c*i + 8*a^3*b^3*c^2*i - 8*a^2*b^4*c^2*i - 16*a^4*b*c^
3*i + 32*a^3*b^2*c^3*i - 12*a^2*b^3*c^3*i + 2*a^2*b^3*c^2*i - 8*a^3*b*c^3*i
 + 6*a^2*b^2*c^3*i - a^2*b*c^3*i)*g - 96*(2*a^2*b^5*c^2*i - 16*a^3*b^3*c^3*
i + 8*a^2*b^4*c^3*i + 32*a^4*b*c^4*i - 32*a^3*b^2*c^4*i + 8*a^2*b^3*c^4*i -
 a^2*b^4*c^2*i + 8*a^3*b^2*c^3*i - 8*a^2*b^3*c^3*i - 16*a^4*c^4*i + 32*a^3*
b*c^4*i - 12*a^2*b^2*c^4*i + 2*a^2*b^2*c^3*i - 8*a^3*c^4*i + 6*a^2*b*c^4*i
- a^2*c^4*i)*e)*log(x + 1/4*sqrt(-(8*a^2*b^5*i - 64*a^3*b^3*c*i + 128*a^4*b
*c^2*i + sqrt(-64*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 + 256*(a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*
b^4*c*i - 8*a^3*b^2*c^2*i + 16*a^4*c^3*i)))/(a^2*b^10*i - 20*a^3*b^8*c*i +
4*a^2*b^9*c*i + 160*a^4*b^6*c^2*i - 64*a^3*b^7*c^2*i + 4*a^2*b^8*c^2*i - 64
0*a^5*b^4*c^3*i + 384*a^4*b^5*c^3*i - 48*a^3*b^6*c^3*i + 1280*a^6*b^2*c^4*i
 - 1024*a^5*b^3*c^4*i + 192*a^4*b^4*c^4*i - 1024*a^7*c^5*i + 1024*a^6*b*c^5
*i - 256*a^5*b^2*c^5*i - 2*a^2*b^8*c*i + 32*a^3*b^6*c^2*i - 4*a^2*b^7*c^2*i
 - 192*a^4*b^4*c^3*i + 48*a^3*b^5*c^3*i + 512*a^5*b^2*c^4*i - 192*a^4*b^3*c
^4*i - 512*a^6*c^5*i + 256*a^5*b*c^5*i + a^2*b^6*c^2*i - 12*a^3*b^4*c^3*i +
 48*a^4*b^2*c^4*i - 64*a^5*c^5*i + sqrt(-b^18 + 36*a*b^16*c - 8*b^17*c - 57
6*a^2*b^14*c^2 + 256*a*b^15*c^2 - 24*b^16*c^2 + 5376*a^3*b^12*c^3 - 3584*a^
2*b^13*c^3 + 672*a*b^14*c^3 - 32*b^15*c^3 - 32256*a^4*b^10*c^4 + 28672*a^3*
```

b^11*c^4 - 8064*a^2*b^12*c^4 + 768*a*b^13*c^4 - 16*b^14*c^4 + 129024*a^5*b^
8*c^5 - 143360*a^4*b^9*c^5 + 53760*a^3*b^10*c^5 - 7680*a^2*b^11*c^5 + 320*a
*b^12*c^5 - 344064*a^6*b^6*c^6 + 458752*a^5*b^7*c^6 - 215040*a^4*b^8*c^6 +
40960*a^3*b^9*c^6 - 2560*a^2*b^10*c^6 + 589824*a^7*b^4*c^7 - 917504*a^6*b^5
*c^7 - 1125899906064384*a^5*b^6*c^7 - 122880*a^4*b^7*c^7 + 10240*a^3*b^8*c^
7 - 589824*a^8*b^2*c^8 + 1048576*a^7*b^3*c^8 - 688128*a^6*b^4*c^8 + 196608*
a^5*b^5*c^8 - 20480*a^4*b^6*c^8 + 262144*a^9*c^9 - 524288*a^8*b*c^9 + 39321
6*a^7*b^2*c^9 - 131072*a^6*b^3*c^9 + 16384*a^5*b^4*c^9 + 4*b^16*c - 128*a*b
^14*c^2 + 24*b^15*c^2 + 1792*a^2*b^12*c^3 - 672*a*b^13*c^3 + 48*b^14*c^3 -
14336*a^3*b^10*c^4 + 8064*a^2*b^11*c^4 - 1152*a*b^12*c^4 + 32*b^13*c^4 + 71
680*a^4*b^8*c^5 - 53760*a^3*b^9*c^5 + 11520*a^2*b^10*c^5 - 640*a*b^11*c^5 -
229376*a^5*b^6*c^6 + 215040*a^4*b^7*c^6 - 61440*a^3*b^8*c^6 + 5120*a^2*b^9
*c^6 + 458752*a^6*b^4*c^7 - 516096*a^5*b^5*c^7 + 184320*a^4*b^6*c^7 - 20480
*a^3*b^7*c^7 - 524288*a^7*b^2*c^8 + 688128*a^6*b^3*c^8 - 294912*a^5*b^4*c^8
+ 40960*a^4*b^5*c^8 + 262144*a^8*c^9 - 393216*a^7*b*c^9 + 196608*a^6*b^2*c
^9 - 32768*a^5*b^3*c^9 - 6*b^14*c^2 + 168*a*b^12*c^3 - 24*b^13*c^3 - 2016*a
^2*b^10*c^4 + 576*a*b^11*c^4 - 24*b^12*c^4 + 13440*a^3*b^8*c^5 - 5760*a^2*b
^9*c^5 + 480*a*b^10*c^5 - 53760*a^4*b^6*c^6 + 30720*a^3*b^7*c^6 - 3840*a^2*
b^8*c^6 + 129024*a^5*b^4*c^7 - 92160*a^4*b^5*c^7 + 15360*a^3*b^6*c^7 - 1720
32*a^6*b^2*c^8 + 147456*a^5*b^3*c^8 - 30720*a^4*b^4*c^8 + 98304*a^7*c^9 - 9
8304*a^6*b*c^9 + 24576*a^5*b^2*c^9 + 4*b^12*c^3 - 96*a*b^10*c^4 + 8*b^11*c^
4 + 960*a^2*b^8*c^5 - 160*a*b^9*c^5 - 5120*a^3*b^6*c^6 + 1280*a^2*b^7*c^6 +
15360*a^4*b^4*c^7 - 5120*a^3*b^5*c^7 - 24576*a^5*b^2*c^8 + 10240*a^4*b^3*c
^8 + 16384*a^6*c^9 - 8192*a^5*b*c^9 - b^10*c^4 + 20*a*b^8*c^5 - 160*a^2*b^6
*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8 + 1024*a^5*c^9)*a^2*b) + 1/64*(32
*a^2*b^7 - 192*a^3*b^5*c + 128*a^2*b^6*c - 256*a^3*b^4*c^2 + 128*a^2*b^5*c^
2 + 1024*a^5*b*c^3 - 1024*a^4*b^2*c^3 + 256*a^3*b^3*c^3 - 16*a^2*b^6 + 96*a
^3*b^4*c - 128*a^2*b^5*c + 256*a^3*b^3*c^2 - 192*a^2*b^4*c^2 - 512*a^5*c^3
+ 1024*a^4*b*c^3 - 384*a^3*b^2*c^3 + 32*a^2*b^4*c - 64*a^3*b^2*c^2 + 96*a^2
*b^3*c^2 - 256*a^4*c^3 + 192*a^3*b*c^3 - 16*a^2*b^2*c^2 - 32*a^3*c^3 - 3*(2
*b^8 - 36*a*b^6*c + 8*b^7*c + 304*a^2*b^4*c^2 - 112*a*b^5*c^2 + 8*b^6*c^2 -
1216*a^3*b^2*c^3 + 768*a^2*b^3*c^3 - 80*a*b^4*c^3 + 1792*a^4*c^4 - 1792*a^
3*b*c^4 + 448*a^2*b^2*c^4 + b^7 - 16*a*b^5*c + 80*a^2*b^3*c^2 + 8*a*b^4*c^2
- 4*b^5*c^2 - 128*a^3*b*c^3 - 256*a^2*b^2*c^3 + 48*a*b^3*c^3 + 896*a^3*c^4
- 448*a^2*b*c^4 - 2*b^5*c + 24*a*b^3*c^2 - 2*b^4*c^2 - 64*a^2*b*c^3 + 12*a
*b^2*c^3 + 112*a^2*c^4 + b^3*c^2 - 8*a*b*c^3)*sqrt(2*b*c - c)*d - (2*a*b^7
- 120*a^2*b^5*c + 8*a*b^6*c + 864*a^3*b^3*c^2 - 448*a^2*b^4*c^2 + 8*a*b^5*c
^2 - 1664*a^4*b*c^3 + 1664*a^3*b^2*c^3 - 416*a^2*b^3*c^3 + a*b^6 + 12*a^2*b
^4*c - 144*a^3*b^2*c^2 + 288*a^2*b^3*c^2 - 4*a*b^4*c^2 + 320*a^4*c^3 - 1152
*a^3*b*c^3 + 496*a^2*b^2*c^3 - 2*a*b^4*c - 32*a^2*b^2*c^2 - 2*a*b^3*c^2 + 1
60*a^3*c^3 - 184*a^2*b*c^3 + a*b^2*c^2 + 20*a^2*c^3)*sqrt(2*b*c - c)*f - 12
*(3*a^2*b^6 - 20*a^3*b^4*c + 12*a^2*b^5*c + 16*a^4*b^2*c^2 - 32*a^3*b^3*c^2
+ 12*a^2*b^4*c^2 + 64*a^5*c^3 - 64*a^4*b*c^3 + 16*a^3*b^2*c^3 - a^2*b^5 +
8*a^3*b^3*c - 10*a^2*b^4*c - 16*a^4*b*c^2 + 32*a^3*b^2*c^2 - 16*a^2*b^3*c^2
+ 32*a^4*c^3 - 16*a^3*b*c^3 + 2*a^2*b^3*c - 8*a^3*b*c^2 + 7*a^2*b^2*c^2 +

4*a^3*c^3 - a^2*b*c^2)*sqrt(2*b*c - c)*h + 48*(2*a^2*b^6*c*i - 16*a^3*b^4*c
^2*i + 8*a^2*b^5*c^2*i + 32*a^4*b^2*c^3*i - 32*a^3*b^3*c^3*i + 8*a^2*b^4*c^
3*i - a^2*b^5*c*i + 8*a^3*b^3*c^2*i - 8*a^2*b^4*c^2*i - 16*a^4*b*c^3*i + 32
*a^3*b^2*c^3*i - 12*a^2*b^3*c^3*i + 2*a^2*b^3*c^2*i - 8*a^3*b*c^3*i + 6*a^2
*b^2*c^3*i - a^2*b*c^3*i)*g - 96*(2*a^2*b^5*c^2*i - 16*a^3*b^3*c^3*i + 8*a^
2*b^4*c^3*i + 32*a^4*b*c^4*i - 32*a^3*b^2*c^4*i + 8*a^2*b^3*c^4*i - a^2*b^4
*c^2*i + 8*a^3*b^2*c^3*i - 8*a^2*b^3*c^3*i - 16*a^4*c^4*i + 32*a^3*b*c^4*i
- 12*a^2*b^2*c^4*i + 2*a^2*b^2*c^3*i - 8*a^3*c^4*i + 6*a^2*b*c^4*i - a^2*c^
4*i)*e)*log(x - 1/4*sqrt(-(8*a^2*b^5*i - 64*a^3*b^3*c*i + 128*a^4*b*c^2*i +
sqrt(-64*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 + 256*(a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c*i
- 8*a^3*b^2*c^2*i + 16*a^4*c^3*i)))/(a^2*b^10*i - 20*a^3*b^8*c*i + 4*a^2*b^
9*c*i + 160*a^4*b^6*c^2*i - 64*a^3*b^7*c^2*i + 4*a^2*b^8*c^2*i - 640*a^5*b^
4*c^3*i + 384*a^4*b^5*c^3*i - 48*a^3*b^6*c^3*i + 1280*a^6*b^2*c^4*i - 1024*
a^5*b^3*c^4*i + 192*a^4*b^4*c^4*i - 1024*a^7*c^5*i + 1024*a^6*b*c^5*i - 256
*a^5*b^2*c^5*i - 2*a^2*b^8*c*i + 32*a^3*b^6*c^2*i - 4*a^2*b^7*c^2*i - 192*a
^4*b^4*c^3*i + 48*a^3*b^5*c^3*i + 512*a^5*b^2*c^4*i - 192*a^4*b^3*c^4*i - 5
12*a^6*c^5*i + 256*a^5*b*c^5*i + a^2*b^6*c^2*i - 12*a^3*b^4*c^3*i + 48*a^4*
b^2*c^4*i - 64*a^5*c^5*i + sqrt(-b^18 + 36*a*b^16*c - 8*b^17*c - 576*a^2*b^
14*c^2 + 256*a*b^15*c^2 - 24*b^16*c^2 + 5376*a^3*b^12*c^3 - 3584*a^2*b^13*c
^3 + 672*a*b^14*c^3 - 32*b^15*c^3 - 32256*a^4*b^10*c^4 + 28672*a^3*b^11*c^4
- 8064*a^2*b^12*c^4 + 768*a*b^13*c^4 - 16*b^14*c^4 + 129024*a^5*b^8*c^5 -
143360*a^4*b^9*c^5 + 53760*a^3*b^10*c^5 - 7680*a^2*b^11*c^5 + 320*a*b^12*c^
5 - 344064*a^6*b^6*c^6 + 458752*a^5*b^7*c^6 - 215040*a^4*b^8*c^6 + 40960*a^
3*b^9*c^6 - 2560*a^2*b^10*c^6 + 589824*a^7*b^4*c^7 - 917504*a^6*b^5*c^7 - 1
125899906064384*a^5*b^6*c^7 - 122880*a^4*b^7*c^7 + 10240*a^3*b^8*c^7 - 5898
24*a^8*b^2*c^8 + 1048576*a^7*b^3*c^8 - 688128*a^6*b^4*c^8 + 196608*a^5*b^5*
c^8 - 20480*a^4*b^6*c^8 + 262144*a^9*c^9 - 524288*a^8*b*c^9 + 393216*a^7*b^
2*c^9 - 131072*a^6*b^3*c^9 + 16384*a^5*b^4*c^9 + 4*b^16*c - 128*a*b^14*c^2
+ 24*b^15*c^2 + 1792*a^2*b^12*c^3 - 672*a*b^13*c^3 + 48*b^14*c^3 - 14336*a^
3*b^10*c^4 + 8064*a^2*b^11*c^4 - 1152*a*b^12*c^4 + 32*b^13*c^4 + 71680*a^4*
b^8*c^5 - 53760*a^3*b^9*c^5 + 11520*a^2*b^10*c^5 - 640*a*b^11*c^5 - 229376*
a^5*b^6*c^6 + 215040*a^4*b^7*c^6 - 61440*a^3*b^8*c^6 + 5120*a^2*b^9*c^6 + 4
58752*a^6*b^4*c^7 - 516096*a^5*b^5*c^7 + 184320*a^4*b^6*c^7 - 20480*a^3*b^7
*c^7 - 524288*a^7*b^2*c^8 + 688128*a^6*b^3*c^8 - 294912*a^5*b^4*c^8 + 40960
*a^4*b^5*c^8 + 262144*a^8*c^9 - 393216*a^7*b*c^9 + 196608*a^6*b^2*c^9 - 327
68*a^5*b^3*c^9 - 6*b^14*c^2 + 168*a*b^12*c^3 - 24*b^13*c^3 - 2016*a^2*b^10*
c^4 + 576*a*b^11*c^4 - 24*b^12*c^4 + 13440*a^3*b^8*c^5 - 5760*a^2*b^9*c^5 +
480*a*b^10*c^5 - 53760*a^4*b^6*c^6 + 30720*a^3*b^7*c^6 - 3840*a^2*b^8*c^6
+ 129024*a^5*b^4*c^7 - 92160*a^4*b^5*c^7 + 15360*a^3*b^6*c^7 - 172032*a^6*b
^2*c^8 + 147456*a^5*b^3*c^8 - 30720*a^4*b^4*c^8 + 98304*a^7*c^9 - 98304*a^6
*b*c^9 + 24576*a^5*b^2*c^9 + 4*b^12*c^3 - 96*a*b^10*c^4 + 8*b^11*c^4 + 960*
a^2*b^8*c^5 - 160*a*b^9*c^5 - 5120*a^3*b^6*c^6 + 1280*a^2*b^7*c^6 + 15360*a
^4*b^4*c^7 - 5120*a^3*b^5*c^7 - 24576*a^5*b^2*c^8 + 10240*a^4*b^3*c^8 + 163
84*a^6*c^9 - 8192*a^5*b*c^9 - b^10*c^4 + 20*a*b^8*c^5 - 160*a^2*b^6*c^6 + 6

40*a^3*b^4*c^7 - 1280*a^4*b^2*c^8 + 1024*a^5*c^9)*a^2*b) + 1/64*(32*a^2*b^7 - 192*a^3*b^5*c + 128*a^2*b^6*c - 256*a^3*b^4*c^2 + 128*a^2*b^5*c^2 + 1024*a^5*b*c^3 - 1024*a^4*b^2*c^3 + 256*a^3*b^3*c^3 + 16*a^2*b^6 - 96*a^3*b^4*c + 128*a^2*b^5*c - 256*a^3*b^3*c^2 + 192*a^2*b^4*c^2 + 512*a^5*c^3 - 1024*a^4*b*c^3 + 384*a^3*b^2*c^3 + 32*a^2*b^4*c - 64*a^3*b^2*c^2 + 96*a^2*b^3*c^2 - 256*a^4*c^3 + 192*a^3*b*c^3 + 16*a^2*b^2*c^2 + 32*a^3*c^3 - 3*(2*b^8 - 36*a*b^6*c + 8*b^7*c + 304*a^2*b^4*c^2 - 112*a*b^5*c^2 + 8*b^6*c^2 - 1216*a^3*b^2*c^3 + 768*a^2*b^3*c^3 - 80*a*b^4*c^3 + 1792*a^4*c^4 - 1792*a^3*b*c^4 + 448*a^2*b^2*c^4 - b^7 + 16*a*b^5*c - 80*a^2*b^3*c^2 - 8*a*b^4*c^2 + 4*b^5*c^2 + 128*a^3*b*c^3 + 256*a^2*b^2*c^3 - 48*a*b^3*c^3 - 896*a^3*c^4 + 448*a^2*b*c^4 - 2*b^5*c + 24*a*b^3*c^2 - 2*b^4*c^2 - 64*a^2*b*c^3 + 12*a*b^2*c^3 + 112*a^2*c^4 - b^3*c^2 + 8*a*b*c^3)*sqrt(2*b*c + c)*d - (2*a*b^7 - 120*a^2*b^5*c + 8*a*b^6*c + 864*a^3*b^3*c^2 - 448*a^2*b^4*c^2 + 8*a*b^5*c^2 - 1664*a^4*b*c^3 + 1664*a^3*b^2*c^3 - 416*a^2*b^3*c^3 - a*b^6 - 12*a^2*b^4*c + 144*a^3*b^2*c^2 - 288*a^2*b^3*c^2 + 4*a*b^4*c^2 - 320*a^4*c^3 + 1152*a^3*b*c^3 - 496*a^2*b^2*c^3 - 2*a*b^4*c - 32*a^2*b^2*c^2 - 2*a*b^3*c^2 + 160*a^3*c^3 - 184*a^2*b*c^3 - a*b^2*c^2 - 20*a^2*c^3)*sqrt(2*b*c + c)*f - 12*(3*a^2*b^6 - 20*a^3*b^4*c + 12*a^2*b^5*c + 16*a^4*b^2*c^2 - 32*a^3*b^3*c^2 + 12*a^2*b^4*c^2 + 64*a^5*c^3 - 64*a^4*b*c^3 + 16*a^3*b^2*c^3 + a^2*b^5 - 8*a^3*b^3*c + 10*a^2*b^4*c + 16*a^4*b*c^2 - 32*a^3*b^2*c^2 + 16*a^2*b^3*c^2 - 32*a^4*c^3 + 16*a^3*b*c^3 + 2*a^2*b^3*c - 8*a^3*b*c^2 + 7*a^2*b^2*c^2 + 4*a^3*c^3 + a^2*b*c^2)*sqrt(2*b*c + c)*h + 48*(2*a^2*b^6*c*i - 16*a^3*b^4*c^2*i + 8*a^2*b^5*c^2*i + 32*a^4*b^2*c^3*i - 32*a^3*b^3*c^3*i + 8*a^2*b^4*c^3*i + a^2*b^5*c*i - 8*a^3*b^3*c^2*i + 8*a^2*b^4*c^2*i + 16*a^4*b*c^3*i - 32*a^3*b^2*c^3*i + 12*a^2*b^3*c^3*i + 2*a^2*b^3*c^2*i - 8*a^3*b*c^3*i + 6*a^2*b^2*c^3*i + a^2*b*c^3*i)*g - 96*(2*a^2*b^5*c^2*i - 16*a^3*b^3*c^3*i + 8*a^2*b^4*c^3*i + 32*a^4*b*c^4*i - 32*a^3*b^2*c^4*i + 8*a^2*b^3*c^4*i + a^2*b^4*c^2*i - 8*a^3*b^2*c^3*i + 8*a^2*b^3*c^3*i + 16*a^4*c^4*i - 32*a^3*b*c^4*i + 12*a^2*b^2*c^4*i + 2*a^2*b^2*c^3*i - 8*a^3*c^4*i + 6*a^2*b*c^4*i + a^2*c^4*i)*e)*log(x + 1/4*sqrt(-(8*a^2*b^5*i - 64*a^3*b^3*c*i + 128*a^4*b*c^2*i - sqrt(-64*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 + 256*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c*i - 8*a^3*b^2*c^2*i + 16*a^4*c^3*i)))/(a^2*b^10*i - 20*a^3*b^8*c*i + 4*a^2*b^9*c*i + 160*a^4*b^6*c^2*i - 64*a^3*b^7*c^2*i + 4*a^2*b^8*c^2*i - 640*a^5*b^4*c^3*i + 384*a^4*b^5*c^3*i - 48*a^3*b^6*c^3*i + 1280*a^6*b^2*c^4*i - 1024*a^5*b^3*c^4*i + 192*a^4*b^4*c^4*i - 1024*a^7*c^5*i + 1024*a^6*b*c^5*i - 256*a^5*b^2*c^5*i + 2*a^2*b^8*c*i - 32*a^3*b^6*c^2*i + 4*a^2*b^7*c^2*i + 192*a^4*b^4*c^3*i - 48*a^3*b^5*c^3*i - 512*a^5*b^2*c^4*i + 192*a^4*b^3*c^4*i + 512*a^6*c^5*i - 256*a^5*b*c^5*i + a^2*b^6*c^2*i - 12*a^3*b^4*c^3*i + 48*a^4*b^2*c^4*i - 64*a^5*c^5*i - sqrt(-b^18 + 36*a*b^16*c - 8*b^17*c - 576*a^2*b^14*c^2 + 256*a*b^15*c^2 - 24*b^16*c^2 + 5376*a^3*b^12*c^3 - 3584*a^2*b^13*c^3 + 672*a*b^14*c^3 - 32*b^15*c^3 - 32256*a^4*b^10*c^4 + 28672*a^3*b^11*c^4 - 8064*a^2*b^12*c^4 + 768*a*b^13*c^4 - 16*b^14*c^4 + 129024*a^5*b^8*c^5 - 143360*a^4*b^9*c^5 + 53760*a^3*b^10*c^5 - 7680*a^2*b^11*c^5 + 320*a*b^12*c^5 - 344064*a^6*b^6*c^6 + 458752*a^5*b^7*c^6 - 215040*a^4*b^8*c^6 + 40960*a^3*b^9*c^

```
6 - 2560*a^2*b^10*c^6 + 589824*a^7*b^4*c^7 - 917504*a^6*b^5*c^7 - 112589990
6064384*a^5*b^6*c^7 - 122880*a^4*b^7*c^7 + 10240*a^3*b^8*c^7 - 589824*a^8*b
^2*c^8 + 1048576*a^7*b^3*c^8 - 688128*a^6*b^4*c^8 + 196608*a^5*b^5*c^8 - 20
480*a^4*b^6*c^8 + 262144*a^9*c^9 - 524288*a^8*b*c^9 + 393216*a^7*b^2*c^9 -
131072*a^6*b^3*c^9 + 16384*a^5*b^4*c^9 - 4*b^16*c + 128*a*b^14*c^2 - 24*b^1
5*c^2 - 1792*a^2*b^12*c^3 + 672*a*b^13*c^3 - 48*b^14*c^3 + 14336*a^3*b^10*c
^4 - 8064*a^2*b^11*c^4 + 1152*a*b^12*c^4 - 32*b^13*c^4 - 71680*a^4*b^8*c^5
+ 53760*a^3*b^9*c^5 - 11520*a^2*b^10*c^5 + 640*a*b^11*c^5 + 229376*a^5*b^6*
c^6 - 215040*a^4*b^7*c^6 + 61440*a^3*b^8*c^6 - 5120*a^2*b^9*c^6 - 458752*a^
6*b^4*c^7 + 516096*a^5*b^5*c^7 - 184320*a^4*b^6*c^7 + 20480*a^3*b^7*c^7 + 5
24288*a^7*b^2*c^8 - 688128*a^6*b^3*c^8 + 294912*a^5*b^4*c^8 - 40960*a^4*b^5
*c^8 - 262144*a^8*c^9 + 393216*a^7*b*c^9 - 196608*a^6*b^2*c^9 + 32768*a^5*b
^3*c^9 - 2251799813160960*a^6*b*c^10 - 6*b^14*c^2 + 168*a*b^12*c^3 - 24*b^1
3*c^3 - 2016*a^2*b^10*c^4 + 576*a*b^11*c^4 - 24*b^12*c^4 + 13440*a^3*b^8*c^
5 - 5760*a^2*b^9*c^5 + 480*a*b^10*c^5 - 53760*a^4*b^6*c^6 + 30720*a^3*b^7*c
^6 - 3840*a^2*b^8*c^6 + 129024*a^5*b^4*c^7 - 92160*a^4*b^5*c^7 + 15360*a^3*
b^6*c^7 - 172032*a^6*b^2*c^8 + 147456*a^5*b^3*c^8 - 30720*a^4*b^4*c^8 + 983
04*a^7*c^9 - 98304*a^6*b*c^9 + 24576*a^5*b^2*c^9 - 4*b^12*c^3 + 96*a*b^10*c
^4 - 8*b^11*c^4 - 960*a^2*b^8*c^5 + 160*a*b^9*c^5 + 5120*a^3*b^6*c^6 - 1280
*a^2*b^7*c^6 - 15360*a^4*b^4*c^7 + 5120*a^3*b^5*c^7 + 24576*a^5*b^2*c^8 - 1
0240*a^4*b^3*c^8 - 16384*a^6*c^9 + 8192*a^5*b*c^9 - b^10*c^4 + 20*a*b^8*c^5
 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8 + 1024*a^5*c^9)*a^2
*b) + 1/64*(32*a^2*b^7 - 192*a^3*b^5*c + 128*a^2*b^6*c - 256*a^3*b^4*c^2 +
128*a^2*b^5*c^2 + 1024*a^5*b*c^3 - 1024*a^4*b^2*c^3 + 256*a^3*b^3*c^3 + 16*
a^2*b^6 - 96*a^3*b^4*c + 128*a^2*b^5*c - 256*a^3*b^3*c^2 + 192*a^2*b^4*c^2
+ 512*a^5*c^3 - 1024*a^4*b*c^3 + 384*a^3*b^2*c^3 + 32*a^2*b^4*c - 64*a^3*b^
2*c^2 + 96*a^2*b^3*c^2 - 256*a^4*c^3 + 192*a^3*b*c^3 + 16*a^2*b^2*c^2 + 32*
a^3*c^3 + 3*(2*b^8 - 36*a*b^6*c + 8*b^7*c + 304*a^2*b^4*c^2 - 112*a*b^5*c^2
 + 8*b^6*c^2 - 1216*a^3*b^2*c^3 + 768*a^2*b^3*c^3 - 80*a*b^4*c^3 + 1792*a^4
*c^4 - 1792*a^3*b*c^4 + 448*a^2*b^2*c^4 - b^7 + 16*a*b^5*c - 80*a^2*b^3*c^2
 - 8*a*b^4*c^2 + 4*b^5*c^2 + 128*a^3*b*c^3 + 256*a^2*b^2*c^3 - 48*a*b^3*c^3
 - 896*a^3*c^4 + 448*a^2*b*c^4 - 2*b^5*c + 24*a*b^3*c^2 - 2*b^4*c^2 - 64*a^
2*b*c^3 + 12*a*b^2*c^3 + 112*a^2*c^4 - b^3*c^2 + 8*a*b*c^3)*sqrt(2*b*c + c)
*d + (2*a*b^7 - 120*a^2*b^5*c + 8*a*b^6*c + 864*a^3*b^3*c^2 - 448*a^2*b^4*c
^2 + 8*a*b^5*c^2 - 1664*a^4*b*c^3 + 1664*a^3*b^2*c^3 - 416*a^2*b^3*c^3 - a*
b^6 - 12*a^2*b^4*c + 144*a^3*b^2*c^2 - 288*a^2*b^3*c^2 + 4*a*b^4*c^2 - 320*
a^4*c^3 + 1152*a^3*b*c^3 - 496*a^2*b^2*c^3 - 2*a*b^4*c - 32*a^2*b^2*c^2 - 2
*a*b^3*c^2 + 160*a^3*c^3 - 184*a^2*b*c^3 - a*b^2*c^2 - 20*a^2*c^3)*sqrt(2*b
*c + c)*f + 12*(3*a^2*b^6 - 20*a^3*b^4*c + 12*a^2*b^5*c + 16*a^4*b^2*c^2 -
32*a^3*b^3*c^2 + 12*a^2*b^4*c^2 + 64*a^5*c^3 - 64*a^4*b*c^3 + 16*a^3*b^2*c^
3 + a^2*b^5 - 8*a^3*b^3*c + 10*a^2*b^4*c + 16*a^4*b*c^2 - 32*a^3*b^2*c^2 +
16*a^2*b^3*c^2 - 32*a^4*c^3 + 16*a^3*b*c^3 + 2*a^2*b^3*c - 8*a^3*b*c^2 + 7*
a^2*b^2*c^2 + 4*a^3*c^3 + a^2*b*c^2)*sqrt(2*b*c + c)*h + 48*(2*a^2*b^6*c*i
- 16*a^3*b^4*c^2*i + 8*a^2*b^5*c^2*i + 32*a^4*b^2*c^3*i - 32*a^3*b^3*c^3*i
+ 8*a^2*b^4*c^3*i + a^2*b^5*c*i - 8*a^3*b^3*c^2*i + 8*a^2*b^4*c^2*i + 16*a^
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4*b*c^3*i - 32*a^3*b^2*c^3*i + 12*a^2*b^3*c^3*i + 2*a^2*b^3*c^2*i - 8*a^3*b
*c^3*i + 6*a^2*b^2*c^3*i + a^2*b*c^3*i)*g - 96*(2*a^2*b^5*c^2*i - 16*a^3*b^
3*c^3*i + 8*a^2*b^4*c^3*i + 32*a^4*b*c^4*i - 32*a^3*b^2*c^4*i + 8*a^2*b^3*c
^4*i + a^2*b^4*c^2*i - 8*a^3*b^2*c^3*i + 8*a^2*b^3*c^3*i + 16*a^4*c^4*i - 3
2*a^3*b*c^4*i + 12*a^2*b^2*c^4*i + 2*a^2*b^2*c^3*i - 8*a^3*c^4*i + 6*a^2*b*
c^4*i + a^2*c^4*i)*e)*log(x - 1/4*sqrt(-(8*a^2*b^5*i - 64*a^3*b^3*c*i + 128
*a^4*b*c^2*i - sqrt(-64*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 + 256*(a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))
/(a^2*b^4*c*i - 8*a^3*b^2*c^2*i + 16*a^4*c^3*i)))/(a^2*b^10*i - 20*a^3*b^8*
c*i + 4*a^2*b^9*c*i + 160*a^4*b^6*c^2*i - 64*a^3*b^7*c^2*i + 4*a^2*b^8*c^2*
i - 640*a^5*b^4*c^3*i + 384*a^4*b^5*c^3*i - 48*a^3*b^6*c^3*i + 1280*a^6*b^2
*c^4*i - 1024*a^5*b^3*c^4*i + 192*a^4*b^4*c^4*i - 1024*a^7*c^5*i + 1024*a^6
*b*c^5*i - 256*a^5*b^2*c^5*i + 2*a^2*b^8*c*i - 32*a^3*b^6*c^2*i + 4*a^2*b^7
*c^2*i + 192*a^4*b^4*c^3*i - 48*a^3*b^5*c^3*i - 512*a^5*b^2*c^4*i + 192*a^4
*b^3*c^4*i + 512*a^6*c^5*i - 256*a^5*b*c^5*i + a^2*b^6*c^2*i - 12*a^3*b^4*c
^3*i + 48*a^4*b^2*c^4*i - 64*a^5*c^5*i - sqrt(-b^18 + 36*a*b^16*c - 8*b^17*
c - 576*a^2*b^14*c^2 + 256*a*b^15*c^2 - 24*b^16*c^2 + 5376*a^3*b^12*c^3 - 3
584*a^2*b^13*c^3 + 672*a*b^14*c^3 - 32*b^15*c^3 - 32256*a^4*b^10*c^4 + 2867
2*a^3*b^11*c^4 - 8064*a^2*b^12*c^4 + 768*a*b^13*c^4 - 16*b^14*c^4 + 129024*
a^5*b^8*c^5 - 143360*a^4*b^9*c^5 + 53760*a^3*b^10*c^5 - 7680*a^2*b^11*c^5 +
 320*a*b^12*c^5 - 344064*a^6*b^6*c^6 + 458752*a^5*b^7*c^6 - 215040*a^4*b^8*
c^6 + 40960*a^3*b^9*c^6 - 2560*a^2*b^10*c^6 + 589824*a^7*b^4*c^7 - 917504*a
^6*b^5*c^7 - 1125899906064384*a^5*b^6*c^7 - 122880*a^4*b^7*c^7 + 10240*a^3*
b^8*c^7 - 589824*a^8*b^2*c^8 + 1048576*a^7*b^3*c^8 - 688128*a^6*b^4*c^8 + 1
96608*a^5*b^5*c^8 - 20480*a^4*b^6*c^8 + 262144*a^9*c^9 - 524288*a^8*b*c^9 +
 393216*a^7*b^2*c^9 - 131072*a^6*b^3*c^9 + 16384*a^5*b^4*c^9 - 4*b^16*c + 1
28*a*b^14*c^2 - 24*b^15*c^2 - 1792*a^2*b^12*c^3 + 672*a*b^13*c^3 - 48*b^14*
c^3 + 14336*a^3*b^10*c^4 - 8064*a^2*b^11*c^4 + 1152*a*b^12*c^4 - 32*b^13*c^
4 - 71680*a^4*b^8*c^5 + 53760*a^3*b^9*c^5 - 11520*a^2*b^10*c^5 + 640*a*b^11
*c^5 + 229376*a^5*b^6*c^6 - 215040*a^4*b^7*c^6 + 61440*a^3*b^8*c^6 - 5120*a
^2*b^9*c^6 - 458752*a^6*b^4*c^7 + 516096*a^5*b^5*c^7 - 184320*a^4*b^6*c^7 +
 20480*a^3*b^7*c^7 + 524288*a^7*b^2*c^8 - 688128*a^6*b^3*c^8 + 294912*a^5*b
^4*c^8 - 40960*a^4*b^5*c^8 - 262144*a^8*c^9 + 393216*a^7*b*c^9 - 196608*a^6
*b^2*c^9 + 32768*a^5*b^3*c^9 - 2251799813160960*a^6*b*c^10 - 6*b^14*c^2 + 1
68*a*b^12*c^3 - 24*b^13*c^3 - 2016*a^2*b^10*c^4 + 576*a*b^11*c^4 - 24*b^12*
c^4 + 13440*a^3*b^8*c^5 - 5760*a^2*b^9*c^5 + 480*a*b^10*c^5 - 53760*a^4*b^6
*c^6 + 30720*a^3*b^7*c^6 - 3840*a^2*b^8*c^6 + 129024*a^5*b^4*c^7 - 92160*a^
4*b^5*c^7 + 15360*a^3*b^6*c^7 - 172032*a^6*b^2*c^8 + 147456*a^5*b^3*c^8 - 3
0720*a^4*b^4*c^8 + 98304*a^7*c^9 - 98304*a^6*b*c^9 + 24576*a^5*b^2*c^9 - 4*
b^12*c^3 + 96*a*b^10*c^4 - 8*b^11*c^4 - 960*a^2*b^8*c^5 + 160*a*b^9*c^5 + 5
120*a^3*b^6*c^6 - 1280*a^2*b^7*c^6 - 15360*a^4*b^4*c^7 + 5120*a^3*b^5*c^7 +
 24576*a^5*b^2*c^8 - 10240*a^4*b^3*c^8 - 16384*a^6*c^9 + 8192*a^5*b*c^9 - b
^10*c^4 + 20*a*b^8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c
^8 + 1024*a^5*c^9)*a^2*b) + 1/8*(3*b^3*c^2*d*i*x^7 - 24*a*b*c^3*d*i*x^7 + a
*b^2*c^2*f*i*x^7 + 20*a^2*c^3*f*i*x^7 - 12*a^2*b*c^2*h*i*x^7 - 12*a^2*b*c^2

```
*g*i*x^6 + 24*a^2*c^3*i*x^6*e + 6*b^4*c*d*i*x^5 - 49*a*b^2*c^2*d*i*x^5 + 28
*a^2*c^3*d*i*x^5 + 2*a*b^3*c*f*i*x^5 + 28*a^2*b*c^2*f*i*x^5 - 19*a^2*b^2*c*
h*i*x^5 + 4*a^3*c^2*h*i*x^5 - 18*a^2*b^2*c*g*i*x^4 - 4*a^2*b^2*c*x^6 - 8*a^
3*c^2*x^6 + 36*a^2*b*c^2*i*x^4*e + 3*b^5*d*i*x^3 - 20*a*b^3*c*d*i*x^3 - 4*a
^2*b*c^2*d*i*x^3 + a*b^4*f*i*x^3 + 5*a^2*b^2*c*f*i*x^3 + 36*a^3*c^2*f*i*x^3
 - 5*a^2*b^3*h*i*x^3 - 16*a^3*b*c*h*i*x^3 - 4*a^2*b^3*g*i*x^2 - 20*a^3*b*c*
g*i*x^2 - 6*a^2*b^3*x^4 - 12*a^3*b*c*x^4 + 8*a^2*b^2*c*i*x^2*e + 40*a^3*c^2
*i*x^2*e + 5*a*b^4*d*i*x - 37*a^2*b^2*c*d*i*x + 44*a^3*c^2*d*i*x - a^2*b^3*
f*i*x + 16*a^3*b*c*f*i*x - 3*a^3*b^2*h*i*x - 12*a^4*c*h*i*x - 2*a^3*b^2*g*i
 - 16*a^4*c*g*i - 20*a^3*b^2*x^2 + 8*a^4*c*x^2 - 2*a^2*b^3*i*e + 20*a^3*b*c
*i*e - 12*a^4*b)/((a^2*b^4*i - 8*a^3*b^2*c*i + 16*a^4*c^2*i)*(c*x^4 + b*x^2
 + a)^2)
```

3.57 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^3} dx$

Optimal. Leaf size=1150

result too large to display

```
[Out] -(b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*(b*g + 2*a
*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(4*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h
 + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h
 - 3*a^2*m))*x^2))/(4*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^3*j)
/c + 2*b*(3*c*e + a*j) - 16*a^2*l - (b^4*l)/c^2 - b^2*(3*g - (5*a*l)/c) + 2
*(6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*x^2)/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (x*(4*a^2*b*c^2*(2*c*f + a*k) + a*b^3*c*(c*f + 2*a*k)
- a*b^2*c*(25*c^2*d + 7*a*c*h - 11*a^2*m) + 4*a^2*c^2*(7*c^2*d + a*c*h - 9*
a^2*m) + b^4*(3*c^2*d - 2*a^2*m) + c*(a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*
c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m
))*x^2))/(8*a^2*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((a*b^2*c*(c*f +
 3*a*k) + 4*a^2*c^2*(5*c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^
2*d + 3*a*c*h + 4*a^2*m) + (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9
*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8
*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5
*c*f + 3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*
m) - (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c*(5*c
^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d + 3
*a*c*h + 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^
2 - 4*a*c]]) - ((6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 8.16355, antiderivative size = 1144, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.164, Rules used = {1673, 1678, 1166, 205, 1663, 1660, 638, 618, 206}

$$\frac{-\frac{lb^4}{c^2} + \frac{jb^3}{c} - \left(3g - \frac{5al}{c}\right)b^2 + 2(3ce + aj)b + 2\left(jb^2 - 3cgb - 3alb + 6c^2e + 2acj\right)x^2 - 16a^2l}{4\left(b^2 - 4ac\right)^2\left(cx^4 + bx^2 + a\right)} + \frac{\left(\left(\frac{ma^2}{c} + 3cd\right)b^3 + a(cf + 3a \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*(b*g + 2*a
*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(4*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c*h
 + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c*h
 - 3*a^2*m))*x^2))/(4*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^3*j)
/c + 2*b*(3*c*e + a*j) - 16*a^2*l - (b^4*l)/c^2 - b^2*(3*g - (5*a*l)/c) + 2
*(6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*x^2)/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (x*(4*a^2*b*c*(2*c*f + a*k) + a*b^3*(c*f + 2*a*k) - a*
b^2*(25*c^2*d + 7*a*c*h - 11*a^2*m) + 4*a^2*c*(7*c^2*d + a*c*h - 9*a^2*m) +
 b^4*(3*c*d - (2*a^2*m)/c) + (a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f + 3*
a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m))*x^2))
/(8*a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((a*b^2*(c*f + 3*a*k) + 4*
a^2*c*(5*c*f + 3*a*k) - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d +
(a^2*m)/c) + (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^
2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c
^2*d + 3*a*c*h + 5*a^2*m))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*(c*f + 3*a*k) + 4*a^2*c*(5*c*f + 3*a*k)
- 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d + (a^2*m)/c) - (a*b^3*c*
(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*c*h
 - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a^2
*m))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) - ((6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*ArcTanh[(b + 2*c*x^2)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{x\left(e+gx^2+jx^4+lx^6\right)}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{d+fx^2+hx^4+kx^6+mx^8}{\left(a+bx^2+cx^4\right)^3}$$

$$= -\frac{x\left(abc(cf+ak)-b^2\left(c^2d+a^2m\right)+2ac\left(c^2d-ach+a^2m\right)+\right.}{4ac^2\left(b^2-4ac\right)}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l\right.}{4c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l\right.}{4c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l\right.}{4c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l\right.}{4c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A] time = 7.84911, size = 1590, normalized size = 1.38

$$\frac{2cla^3 + 2cmxa^3 - 2c^2kx^3a^2 + 3bcmx^3a^2 - 2c^2jx^2a^2 + 3bclx^2a^2 - 2c^2ga^2 + bcja^2 - b^2la^2 - 2c^2hxa^2 + bckxa^2 - b^2mxa^2 + \cdots}{4ac^2\left(4ac - \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - a^2*b^2*l + 2*a^3*c*l - b^2*c^2*d*x
+ 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x + a^2*b*c*k*x - a^2*b^2*m*x + 2
*a^3*c*m*x + 2*a*c^3*e*x^2 - a*b*c^2*g*x^2 + a*b^2*c*j*x^2 - 2*a^2*c^2*j*x^
2 - a*b^3*l*x^2 + 3*a^2*b*c*l*x^2 - b*c^3*d*x^3 + 2*a*c^3*f*x^3 - a*b*c^2*h
*x^3 + a*b^2*c*k*x^3 - 2*a^2*c^2*k*x^3 - a*b^3*m*x^3 + 3*a^2*b*c*m*x^3)/(4*
a*c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^3*e - 6*a^2*b^2*c
^2*g + 2*a^2*b^3*c*j + 4*a^3*b*c^2*j - 2*a^2*b^4*l + 10*a^3*b^2*c*l - 32*a^
4*c^2*l + 3*b^4*c^2*d*x - 25*a*b^2*c^3*d*x + 28*a^2*c^4*d*x + a*b^3*c^2*f*x
 + 8*a^2*b*c^3*f*x - 7*a^2*b^2*c^2*h*x + 4*a^3*c^3*h*x + 2*a^2*b^3*c*k*x +
4*a^3*b*c^2*k*x - 2*a^2*b^4*m*x + 11*a^3*b^2*c*m*x - 36*a^4*c^2*m*x + 24*a^
2*c^4*e*x^2 - 12*a^2*b*c^3*g*x^2 + 4*a^2*b^2*c^2*j*x^2 + 8*a^3*c^3*j*x^2 -
12*a^3*b*c^2*l*x^2 + 3*b^3*c^3*d*x^3 - 24*a*b*c^4*d*x^3 + a*b^2*c^3*f*x^3 +
 20*a^2*c^4*f*x^3 - 12*a^2*b*c^3*h*x^3 + 3*a^2*b^2*c^2*k*x^3 + 12*a^3*c^3*k
*x^3 + a^2*b^3*c*m*x^3 - 16*a^3*b*c^2*m*x^3)/(8*a^2*c^2*(-b^2 + 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 168*a^2*c^4*d + 3*b^3
*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d + a*b^3*c^2*f - 5
2*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c
]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h -
3*a^2*b^3*c*k - 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 - 4*a*c]*k + 12*a^3*c
^2*Sqrt[b^2 - 4*a*c]*k - a^2*b^4*m + 18*a^3*b^2*c*m + 40*a^4*c^2*m + a^2*b^
3*Sqrt[b^2 - 4*a*c]*m - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*m)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((-3*b^4*c^2*d + 30*a*b^2*c^3*d - 168*
a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d
- a*b^3*c^2*f + 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3
*Sqrt[b^2 - 4*a*c]*f - 18*a^2*b^2*c^2*h - 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[
b^2 - 4*a*c]*h + 3*a^2*b^3*c*k + 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 - 4*
a*c]*k + 12*a^3*c^2*Sqrt[b^2 - 4*a*c]*k + a^2*b^4*m - 18*a^3*b^2*c*m - 40*a
^4*c^2*m + a^2*b^3*Sqrt[b^2 - 4*a*c]*m - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*m)*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/
2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g +
 b^2*j + 2*a*c*j - 3*a*b*l)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2
```

```
- 4*a*c)^(5/2)) + ((-6*c^2*e + 3*b*c*g - b^2*j - 2*a*c*j + 3*a*b*l)*Log[b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.086, size = 6026, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.58 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=645

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(cd-ah)-ab^3j+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+\frac{ab^2j}{c}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(c\cdots}{}\right.}{}$$

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*h) - (a*b*(c*f + a*j))/c) + (b*c*(c*d + a*h) -
a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) - (b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g - a*k) + (2*c^3*e - c^2*(b*g +
2*a*i) - b^3*k + b*c*(b*i + 3*a*k))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + ((b*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d -
 a*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*h
) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) - (b^2*c*(c*d - a*h) - 4*a*c^2*(3*c*d +
 a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4
*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*i) + b^3*
k - 6*a*b*c*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*
c)^(3/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rubi [A] time = 3.36662, antiderivative size = 645, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1673, 1678, 1166, 205, 1663, 1660, 634, 618, 206, 628}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(cd-ah)-ab^3j+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+\frac{ab^2j}{c}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2c(c\cdots}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 +
c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*h) - (a*b*(c*f + a*j))/c) + (b*c*(c*d + a*h) -
a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
```

```
) - (b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g - a*k) + (2*c^3*e - c^2*(b*g +
2*a*i) - b^3*k + b*c*(b*i + 3*a*k))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + ((b*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d -
 a*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*h
) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) - (b^2*c*(c*d - a*h) - 4*a*c^2*(3*c*d +
 a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4
*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*i) + b^3*
k - 6*a*b*c*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*
c)^(3/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+58x^5+jx^6+kx^7}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{d+fx^2+hx^4+jx^6}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x\left(e+gx^2+58x^4+kx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf\right.\right.}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 6.14003, size = 775, normalized size = 1.2

$$\frac{\frac{2\left(a^2\left(b^2(-k)+bc(i+x(j+3kx))-2c^2(g+x(h+x(i+jx)))\right)+2a^3ck+a\left(b^2cx^2(i+jx)+b^3(-k)x^2+bc^2(e+x(f-x(g+hx)))+2c^3x(d+x(e+fx))\right)-bc^2dx\left(b+cx^2\right)\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*
x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a^3*c*k - b*c^2*d*x*(b + c*x^2) + a*(-(b^3*k*x^2) + b^2*c*x^2*(i + j
*x) + 2*c^3*x*(d + x*(e + f*x)) + b*c^2*(e + x*(f - x*(g + h*x)))) + a^2*(-
```

```
(b^2*k) + b*c*(i + x*(j + 3*k*x)) - 2*c^2*(g + x*(h + x*(i + j*x))))))/(a*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*Sqrt[c]*(a*b^3*j - b*c*(c*Sqr
t[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h + 8*a^2*j) - b^2*(c^2*d
- a*c*h + a*Sqrt[b^2 - 4*a*c]*j) + 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f +
 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*j))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*Sqrt[c]*(a*b^3*j + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2
 - 4*a*c]*h - 8*a^2*j) + 2*a*c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*f + 2*a*c*h -
 3*a*Sqrt[b^2 - 4*a*c]*j) + b^2*(-(c^2*d) + a*c*h + a*Sqrt[b^2 - 4*a*c]*j))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-4*c^3*e + 2*c^2*(b*g - 2*a*i) + b^2
*(-b + Sqrt[b^2 - 4*a*c])*k + a*c*(6*b*k - 4*Sqrt[b^2 - 4*a*c]*k))*Log[-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((4*c^3*e + c^2*(-2*b*
g + 4*a*i) + b^2*(b + Sqrt[b^2 - 4*a*c])*k - 2*a*c*(3*b + 2*Sqrt[b^2 - 4*a*
c])*k)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*c^2)
```

Maple [B] time = 0.061, size = 3107, normalized size = 4.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2
)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2))*b^3*d-c/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*
b*f-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-1/4*c/(4*a*c-
b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d+a/(4*a*c-b^2)^2*c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*(-4*a*c+b^2)^(1/2)*h-a/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*h+a/(4*a
*c-b^2)^2*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*h+a/(4*a*c-b^2)^2*c*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2))*b*h+(-1/2/a*(2*a^2*c*j-a*b^2*j+a*b*c*h-2*a*c^2*f+b*c^2*d)/(
```

```
4*a*c-b^2)/c*x^3+1/2*(3*a*b*c*k-2*a*c^2*i-b^3*k+b^2*c*i-b*c^2*g+2*c^3*e)/(4
*a*c-b^2)/c^2*x^2+1/2*(a^2*b*j-2*a^2*c*h+a*b*c*f+2*a*c^2*d-b^2*c*d)/a/c/(4*
a*c-b^2)*x+1/2*(2*a^2*c*k-a*b^2*k+a*b*c*i-2*a*c^2*g+b*c^2*e)/(4*a*c-b^2)/c^
2)/(c*x^4+b*x^2+a)+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2
)*d-c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*
c)^(1/2))*(-4*a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/
2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d-2*
a/(4*a*c-b^2)^2/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^2*k-2*a/(4*a*c-b^2)^2/
c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*b^2*k-1/4/(4*a*c-b^2)^2/c^2*ln(-2*c*x^2
+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b^3*k+1/4/(4*a*c-b^2)^2/c^2*ln(2*
c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b^3*k+1/4/(4*a*c-b^2)^2/c^2*
ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*b^4*k+1/4/(4*a*c-b^2)^2/c^2*ln(-2*c*x^2+(-
4*a*c+b^2)^(1/2)-b)*b^4*k+1/2/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-
b)*(-4*a*c+b^2)^(1/2)*b*g-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b
)*(-4*a*c+b^2)^(1/2)*b*g+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*j-1/4/(
4*a*c-b^2)^2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^4*j+3/2*a/(4*a*c-b^2)^2/c*ln(-2*c*x^2+
(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^2)^(1/2)*b*k-6*a^2/(4*a*c-b^2)^2*c*2^(1/2)/
(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2))*j+5/2*a/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*
arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^2*j-5/2*a/(4*a*c-b^
2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b^2*j-3/2*a/(4*a*c-b^2)^2/c*ln(2*c*x^2+(-4*a*c+b^2)
^(1/2)+b)*(-4*a*c+b^2)^(1/2)*b*k+6*a^2/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*j
+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2*f-2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2
)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-b^2)
^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)*c)^(1/2))*b^2*f+1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+
b^2)^(1/2)*b^2*h+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^
2*h+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)-c
/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/4/(
4*a*c-b^2)^2/c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*j+1/4/(4*a*c-b^2)
^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b^3*j-2*a/(4*a*c-b^2)^2*2^(1/2)/
```

```
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*j-2*a/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^
(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4
*a*c+b^2)^(1/2)*b*j+a/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-4*a*
c+b^2)^(1/2)*i-a/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-4*a*c+b^
2)^(1/2)*i-1/4/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arcta
nh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*h+1/4/(4*a*c-b^2)^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*h+4*a^2/(4*a*c-b^2)^2*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)
*k+4*a^2/(4*a*c-b^2)^2*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*k
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{abc^2e-2a^2c^2g+a^2bci-\left(bc^3d-2ac^3f+abc^2h-\left(ab^2c-2a^2c^2\right)j\right)x^3+\left(2ac^3e-abc^2g+\left(ab^2c-2a^2c^2\right)i-\left(ab^3-3a^2\right.\right.}{2\left(a^2b^2c^2-4a^3c^3+\left(ab^2c^3-4a^2c^4\right)x^4+\left(ab^3c^2-4a^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
algorithm="maxima")
```

```
[Out] -1/2*(a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*i - (b*c^3*d - 2*a*c^3*f + a*b*c^2*
h - (a*b^2*c - 2*a^2*c^2)*j)*x^3 + (2*a*c^3*e - a*b*c^2*g + (a*b^2*c - 2*a^
2*c^2)*i - (a*b^3 - 3*a^2*b*c)*k)*x^2 - (a^2*b^2 - 2*a^3*c)*k + (a*b*c^2*f
- 2*a^2*c^2*h + a^2*b*c*j - (b^2*c^2 - 2*a*c^3)*d)*x)/(a^2*b^2*c^2 - 4*a^3*
c^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) - 1/2*in
tegrate(-(2*(a*b^2 - 4*a^2*c)*k*x^3 + a*b*c*f - 2*a^2*c*h + a^2*b*j + (b*c^
2*d - 2*a*c^2*f + a*b*c*h + (a*b^2 - 6*a^2*c)*j)*x^2 + (b^2*c - 6*a*c^2)*d
- 2*(2*a*c^2*e - a*b*c*g + 2*a^2*c*i - a^2*b*k)*x)/(c*x^4 + b*x^2 + a), x)/
(a*b^2*c - 4*a^2*c^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**7+j*x**6+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+
a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
algorithm="giac")
```

```
[Out] Timed out
```

3.59 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=1177

result too large to display

```
[Out] -(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j)/c)) + (2
*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2))/(4*a*c^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) - (b*c^3*(c*e + a*i) - a*b^4*k + 4*a^2*b^2*c*k
 - 2*a*c^2*(c^2*g + a^2*k) + (2*c^5*e + b^2*c^3*i - c^4*(b*g + 2*a*i) - b^5
*k + 5*a*b^3*c*k - 5*a^2*b*c^2*k)*x^2)/(4*c^4*(b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (x*(c*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(7*c^2*d + a*c*h - 9*a^2*j)
+ b^4*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*j)/c)) + (a*b
^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6*c^2*d + 3*a*c*
h + 4*a^2*j))*x^2))/(8*a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*c^
2*i + 2*b*c^3*(3*c*e + a*i) + 11*a*b^4*k - (b^6*k)/c + 32*a^3*c^2*k - 3*b^2
*(c^3*g + 13*a^2*c*k) + 2*(6*c^5*e + b^2*c^3*i - c^4*(3*b*g - 2*a*i) + 2*b^
5*k - 15*a*b^3*c*k + 25*a^2*b*c^2*k)*x^2)/(4*c^3*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + ((a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c
*(6*c^2*d + 3*a*c*h + 4*a^2*j) + (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c*
(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21*c^2*d
 + 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + ((a*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4
*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*j) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*
b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21
*c^2*d + 3*a*c*h + 5*a^2*j))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]) - ((12*c^5*e + 2*b^2*c^3*i - c^4*(6*b*g - 4*a*i) - b
^5*k + 10*a*b^3*c*k - 30*a^2*b*c^2*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*
c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 7.92648, antiderivative size = 1179, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1673, 1678, 1166, 205, 1663, 1660, 634, 618, 206, 628}

$$-\frac{x\left(\left(-\left(\frac{ja^2}{c^2}+d\right)b^2+afb+2a\left(\frac{ja^2}{c}-ha+cd\right)\right)c^2+\left(-ajb^3-c\left(-3ja^2+cha+c^2d\right)b+2ac^3f\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(cx^4+bx^2+a\right)^2}+\frac{\left(\left(\frac{ja^2}{c}+3cd\right)b^3+ac\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2 +
 c*x^4)^3,x]
```

```
[Out] -(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j)/c)) + (2
*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2))/(4*a*c^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^2) - (b*c^3*(c*e + a*i) - a*b^4*k + 4*a^2*b^2*c*k
 - 2*a*c^2*(c^2*g + a^2*k) + (2*c^5*e + b^2*c^3*i - c^4*(b*g + 2*a*i) - b^5
*k + 5*a*b^3*c*k - 5*a^2*b*c^2*k)*x^2)/(4*c^4*(b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (x*(c*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(7*c^2*d + a*c*h - 9*a^2*j)
+ b^4*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*j)/c)) + (a*b
^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6*c^2*d + 3*a*c*
h + 4*a^2*j))*x^2))/(8*a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*c^
2*i + 2*b*c^3*(3*c*e + a*i) + 11*a*b^4*k - (b^6*k)/c + 32*a^3*c^2*k - 3*b^2
*(c^3*g + 13*a^2*c*k) + 2*(6*c^5*e + b^2*c^3*i - c^4*(3*b*g - 2*a*i) + 2*b^
5*k - 15*a*b^3*c*k + 25*a^2*b*c^2*k)*x^2)/(4*c^3*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + ((a*b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*
j) + b^3*(3*c*d + (a^2*j)/c) + (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c*(5
*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21*c^2*d +
 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((a*b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*h
+ 4*a^2*j) + b^3*(3*c*d + (a^2*j)/c) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*
b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21
*c^2*d + 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sq
rt[b + Sqrt[b^2 - 4*a*c]]) - ((12*c^5*e + 2*b^2*c^3*i - c^4*(6*b*g - 4*a*i)
 - b^5*k + 10*a*b^3*c*k - 30*a^2*b*c^2*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+59x^5+jx^8+kx^{11}}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{d+fx^2+hx^4+jx^8}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{x\left(e+gx^2+59x^4+kx^{10}\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-\right.\right.}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A] time = 7.6312, size = 1649, normalized size = 1.4

$$\frac{-akx^2b^5-a^2kb^4-ac^2jx^3b^3+5a^2ckx^2b^3+ac^3ix^2b^2+4a^3ckb^2-c^4dxb^2-a^2c^2jxb^2-c^5dx^3b-ac^4hx^3b+3a^2c^3jx^3b-ac^4g}{4ac^4\left(4ac-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b
```

```
*x^2 + c*x^4)^3,x]

[Out] (a*b*c^4*e - 2*a^2*c^4*g + a^2*b*c^3*i - a^2*b^4*k + 4*a^3*b^2*c*k - 2*a^4*
c^2*k - b^2*c^4*d*x + 2*a*c^5*d*x + a*b*c^4*f*x - 2*a^2*c^4*h*x - a^2*b^2*c
^2*j*x + 2*a^3*c^3*j*x + 2*a*c^5*e*x^2 - a*b*c^4*g*x^2 + a*b^2*c^3*i*x^2 -
2*a^2*c^4*i*x^2 - a*b^5*k*x^2 + 5*a^2*b^3*c*k*x^2 - 5*a^3*b*c^2*k*x^2 - b*c
^5*d*x^3 + 2*a*c^5*f*x^3 - a*b*c^4*h*x^3 - a*b^3*c^2*j*x^3 + 3*a^2*b*c^3*j*
x^3)/(4*a*c^4*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^5*e - 6*a
^2*b^2*c^4*g + 2*a^2*b^3*c^3*i + 4*a^3*b*c^4*i - 2*a^2*b^6*k + 22*a^3*b^4*c
*k - 78*a^4*b^2*c^2*k + 64*a^5*c^3*k + 3*b^4*c^4*d*x - 25*a*b^2*c^5*d*x + 2
8*a^2*c^6*d*x + a*b^3*c^4*f*x + 8*a^2*b*c^5*f*x - 7*a^2*b^2*c^4*h*x + 4*a^3
*c^5*h*x - 2*a^2*b^4*c^2*j*x + 11*a^3*b^2*c^3*j*x - 36*a^4*c^4*j*x + 24*a^2
*c^6*e*x^2 - 12*a^2*b*c^5*g*x^2 + 4*a^2*b^2*c^4*i*x^2 + 8*a^3*c^5*i*x^2 + 8
*a^2*b^5*c*k*x^2 - 60*a^3*b^3*c^2*k*x^2 + 100*a^4*b*c^3*k*x^2 + 3*b^3*c^5*d
*x^3 - 24*a*b*c^6*d*x^3 + a*b^2*c^5*f*x^3 + 20*a^2*c^6*f*x^3 - 12*a^2*b*c^5
*h*x^3 + a^2*b^3*c^3*j*x^3 - 16*a^3*b*c^4*j*x^3)/(8*a^2*c^4*(-b^2 + 4*a*c)^
2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 168*a^2*c^4*d + 3
*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d + a*b^3*c^2*f
 - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4
*a*c]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*
h - a^2*b^4*j + 18*a^3*b^2*c*j + 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j
 - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + ((-3*b^4*c^2*d + 30*a*b^2*c^3*d - 168*a^2*c^4*d + 3*b^3*c^2
*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d - a*b^3*c^2*f + 52*a^
2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c]*f
- 18*a^2*b^2*c^2*h - 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h + a^2*
b^4*j - 18*a^3*b^2*c*j - 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j - 16*a^
3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) + ((12*c^5*e - 6*b*c^4*g + 2*b^2*c^3*i + 4*a*c^4*i - b^5*k + 10*a*b^3
*c*k - 30*a^2*b*c^2*k + b^4*Sqrt[b^2 - 4*a*c]*k - 8*a*b^2*c*Sqrt[b^2 - 4*a*
c]*k + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]*k)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
])/(4*c^3*(b^2 - 4*a*c)^(5/2)) + ((-12*c^5*e + 6*b*c^4*g - 2*b^2*c^3*i - 4*
a*c^4*i + b^5*k - 10*a*b^3*c*k + 30*a^2*b*c^2*k + b^4*Sqrt[b^2 - 4*a*c]*k -
 8*a*b^2*c*Sqrt[b^2 - 4*a*c]*k + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]*k)*Log[b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^3*(b^2 - 4*a*c)^(5/2))
```

Maple [B] time = 0.089, size = 6130, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**11+j*x**8+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2
+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.60 $\int \left(a + bx^2 + cx^4\right)^3 \left(ad + aex + (bd + af)x^2 + bex^3 + (cd + b\right.$

Optimal. Leaf size=416

$$\frac{1}{11}x^{11}\left(6a^2c^2f + 12ab^2cf + 12abc^2d + 4b^3cd + b^4f\right) + \frac{1}{9}x^9\left(12a^2bcf + 6a^2c^2d + 12ab^2cd + 4ab^3f + b^4d\right) + \frac{1}{10}ex^{10}\left(6a^2c^2\right.$$

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2*
(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2*a
*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*c)*
e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)
*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b
*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*
e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/13 + (
c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f)*x^15)/
15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)/18 + (c^
4*f*x^19)/19
```

Rubi [A] time = 0.628971, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016, Rules used = {1671}

$$\frac{1}{11}x^{11}\left(6a^2c^2f + 12ab^2cf + 12abc^2d + 4b^3cd + b^4f\right) + \frac{1}{9}x^9\left(12a^2bcf + 6a^2c^2d + 12ab^2cd + 4ab^3f + b^4d\right) + \frac{1}{10}ex^{10}\left(6a^2c^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d +
 b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```



```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2*
(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2*a
*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*c)*
e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)
*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b
*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*
e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/13 + (
c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f)*x^15)/
15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)/18 + (c^
4*f*x^19)/19
```


Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx = \int \left(a^4d+a^4ex+a^3(4bd+af)x^2\right.$$
$$= a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3(4bd+af)$$

Mathematica [A] time = 0.132671, size = 416, normalized size = 1.

$$\frac{1}{11}x^{11}\left(6a^2c^2f+12ab^2cf+12abc^2d+4b^3cd+b^4f\right)+\frac{1}{9}x^9\left(12a^2bcf+6a^2c^2d+12ab^2cd+4ab^3f+b^4d\right)+\frac{1}{10}ex^{10}\left(6a^2c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
(c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2*
(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2*a
*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*c)*
e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)
*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b
*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*
e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/13 + (
c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f)*x^15)/
15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)/18 + (c^
4*f*x^19)/19
```

Maple [B] time = 0.001, size = 829, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^
5+c*f*x^6),x)
```

```
[Out] 1/19*c^4*f*x^19+1/18*c^4*e*x^18+1/17*(3*b*c^3*f+c^3*(b*f+c*d))*x^17+1/4*b*c
^3*e*x^16+1/15*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*c*f+3*c^2*b*(b*f+c*d)+c^3*(a*
f+b*d))*x^15+1/14*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*c*e+3*c^2*b^2*e+a*c^3*e)*x
^14+1/13*((4*a*b*c+b*(2*a*c+b^2))*c*f+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*(b*f+c*
d)+3*c^2*b*(a*f+b*d)+a*c^3*d)*x^13+1/12*((4*a*b*c+b*(2*a*c+b^2))*c*e+(a*c^2
+2*b^2*c+c*(2*a*c+b^2))*b*e+3*a*b*c^2*e)*x^12+1/11*((a*(2*a*c+b^2)+2*a*b^2+
a^2*c)*c*f+(4*a*b*c+b*(2*a*c+b^2))*(b*f+c*d)+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*
(a*f+b*d)+3*a*b*c^2*d)*x^11+1/10*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*c*e+(4*a*b*
c+b*(2*a*c+b^2))*b*e+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*a*e)*x^10+1/9*(3*a^2*b*c
*f+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*(b*f+c*d)+(4*a*b*c+b*(2*a*c+b^2))*(a*f+b*d
)+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*a*d)*x^9+1/8*(3*a^2*b*c*e+(a*(2*a*c+b^2)+2*
a*b^2+a^2*c)*b*e+(4*a*b*c+b*(2*a*c+b^2))*a*e)*x^8+1/7*(a^3*c*f+3*a^2*b*(b*f
+c*d)+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*(a*f+b*d)+(4*a*b*c+b*(2*a*c+b^2))*a*d)*
x^7+1/6*(a^3*c*e+3*a^2*b^2*e+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*a*e)*x^6+1/5*(a^
3*(b*f+c*d)+3*a^2*b*(a*f+b*d)+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*a*d)*x^5+a^3*b*
e*x^4+1/3*(a^3*(a*f+b*d)+3*a^3*b*d)*x^3+1/2*a^4*e*x^2+a^4*d*x
```

Maxima [A] time = 0.981567, size = 564, normalized size = 1.36

$$\frac{1}{19}c^4fx^{19} + \frac{1}{18}c^4ex^{18} + \frac{1}{4}bc^3ex^{16} + \frac{1}{17}\left(c^4d + 4bc^3f\right)x^{17} + \frac{1}{7}\left(3b^2c^2 + 2ac^3\right)ex^{14} + \frac{2}{15}\left(2bc^3d + \left(3b^2c^2 + 2ac^3\right)f\right)x^{15} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/4*b*c^3*e*x^16 + 1/17*(c^4*d + 4*b*c^
3*f)*x^17 + 1/7*(3*b^2*c^2 + 2*a*c^3)*e*x^14 + 2/15*(2*b*c^3*d + (3*b^2*c^2
 + 2*a*c^3)*f)*x^15 + 1/3*(b^3*c + 3*a*b*c^2)*e*x^12 + 2/13*((3*b^2*c^2 + 2
*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*f)*x^13 + 1/10*(b^4 + 12*a*b^2*c + 6*a^2*
c^2)*e*x^10 + 1/11*(4*(b^3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^2*c^2
)*f)*x^11 + 1/2*(a*b^3 + 3*a^2*b*c)*e*x^8 + 1/9*((b^4 + 12*a*b^2*c + 6*a^2*
c^2)*d + 4*(a*b^3 + 3*a^2*b*c)*f)*x^9 + a^3*b*e*x^4 + 1/3*(3*a^2*b^2 + 2*a^
3*c)*e*x^6 + 2/7*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*f)*x^7 +
1/2*a^4*e*x^2 + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*b^2 + 2*a^3*c)*d)*x^5 + 1
/3*(4*a^3*b*d + a^4*f)*x^3
```

Fricas [A] time = 1.54139, size = 1152, normalized size = 2.77

$$\frac{1}{19}x^{19}fc^4 + \frac{1}{18}x^{18}ec^4 + \frac{1}{17}x^{17}dc^4 + \frac{4}{17}x^{17}fc^3b + \frac{1}{4}x^{16}ec^3b + \frac{4}{15}x^{15}dc^3b + \frac{2}{5}x^{15}fc^2b^2 + \frac{4}{15}x^{15}fc^3a + \frac{3}{7}x^{14}ec^2b^2 + \frac{2}{7}x^{14}ec$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*f*c^4 + 1/18*x^18*e*c^4 + 1/17*x^17*d*c^4 + 4/17*x^17*f*c^3*b + 1
/4*x^16*e*c^3*b + 4/15*x^15*d*c^3*b + 2/5*x^15*f*c^2*b^2 + 4/15*x^15*f*c^3*
a + 3/7*x^14*e*c^2*b^2 + 2/7*x^14*e*c^3*a + 6/13*x^13*d*c^2*b^2 + 4/13*x^13
*f*c*b^3 + 4/13*x^13*d*c^3*a + 12/13*x^13*f*c^2*b*a + 1/3*x^12*e*c*b^3 + x^
12*e*c^2*b*a + 4/11*x^11*d*c*b^3 + 1/11*x^11*f*b^4 + 12/11*x^11*d*c^2*b*a +
 12/11*x^11*f*c*b^2*a + 6/11*x^11*f*c^2*a^2 + 1/10*x^10*e*b^4 + 6/5*x^10*e*
c*b^2*a + 3/5*x^10*e*c^2*a^2 + 1/9*x^9*d*b^4 + 4/3*x^9*d*c*b^2*a + 4/9*x^9*
f*b^3*a + 2/3*x^9*d*c^2*a^2 + 4/3*x^9*f*c*b*a^2 + 1/2*x^8*e*b^3*a + 3/2*x^8
*e*c*b*a^2 + 4/7*x^7*d*b^3*a + 12/7*x^7*d*c*b*a^2 + 6/7*x^7*f*b^2*a^2 + 4/7
*x^7*f*c*a^3 + x^6*e*b^2*a^2 + 2/3*x^6*e*c*a^3 + 6/5*x^5*d*b^2*a^2 + 4/5*x^
5*d*c*a^3 + 4/5*x^5*f*b*a^3 + x^4*e*b*a^3 + 4/3*x^3*d*b*a^3 + 1/3*x^3*f*a^4
 + 1/2*x^2*e*a^4 + x*d*a^4
```

Sympy [A] time = 0.145846, size = 503, normalized size = 1.21

$$a^4dx + \frac{a^4ex^2}{2} + a^3bex^4 + \frac{bc^3ex^{16}}{4} + \frac{c^4ex^{18}}{18} + \frac{c^4fx^{19}}{19} + x^{17}\left(\frac{4bc^3f}{17} + \frac{c^4d}{17}\right) + x^{15}\left(\frac{4ac^3f}{15} + \frac{2b^2c^2f}{5} + \frac{4bc^3d}{15}\right) + x^{14}\left(\frac{2ac}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)
*x**4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**4*d*x + a**4*e*x**2/2 + a**3*b*e*x**4 + b*c**3*e*x**16/4 + c**4*e*x**18/
18 + c**4*f*x**19/19 + x**17*(4*b*c**3*f/17 + c**4*d/17) + x**15*(4*a*c**3*
f/15 + 2*b**2*c**2*f/5 + 4*b*c**3*d/15) + x**14*(2*a*c**3*e/7 + 3*b**2*c**2
*e/7) + x**13*(12*a*b*c**2*f/13 + 4*a*c**3*d/13 + 4*b**3*c*f/13 + 6*b**2*c*
*2*d/13) + x**12*(a*b*c**2*e + b**3*c*e/3) + x**11*(6*a**2*c**2*f/11 + 12*a
*b**2*c*f/11 + 12*a*b*c**2*d/11 + b**4*f/11 + 4*b**3*c*d/11) + x**10*(3*a**
2*c**2*e/5 + 6*a*b**2*c*e/5 + b**4*e/10) + x**9*(4*a**2*b*c*f/3 + 2*a**2*c*
*2*d/3 + 4*a*b**3*f/9 + 4*a*b**2*c*d/3 + b**4*d/9) + x**8*(3*a**2*b*c*e/2 +
 a*b**3*e/2) + x**7*(4*a**3*c*f/7 + 6*a**2*b**2*f/7 + 12*a**2*b*c*d/7 + 4*a
*b**3*d/7) + x**6*(2*a**3*c*e/3 + a**2*b**2*e) + x**5*(4*a**3*b*f/5 + 4*a**
3*c*d/5 + 6*a**2*b**2*d/5) + x**3*(a**4*f/3 + 4*a**3*b*d/3)
```

Giac [A] time = 1.14906, size = 645, normalized size = 1.55

$$\frac{1}{19}\,c^4fx^{19}+\frac{1}{18}\,c^4x^{18}e+\frac{1}{17}\,c^4dx^{17}+\frac{4}{17}\,bc^3fx^{17}+\frac{1}{4}\,bc^3x^{16}e+\frac{4}{15}\,bc^3dx^{15}+\frac{2}{5}\,b^2c^2fx^{15}+\frac{4}{15}\,ac^3fx^{15}+\frac{3}{7}\,b^2c^2x^{14}e+\frac{2}{7}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*x^18*e + 1/17*c^4*d*x^17 + 4/17*b*c^3*f*x^17 + 1
/4*b*c^3*x^16*e + 4/15*b*c^3*d*x^15 + 2/5*b^2*c^2*f*x^15 + 4/15*a*c^3*f*x^1
5 + 3/7*b^2*c^2*x^14*e + 2/7*a*c^3*x^14*e + 6/13*b^2*c^2*d*x^13 + 4/13*a*c^
3*d*x^13 + 4/13*b^3*c*f*x^13 + 12/13*a*b*c^2*f*x^13 + 1/3*b^3*c*x^12*e + a*
b*c^2*x^12*e + 4/11*b^3*c*d*x^11 + 12/11*a*b*c^2*d*x^11 + 1/11*b^4*f*x^11 +
 12/11*a*b^2*c*f*x^11 + 6/11*a^2*c^2*f*x^11 + 1/10*b^4*x^10*e + 6/5*a*b^2*c
*x^10*e + 3/5*a^2*c^2*x^10*e + 1/9*b^4*d*x^9 + 4/3*a*b^2*c*d*x^9 + 2/3*a^2*
c^2*d*x^9 + 4/9*a*b^3*f*x^9 + 4/3*a^2*b*c*f*x^9 + 1/2*a*b^3*x^8*e + 3/2*a^2
*b*c*x^8*e + 4/7*a*b^3*d*x^7 + 12/7*a^2*b*c*d*x^7 + 6/7*a^2*b^2*f*x^7 + 4/7
*a^3*c*f*x^7 + a^2*b^2*x^6*e + 2/3*a^3*c*x^6*e + 6/5*a^2*b^2*d*x^5 + 4/5*a^
3*c*d*x^5 + 4/5*a^3*b*f*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 1/3*a^4*f*x^3
 + 1/2*a^4*x^2*e + a^4*d*x
```

3.61 $\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+\right.$

Optimal. Leaf size=259

$$\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{9}x^9\left(6abcf+3ac^2d+3b^2cd+b\right.$$

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 + (
3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d + 6*
a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((3*b
^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/1
0 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d +
3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Rubi [A] time = 0.332331, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016, Rules used = {1671}

$$\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{9}x^9\left(6abcf+3ac^2d+3b^2cd+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d +
 b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```



```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 + (
3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d + 6*
a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((3*b
^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/1
0 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d +
3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```


Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx = \int \left(a^3d+a^3ex+a^2(3bd+af)x^2\right.$$
$$= a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bd+af)x$$

Mathematica [A] time = 0.054027, size = 259, normalized size = 1.

$$\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{9}x^9\left(6abcf+3ac^2d+3b^2cd+b^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
(c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 + (
3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d + 6*
a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((3*b
^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/1
0 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d +
3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Maple [A] time = 0.002, size = 354, normalized size = 1.4

$$\frac{c^3fx^{15}}{15}+\frac{c^3ex^{14}}{14}+\frac{\left(2\,bc^2f+c^2\left(bf+cd\right)\right)x^{13}}{13}+\frac{bc^2ex^{12}}{4}+\frac{\left(\left(2\,ac+b^2\right)cf+2\,bc\left(bf+cd\right)+c^2\left(af+bd\right)\right)x^{11}}{11}+\frac{\left(\left(2\,ac\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^
5+c*f*x^6),x)
```

```
[Out] 1/15*c^3*f*x^15+1/14*c^3*e*x^14+1/13*(2*b*c^2*f+c^2*(b*f+c*d))*x^13+1/4*b*c
^2*e*x^12+1/11*((2*a*c+b^2)*c*f+2*b*c*(b*f+c*d)+c^2*(a*f+b*d))*x^11+1/10*((
2*a*c+b^2)*c*e+2*b^2*c*e+a*c^2*e)*x^10+1/9*(2*a*b*c*f+(2*a*c+b^2)*(b*f+c*d)
+2*b*c*(a*f+b*d)+a*c^2*d)*x^9+1/8*(4*a*b*c*e+(2*a*c+b^2)*b*e)*x^8+1/7*(a^2*
c*f+2*a*b*(b*f+c*d)+(2*a*c+b^2)*(a*f+b*d)+2*a*b*c*d)*x^7+1/6*(a^2*c*e+2*a*b
^2*e+(2*a*c+b^2)*a*e)*x^6+1/5*(a^2*(b*f+c*d)+2*a*b*(a*f+b*d)+(2*a*c+b^2)*a*
d)*x^5+3/4*a^2*b*e*x^4+1/3*(a^2*(a*f+b*d)+2*a^2*b*d)*x^3+1/2*a^3*e*x^2+a^3*
```

```
d*x
```

Maxima [A] time = 0.965589, size = 339, normalized size = 1.31

$$\frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3ex^{14}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}\left(c^3d+3\,bc^2f\right)x^{13}+\frac{3}{10}\left(b^2c+ac^2\right)ex^{10}+\frac{3}{11}\left(bc^2d+\left(b^2c+ac^2\right)f\right)x^{11}+\frac{1}{8}\left(b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/4*b*c^2*e*x^12 + 1/13*(c^3*d + 3*b*c^
2*f)*x^13 + 3/10*(b^2*c + a*c^2)*e*x^10 + 3/11*(b*c^2*d + (b^2*c + a*c^2)*f
)*x^11 + 1/8*(b^3 + 6*a*b*c)*e*x^8 + 1/9*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*f)*x^9 + 3/4*a^2*b*e*x^4 + 1/2*(a*b^2 + a^2*c)*e*x^6 + 1/7*((b^3 + 6*a
*b*c)*d + 3*(a*b^2 + a^2*c)*f)*x^7 + 1/2*a^3*e*x^2 + 3/5*(a^2*b*f + (a*b^2
+ a^2*c)*d)*x^5 + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Fricas [A] time = 1.57373, size = 716, normalized size = 2.76

$$\frac{1}{15}x^{15}fc^3+\frac{1}{14}x^{14}ec^3+\frac{1}{13}x^{13}dc^3+\frac{3}{13}x^{13}fc^2b+\frac{1}{4}x^{12}ec^2b+\frac{3}{11}x^{11}dc^2b+\frac{3}{11}x^{11}fcb^2+\frac{3}{11}x^{11}fc^2a+\frac{3}{10}x^{10}ecb^2+\frac{3}{10}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f*c^3 + 1/14*x^14*e*c^3 + 1/13*x^13*d*c^3 + 3/13*x^13*f*c^2*b + 1
/4*x^12*e*c^2*b + 3/11*x^11*d*c^2*b + 3/11*x^11*f*c*b^2 + 3/11*x^11*f*c^2*a
 + 3/10*x^10*e*c*b^2 + 3/10*x^10*e*c^2*a + 1/3*x^9*d*c*b^2 + 1/9*x^9*f*b^3
+ 1/3*x^9*d*c^2*a + 2/3*x^9*f*c*b*a + 1/8*x^8*e*b^3 + 3/4*x^8*e*c*b*a + 1/7
*x^7*d*b^3 + 6/7*x^7*d*c*b*a + 3/7*x^7*f*b^2*a + 3/7*x^7*f*c*a^2 + 1/2*x^6*
e*b^2*a + 1/2*x^6*e*c*a^2 + 3/5*x^5*d*b^2*a + 3/5*x^5*d*c*a^2 + 3/5*x^5*f*b
*a^2 + 3/4*x^4*e*b*a^2 + x^3*d*b*a^2 + 1/3*x^3*f*a^3 + 1/2*x^2*e*a^3 + x*d*
a^3
```

Sympy [A] time = 0.112955, size = 309, normalized size = 1.19

$$a^3dx+\frac{a^3ex^2}{2}+\frac{3a^2bex^4}{4}+\frac{bc^2ex^{12}}{4}+\frac{c^3ex^{14}}{14}+\frac{c^3fx^{15}}{15}+x^{13}\left(\frac{3bc^2f}{13}+\frac{c^3d}{13}\right)+x^{11}\left(\frac{3ac^2f}{11}+\frac{3b^2cf}{11}+\frac{3bc^2d}{11}\right)+x^{10}\left(\frac{3ac^2}{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)
*x**4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**3*d*x + a**3*e*x**2/2 + 3*a**2*b*e*x**4/4 + b*c**2*e*x**12/4 + c**3*e*x*
*14/14 + c**3*f*x**15/15 + x**13*(3*b*c**2*f/13 + c**3*d/13) + x**11*(3*a*c
**2*f/11 + 3*b**2*c*f/11 + 3*b*c**2*d/11) + x**10*(3*a*c**2*e/10 + 3*b**2*c
*e/10) + x**9*(2*a*b*c*f/3 + a*c**2*d/3 + b**3*f/9 + b**2*c*d/3) + x**8*(3*
a*b*c*e/4 + b**3*e/8) + x**7*(3*a**2*c*f/7 + 3*a*b**2*f/7 + 6*a*b*c*d/7 + b
**3*d/7) + x**6*(a**2*c*e/2 + a*b**2*e/2) + x**5*(3*a**2*b*f/5 + 3*a**2*c*d
/5 + 3*a*b**2*d/5) + x**3*(a**3*f/3 + a**2*b*d)
```

Giac [A] time = 1.11794, size = 398, normalized size = 1.54

$$\frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3x^{14}e+\frac{1}{13}c^3dx^{13}+\frac{3}{13}bc^2fx^{13}+\frac{1}{4}bc^2x^{12}e+\frac{3}{11}bc^2dx^{11}+\frac{3}{11}b^2cfx^{11}+\frac{3}{11}ac^2fx^{11}+\frac{3}{10}b^2cx^{10}e+\frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*x^14*e + 1/13*c^3*d*x^13 + 3/13*b*c^2*f*x^13 + 1
/4*b*c^2*x^12*e + 3/11*b*c^2*d*x^11 + 3/11*b^2*c*f*x^11 + 3/11*a*c^2*f*x^11
 + 3/10*b^2*c*x^10*e + 3/10*a*c^2*x^10*e + 1/3*b^2*c*d*x^9 + 1/3*a*c^2*d*x^
9 + 1/9*b^3*f*x^9 + 2/3*a*b*c*f*x^9 + 1/8*b^3*x^8*e + 3/4*a*b*c*x^8*e + 1/7
*b^3*d*x^7 + 6/7*a*b*c*d*x^7 + 3/7*a*b^2*f*x^7 + 3/7*a^2*c*f*x^7 + 1/2*a*b^
2*x^6*e + 1/2*a^2*c*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*c*d*x^5 + 3/5*a^2*b*f
*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*x^2*e + a^3*
d*x
```

3.62 $\int \left(a+bx^2+cx^4\right)\left(ad+aex+(bd+af)x^2+bex^3+(cd+b$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+$$

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Rubi [A] time = 0.151717, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016, Rules used = {1671}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b
*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```


```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```


Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx=\int \left(a^2d+a^2ex+a(2bd+af)x^2+\right.$$

$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3$$

Mathematica [A] time = 0.0353443, size = 154, normalized size = 1.

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c
*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Maple [A] time = 0.002, size = 161, normalized size = 1.1

$$\frac{c^2fx^{11}}{11}+\frac{c^2ex^{10}}{10}+\frac{\left(fbc+c\left(bf+cd\right)\right)x^9}{9}+\frac{bcex^8}{4}+\frac{\left(acf+b\left(bf+cd\right)+c\left(af+bd\right)\right)x^7}{7}+\frac{\left(2\,cea+b^2e\right)x^6}{6}+\frac{\left(a\left(bf+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+
c*f*x^6),x)
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(f*b*c+c*(b*f+c*d))*x^9+1/4*b*c*e*x^8+1
/7*(a*c*f+b*(b*f+c*d)+c*(a*f+b*d))*x^7+1/6*(2*a*c*e+b^2*e)*x^6+1/5*(a*(b*f+
c*d)+b*(a*f+b*d)+a*c*d)*x^5+1/2*a*b*e*x^4+1/3*(a*(a*f+b*d)+b*a*d)*x^3+1/2*a
^2*e*x^2+a^2*d*x
```

Maxima [A] time = 0.962803, size = 186, normalized size = 1.21

$$\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,c^2ex^{10}+\frac{1}{4}\,bcex^8+\frac{1}{9}\left(c^2d+2\,bcf\right)x^9+\frac{1}{6}\left(b^2+2\,ac\right)ex^6+\frac{1}{7}\left(2\,bcd+\left(b^2+2\,ac\right)f\right)x^7+\frac{1}{2}\,abex^4+\frac{1}{5}\left(2\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*a*
b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x + 1
/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A] time = 1.52484, size = 385, normalized size = 2.5

$$\frac{1}{11}x^{11}fc^2 + \frac{1}{10}x^{10}ec^2 + \frac{1}{9}x^9dc^2 + \frac{2}{9}x^9fcb + \frac{1}{4}x^8ecb + \frac{2}{7}x^7dcb + \frac{1}{7}x^7fb^2 + \frac{2}{7}x^7fca + \frac{1}{6}x^6eb^2 + \frac{1}{3}x^6eca + \frac{1}{5}x^5db^2 + \frac{2}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*c^2 + 1/10*x^10*e*c^2 + 1/9*x^9*d*c^2 + 2/9*x^9*f*c*b + 1/4*x^8
*e*c*b + 2/7*x^7*d*c*b + 1/7*x^7*f*b^2 + 2/7*x^7*f*c*a + 1/6*x^6*e*b^2 + 1/
3*x^6*e*c*a + 1/5*x^5*d*b^2 + 2/5*x^5*d*c*a + 2/5*x^5*f*b*a + 1/2*x^4*e*b*a
 + 2/3*x^3*d*b*a + 1/3*x^3*f*a^2 + 1/2*x^2*e*a^2 + x*d*a^2
```

Sympy [A] time = 0.088181, size = 165, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{abex^4}{2} + \frac{bcex^8}{4} + \frac{c^2ex^{10}}{10} + \frac{c^2fx^{11}}{11} + x^9\left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^7\left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x*
*4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/7
+ 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b*
*2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A] time = 1.09983, size = 212, normalized size = 1.38

$$\frac{1}{11}c^2fx^{11} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*c
*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*x^6*e + 1/
3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e
 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

3.63 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} dx$

Optimal. Leaf size=20

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Rubi [A] time = 0.0331228, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016, Rules used = {1586}

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4} dx &= \int \left(d+ex+fx^2\right) dx\\ &= dx + \frac{ex^2}{2} + \frac{fx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0017564, size = 20, normalized size = 1.

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a),x)
```

```
[Out] d*x+1/2*e*x^2+1/3*f*x^3
```

Maxima [A] time = 0.951129, size = 22, normalized size = 1.1

$$\frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Fricas [A] time = 1.68847, size = 39, normalized size = 1.95

$$\frac{1}{3} fx^3 + \frac{1}{2} ex^2 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/3*f*x^3 + 1/2*e*x^2 + d*x
```

Sympy [A] time = 0.079969, size = 15, normalized size = 0.75

$$dx + \frac{ex^2}{2} + \frac{fx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a),x)
```

```
[Out] d*x + e*x**2/2 + f*x**3/3
```

Giac [A] time = 1.18739, size = 23, normalized size = 1.15

$$\frac{1}{3} fx^3 + \frac{1}{2} x^2 e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3 + 1/2*x^2*e + d*x
```

3.64 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.318455, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.127, Rules used = {1586, 1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{\left(a + bx^2 + cx^4\right)^2}\,dx = \int \frac{d + ex + fx^2}{a + bx^2 + cx^4}\,dx$$

$$= \int \frac{ex}{a + bx^2 + cx^4}\,dx + \int \frac{d + fx^2}{a + bx^2 + cx^4}\,dx$$

$$= e\int \frac{x}{a + bx^2 + cx^4}\,dx + \frac{1}{2}\left(f - \frac{2cd - bf}{\sqrt{b^2 - 4ac}}\right)\int \frac{}{\frac{b}{2} + \frac{1}{2}}$$

$$= \frac{\left(f + \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(f - \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\text{ta}}{\sqrt{2}\sqrt{c}\sqrt{b\,}}$$

$$= \frac{\left(f + \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(f - \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\text{ta}}{\sqrt{2}\sqrt{c}\sqrt{b\,}}$$

$$= \frac{\left(f + \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(f - \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\text{ta}}{\sqrt{2}\sqrt{c}\sqrt{b\,}}$$

Mathematica [A] time = 0.254117, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(f\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} + e\log\left(\sqrt{b^2 - 4ac} - b - 2cx^2\right) - e\log\left(\sqrt{b^2 - 4\right.}{2\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [B] time = 0.014, size = 616, normalized size = 2.9

$$-\frac{e}{8ac-2b^2}\sqrt{-4ac+b^2}\ln\left(-2cx^2+\sqrt{-4ac+b^2}-b\right)-2\frac{c\sqrt{2}fa}{\left(4ac-b^2\right)\sqrt{\left(\sqrt{-4ac+b^2}-b\right)c}}\mathrm{Artanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(\sqrt{-4ac+b^2}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^2,x)
```

```
[Out] -1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c
/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*a+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)
^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f*
b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*f+c*(-4*a*c+b^
2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2
^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d+1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*
a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*f*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*f+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cfx^6 + cex^5 + bex^3 + \left(cd + bf\right)x^4 + aex + \left(bd + af\right)x^2 + ad}{\left(cx^4 + bx^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*f*x^6 + c*e*x^5 + b*e*x^3 + (c*d + b*f)*x^4 + a*e*x + (b*d + a
*f)*x^2 + a*d)/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 22.3502, size = 9103, normalized size = 43.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
```

/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(
a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))) + 4*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*s
qrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3
 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2
- 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*s
in(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(
1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^
2)*d*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos
(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4
*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1

```
/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4
)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cosh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*sin(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a
*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + 4*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sq
rt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) + 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3
- sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
```

```
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(1/4*
pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) - 1/
4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*
c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*c
osh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)
*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
 (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a
*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*
f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)
^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqr
t(b^2 - 4*a*c)*b)*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*f*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)
*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2*e + (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4
*a*c)*sqrt(a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 -
 2*(sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c
^2)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c))))) - 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^
3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (sqr
t(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos
```

```
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c^2 - 4*sq
rt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*e*sin(5/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3
 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1
/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
)*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
 + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(
a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*ab
s(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)
^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*f*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a
*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3
)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*f*cos(1/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^3 - (sqrt(a*c)*b^2*c^2 - 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*
sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e - (sqrt
(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*re
```

```
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 2*(sqrt(a*c)*b^2*c^2 - 4*s
qrt(a*c)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) + 2*(sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 + sqrt(b^2 - 4*a*c)*sqrt(
a*c)*b*c^2)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c
)*a*c^3 - sqrt(b^2 - 4*a*c)*sqrt(a*c)*b*c^2)*cos(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2 - (sqrt(a*c)*b^2*c^2 + 4*sqrt(a*c)*a*c^3 - sqrt(b^2 -
4*a*c)*sqrt(a*c)*b*c^2)*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
+ ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 -
 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1
/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2
)*d*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(
1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*
c^3 - 4*a^2*c^4)
```

3.65 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*a*
c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.922801, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1586, 1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af\right.}{2\sqrt{2}a\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*a*
c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d - 12
*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
```

```
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{\left(a + bx^2 + cx^4\right)^3}\,dx = \int \frac{d + ex + fx^2}{\left(a + bx^2 + cx^4\right)^2}\,dx$$

$$= \int \frac{ex}{\left(a + bx^2 + cx^4\right)^2}\,dx + \int \frac{d + fx^2}{\left(a + bx^2 + cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2d - 2acd - abf + c(bd - 2af)x^2\right)}{2a\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} - \frac{\int \frac{-b^2d+6ac}{a}}{2a}$$

$$= \frac{x\left(b^2d - 2acd - abf + c(bd - 2af)x^2\right)}{2a\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} + \frac{1}{2}e\,\mathrm{Subst}$$

$$= -\frac{e\left(b + 2cx^2\right)}{2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} + \frac{x\left(b^2d - 2acd - ab\right.}{2a\left(b^2 - 4ac\right)}$$

$$= -\frac{e\left(b + 2cx^2\right)}{2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} + \frac{x\left(b^2d - 2acd - ab\right.}{2a\left(b^2 - 4ac\right)}$$

$$= -\frac{e\left(b + 2cx^2\right)}{2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)} + \frac{x\left(b^2d - 2acd - ab\right.}{2a\left(b^2 - 4ac\right)}$$

Mathematica [A] time = 1.42731, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e + fx) + 4acx(d + x(e + fx)) - 2bdx\left(b + cx^2\right)}{a\left(4ac - b^2\right)\left(a + bx^2 + cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b\left(d\sqrt{b^2 - 4ac} + 4af\right) - 2a\left(f\sqrt{b^2 - 4ac} + 6cd\right) + b^2d\right.}{a\left(b^2 - 4ac\right)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 -
```

```
4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]
*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
 (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] time = 0.095, size = 1813, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^3,x)
```

```
[Out] 2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*a*x*f+2*c/(4*a*c-b
^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*a*x*f+1/4*c/(4*a*c-b^2)^2/a*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b^3*d-c/(4
*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/((
(-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-c/(4*a*c-b^2)^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*b*f-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*(-4*a*c+b^2)^(1/2)*b^2*d-1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
a*c+b^2)^(1/2)*b^2*d+c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)
*d*x*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b
/c)*d*x*b+1/4/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*b^
3-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*(-4*a*c+b^2)^(
1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*d*x*b+1/4/(4*a*
c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*d*x*b^3-1/4/(4*a*c-b^2)^2
/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*(-4*a*c+b^2)^(1/2)*d-c^2/(4*a*c-b^2)^2*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b*d+1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c
)/a*d*x*(-4*a*c+b^2)^(1/2)*b^2+3*c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(
1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*(-4*
```

```
a*c+b^2)^(1/2)*d+c^2/(4*a*c-b^2)^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)
*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*d+2*c^2/(4*a*c-b^2
)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*f-1/2*c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*f-2*c
^2/(4*a*c-b^2)^2*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(
1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*f+1/2*c/(4*a*c-b^2)^2*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(
1/2))*b^2*f+c/(4*a*c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(2*c*x^2+(-4*a*c+b^2)^(1
/2)+b)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*e*a-c/(4*a*
c-b^2)^2*(-4*a*c+b^2)^(1/2)*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+2*c/(4*a*c-
b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*a-1/2/(4*a*c-b^2)^2/(x^2+1/
2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-
4*a*c+b^2)^(1/2)/c)*e*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(
1/2)/c)*x*b^2*f-1/2/(4*a*c-b^2)^2/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*x*
b^2*f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.66 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{\left(a+bx^2+cx^4\right)^4}\,dx$

Optimal. Leaf size=621

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2c\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\cdots}$$

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(
3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b
*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f)
+ 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3
*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 4.59433, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.159, Rules used = {1586, 1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2\left(20a^2cf+ab^2f-24abcd+3b^3d\right)+8a^2bcf+28a^2c^2d-25ab^2cd+ab^3f+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(-\frac{-52a^2bcf+168a^2c^2d-30ab^2c\cdots}{\sqrt{b^2-4ac}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

```
[Out] -(e*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
```

```
)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(
3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*d
- 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b
*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f)
+ 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f - (3
*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]
*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2
*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{ad + aex + (bd + af)x^2 + bex^3 + (cd + bf)x^4 + cex^5 + cfx^6}{\left(a + bx^2 + cx^4\right)^4} dx = \int \frac{d + ex + fx^2}{\left(a + bx^2 + cx^4\right)^3} dx$$

$$= \int \frac{ex}{\left(a + bx^2 + cx^4\right)^3} dx + \int \frac{d + fx^2}{\left(a + bx^2 + cx^4\right)^3} dx$$

$$= \frac{x\left(b^2d - 2acd - abf + c(bd - 2af)x^2\right)}{4a\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} - \frac{\int \frac{-3b^2d+14ac}{(a}}{4a}$$

$$= \frac{x\left(b^2d - 2acd - abf + c(bd - 2af)x^2\right)}{4a\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{x\left(3b^4d - 2\right.}{}$$

$$= -\frac{e\left(b + 2cx^2\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{x\left(b^2d - 2acd - ab\right.}{4a\left(b^2 - 4ac\right)(}$$

$$= -\frac{e\left(b + 2cx^2\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{x\left(b^2d - 2acd - ab\right.}{4a\left(b^2 - 4ac\right)(}$$

$$= -\frac{e\left(b + 2cx^2\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{x\left(b^2d - 2acd - ab\right.}{4a\left(b^2 - 4ac\right)(}$$

$$= -\frac{e\left(b + 2cx^2\right)}{4\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)^2} + \frac{x\left(b^2d - 2acd - ab\right.}{4a\left(b^2 - 4ac\right)(}$$

Mathematica [A] time = 4.53781, size = 625, normalized size = 1.01

$$\frac{1}{16}\left(\frac{8a^2c\left(b(3e + 2fx) + cx\left(7d + 6ex + 5fx^2\right)\right) + 2abx\left(b^2f - 25bcd + bcfx^2 - 24c^2dx^2\right) + 6b^3dx\left(b + cx^2\right)}{a^2\left(b^2 - 4ac\right)^2\left(a + bx^2 + cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(4a^2\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
```

```
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

```
[Out] ((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-25*
b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c*x*
(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[
b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*
(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*S
qrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2
*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48
*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] time = 0.229, size = 7858, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^4,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.67 $\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=4

$$\log(x+2)$$

```
[Out] Log[2 + x]
```

Rubi [A] time = 0.0105952, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1586, 31}

$$\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx &= \int \frac{1}{2+x}\,dx\\ &= \log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0011245, size = 4, normalized size = 1.

$$\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[2 + x]
```

Maple [A] time = 0.002, size = 5, normalized size = 1.3

$$\ln (2 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

```
[Out] ln(2+x)
```

Maxima [A] time = 0.968118, size = 5, normalized size = 1.25

$$\log (x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] log(x + 2)
```

Fricas [A] time = 1.98294, size = 16, normalized size = 4.

$$\log (x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] log(x + 2)
```

Sympy [A] time = 0.061844, size = 3, normalized size = 0.75

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 2)
```

Giac [A] time = 1.06881, size = 7, normalized size = 1.75

$$\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] log(abs(x + 2))
```

3.68 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=14

$$(d-2e)\log(x+2)+ex$$

```
[Out] e*x + (d - 2*e)*Log[2 + x]
```

Rubi [A] time = 0.0239435, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 43}

$$(d-2e)\log(x+2)+ex$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] e*x + (d - 2*e)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex}{2+x}\,dx \\
&= \int \left(e+\frac{d-2e}{2+x}\right)dx \\
&= ex+(d-2e)\log(2+x)
\end{aligned}
$$

Mathematica [A] time = 0.0044482, size = 16, normalized size = 1.14

$$(d-2e)\log(x+2)+e(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] e*(2 + x) + (d - 2*e)*Log[2 + x]
```

Maple [A] time = 0.002, size = 18, normalized size = 1.3

$$ex+\ln(2+x)\,d-2\,\ln(2+x)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

```
[Out] e*x+ln(2+x)*d-2*ln(2+x)*e
```

Maxima [A] time = 0.948895, size = 19, normalized size = 1.36

$$ex+(d-2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] e*x + (d - 2*e)*log(x + 2)
```

Fricas [A] time = 1.73338, size = 38, normalized size = 2.71

$$ex + (d - 2e)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] e*x + (d - 2*e)*log(x + 2)
```

Sympy [A] time = 0.263152, size = 12, normalized size = 0.86

$$ex + (d - 2e)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] e*x + (d - 2*e)*log(x + 2)
```

Giac [A] time = 1.08089, size = 23, normalized size = 1.64

$$xe + (d - 2e)\log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] x*e + (d - 2*e)*log(abs(x + 2))
```

3.69 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx$

Optimal. Leaf size=31

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

```
[Out] (e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rubi [A] time = 0.0522829, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1586, 698}

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2}{2+x}\,dx$$
$$= \int \left(e-4f+\frac{d-2e+4f}{2+x}+f(2+x)\right)dx$$
$$= (e-4f)x+\frac{1}{2}f(2+x)^2+(d-2e+4f)\log(2+x)$$

Mathematica [A] time = 0.0117812, size = 30, normalized size = 0.97

$$\log(x+2)(d-2e+4f)+\frac{1}{2}(x+2)(2e+f(x-6))$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] ((2*e + f*(-6 + x))*(2 + x))/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Maple [A] time = 0.002, size = 35, normalized size = 1.1

$$\frac{fx^2}{2}+ex-2fx+\ln(2+x)d-2\ln(2+x)e+4\ln(2+x)f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*f*x^2+e*x-2*f*x+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+x)*f
```

Maxima [A] time = 1.03464, size = 36, normalized size = 1.16

$$\frac{1}{2}fx^2+(e-2f)x+(d-2e+4f)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima"
)
```

```
[Out] 1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Fricas [A] time = 1.46327, size = 73, normalized size = 2.35

$$\frac{1}{2} f x^2 + \left(e - 2 f\right)x + \left(d - 2 e + 4 f\right) \log \left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas"
)
```

```
[Out] 1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Sympy [A] time = 0.28747, size = 26, normalized size = 0.84

$$\frac{fx^2}{2} + x \left(e - 2f\right) + \left(d - 2e + 4f\right) \log \left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] f*x**2/2 + x*(e - 2*f) + (d - 2*e + 4*f)*log(x + 2)
```

Giac [A] time = 1.09603, size = 41, normalized size = 1.32

$$\frac{1}{2} f x^2 - 2 f x + x e + \left(d + 4 f - 2 e\right) \log \left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2 - 2*f*x + x*e + (d + 4*f - 2*e)*log(abs(x + 2))
```

3.70 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=51

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

```
[Out] (e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 + (d - 2*e +
 4*f - 8*g)*Log[2 + x]
```

Rubi [A] time = 0.0847926, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {1586, 1850}

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 + (d - 2*e +
 4*f - 8*g)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3}{2+x}\,dx$$
$$= \int \left(e-4f+12g+\frac{d-2e+4f-8g}{2+x}+(f-6g)(2+x)+g(2+x)^2\right) dx$$
$$= (e-4f+12g)x+\frac{1}{2}(f-6g)(2+x)^2+\frac{1}{3}g(2+x)^3+(d-2e+4f-8g$$

Mathematica [A] time = 0.0265888, size = 45, normalized size = 0.88

$$\log(x+2)(d-2e+4f-8g)+\frac{1}{6}(x+2)\left(6e+3f(x-6)+2g\left(x^2-5x+22\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^
4),x]
```

```
[Out] ((2 + x)*(6*e + 3*f*(-6 + x) + 2*g*(22 - 5*x + x^2)))/6 + (d - 2*e + 4*f -
8*g)*Log[2 + x]
```

Maple [A] time = 0.004, size = 58, normalized size = 1.1

$$\frac{gx^3}{3}+\frac{fx^2}{2}-gx^2+ex-2fx+4gx+\ln(2+x)d-2\ln(2+x)e+4\ln(2+x)f-8\ln(2+x)g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*g*x^3+1/2*f*x^2-g*x^2+e*x-2*f*x+4*g*x+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+x)*f
-8*ln(2+x)*g
```

Maxima [A] time = 1.15627, size = 58, normalized size = 1.14

$$\frac{1}{3}gx^3+\frac{1}{2}(f-2g)x^2+(e-2f+4g)x+(d-2e+4f-8g)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="m
axima")
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*f - 8*g)*l
og(x + 2)
```

Fricas [A] time = 1.46849, size = 116, normalized size = 2.27

$$\frac{1}{3}gx^3 + \frac{1}{2}\left(f - 2g\right)x^2 + \left(e - 2f + 4g\right)x + \left(d - 2e + 4f - 8g\right)\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="f
ricas")
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*f - 8*g)*l
og(x + 2)
```

Sympy [A] time = 0.317787, size = 41, normalized size = 0.8

$$\frac{gx^3}{3} + x^2\left(\frac{f}{2} - g\right) + x\left(e - 2f + 4g\right) + \left(d - 2e + 4f - 8g\right)\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**3/3 + x**2*(f/2 - g) + x*(e - 2*f + 4*g) + (d - 2*e + 4*f - 8*g)*log(x
 + 2)
```

Giac [A] time = 1.08436, size = 66, normalized size = 1.29

$$\frac{1}{3}gx^3 + \frac{1}{2}fx^2 - gx^2 - 2fx + 4gx + xe + \left(d + 4f - 8g - 2e\right)\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="g
iac")
```

```
[Out] 1/3*g*x^3 + 1/2*f*x^2 - g*x^2 - 2*f*x + 4*g*x + x*e + (d + 4*f - 8*g - 2*e)
*log(abs(x + 2))
```

3.71 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=68

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h*
x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi [A] time = 0.117464, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1586, 1850}

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h*
x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{2+x}\,dx$$
$$= \int \left(e\left(1-\frac{2(f-2g+4h)}{e}\right)+(f-2g+4h)x+(g-2h)x^2+hx^3$$
$$= (e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2h)x^3+\frac{hx^4}{4}+$$

Mathematica [A] time = 0.0231874, size = 68, normalized size = 1.

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*
x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h*
x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Maple [A] time = 0.003, size = 87, normalized size = 1.3

$$\frac{hx^4}{4}+\frac{gx^3}{3}-\frac{2\,hx^3}{3}+\frac{fx^2}{2}-gx^2+2\,hx^2+ex-2\,fx+4\,gx-8\,hx+\ln(2+x)\,d-2\,\ln(2+x)\,e+4\,\ln(2+x)\,f-8\,\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/4*h*x^4+1/3*g*x^3-2/3*h*x^3+1/2*f*x^2-g*x^2+2*h*x^2+e*x-2*f*x+4*g*x-8*h*x
+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+x)*f-8*ln(2+x)*g+16*ln(2+x)*h
```

Maxima [A] time = 1.03224, size = 84, normalized size = 1.24

$$\frac{1}{4}\,hx^4+\frac{1}{3}\,(g-2\,h)x^3+\frac{1}{2}\,(f-2\,g+4\,h)x^2+(e-2\,f+4\,g-8\,h)x+(d-2\,e+4\,f-8\,g+16\,h)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="maxima")
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*f + 4*g -
8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Fricas [A] time = 1.51703, size = 169, normalized size = 2.49

$$\frac{1}{4}hx^4 + \frac{1}{3}(g-2h)x^3 + \frac{1}{2}(f-2g+4h)x^2 + (e-2f+4g-8h)x + (d-2e+4f-8g+16h)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="fricas")
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*f + 4*g -
8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Sympy [A] time = 0.341934, size = 63, normalized size = 0.93

$$\frac{hx^4}{4} + x^3\left(\frac{g}{3} - \frac{2h}{3}\right) + x^2\left(\frac{f}{2} - g + 2h\right) + x\left(e - 2f + 4g - 8h\right) + \left(d - 2e + 4f - 8g + 16h\right)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**4/4 + x**3*(g/3 - 2*h/3) + x**2*(f/2 - g + 2*h) + x*(e - 2*f + 4*g - 8
*h) + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Giac [A] time = 1.07238, size = 100, normalized size = 1.47

$$\frac{1}{4}hx^4 + \frac{1}{3}gx^3 - \frac{2}{3}hx^3 + \frac{1}{2}fx^2 - gx^2 + 2hx^2 - 2fx + 4gx - 8hx + xe + (d+4f-8g+16h-2e)\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="giac")
```

```
[Out] 1/4*h*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 1/2*f*x^2 - g*x^2 + 2*h*x^2 - 2*f*x + 4
*g*x - 8*h*x + x*e + (d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2))
```

3.72 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=92

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4($$

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
+ 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*Log[2 + x]
```

Rubi [A] time = 0.14863, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {1586, 1850}

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4($$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4),x]
```


```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
+ 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*Log[2 + x]
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{\left(2-x-2x^2+x^3\right)\left(d+ex+fx^2+gx^3+hx^4+72x^5\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+72x^5}{2+x}\,dx$$
$$= \int \left(1152\left(1+\frac{e-2f+4g-8h}{1152}\right)+(-576+f-2g+4h)\right.$$
$$= (1152+e-2f+4g-8h)x-\frac{1}{2}(576-f+2g-4h)x^2+$$

Mathematica [A] time = 0.0350037, size = 92, normalized size = 1.

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))
/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
+ 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*Log[2 + x]
```

Maple [A] time = 0.003, size = 122, normalized size = 1.3

$$\frac{ix^5}{5}+\frac{hx^4}{4}-\frac{ix^4}{2}+\frac{gx^3}{3}-\frac{2\,hx^3}{3}+\frac{4\,ix^3}{3}+\frac{fx^2}{2}-gx^2+2\,hx^2-4\,ix^2+ex-2\,fx+4\,gx-8\,hx+16\,ix+\ln(2+x)\,d-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/5*i*x^5+1/4*h*x^4-1/2*i*x^4+1/3*g*x^3-2/3*h*x^3+4/3*i*x^3+1/2*f*x^2-g*x^2
+2*h*x^2-4*i*x^2+e*x-2*f*x+4*g*x-8*h*x+16*i*x+ln(2+x)*d-2*ln(2+x)*e+4*ln(2+
x)*f-8*ln(2+x)*g+16*ln(2+x)*h-32*ln(2+x)*i
```

Maxima [A] time = 0.967076, size = 113, normalized size = 1.23

$$\frac{1}{5}\,ix^5+\frac{1}{4}\,(h-2\,i)x^4+\frac{1}{3}\left(g-2\,h+4\,i\right)x^3+\frac{1}{2}\left(f-2\,g+4\,h-8\,i\right)x^2+\left(e-2\,f+4\,g-8\,h+16\,i\right)x+\left(d-2\,e+4\,f-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="maxima")
```

```
[Out] 1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f - 2*g + 4*
h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*log(x + 2)
```

Fricas [A] time = 1.52546, size = 231, normalized size = 2.51

$$\frac{1}{5}\,ix^5 + \frac{1}{4}\,(h - 2\,i)x^4 + \frac{1}{3}\left(g - 2\,h + 4\,i\right)x^3 + \frac{1}{2}\left(f - 2\,g + 4\,h - 8\,i\right)x^2 + \left(e - 2\,f + 4\,g - 8\,h + 16\,i\right)x + \left(d - 2\,e + 4\,f - 8\,g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="fricas")
```

```
[Out] 1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f - 2*g + 4*
h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*log(x + 2)
```

Sympy [A] time = 0.37633, size = 88, normalized size = 0.96

$$\frac{ix^5}{5} + x^4\left(\frac{h}{4} - \frac{i}{2}\right) + x^3\left(\frac{g}{3} - \frac{2h}{3} + \frac{4i}{3}\right) + x^2\left(\frac{f}{2} - g + 2h - 4i\right) + x\left(e - 2f + 4g - 8h + 16i\right) + \left(d - 2e + 4f - 8g + 16h - 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**
2+4),x)
```

```
[Out] i*x**5/5 + x**4*(h/4 - i/2) + x**3*(g/3 - 2*h/3 + 4*i/3) + x**2*(f/2 - g +
2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + (d - 2*e + 4*f - 8*g + 16*h -
 32*i)*log(x + 2)
```

Giac [A] time = 1.06095, size = 142, normalized size = 1.54

$$\frac{1}{5}ix^5+\frac{1}{4}hx^4-\frac{1}{2}ix^4+\frac{1}{3}gx^3-\frac{2}{3}hx^3+\frac{4}{3}ix^3+\frac{1}{2}fx^2-gx^2+2hx^2-4ix^2-2fx+4gx-8hx+16ix+xe+\left(d+4f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="giac")
```

```
[Out] 1/5*i*x^5 + 1/4*h*x^4 - 1/2*i*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 4/3*i*x^3 + 1/2
*f*x^2 - g*x^2 + 2*h*x^2 - 4*i*x^2 - 2*f*x + 4*g*x - 8*h*x + 16*i*x + x*e +
 (d + 4*f - 8*g + 16*h - 32*i - 2*e)*log(abs(x + 2))
```

3.73 $\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=11

$$\log(x+1)-\log(x+2)$$

```
[Out] Log[1 + x] - Log[2 + x]
```

Rubi [A] time = 0.0101318, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1586, 616, 31}

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{2-3x+x^2}{4-5x^2+x^4}\,dx = \int \frac{1}{2+3x+x^2}\,dx$$
$$= \int \frac{1}{1+x}\,dx - \int \frac{1}{2+x}\,dx$$
$$= \log(1+x) - \log(2+x)$$

Mathematica [A] time = 0.0027395, size = 11, normalized size = 1.

$$\log(x+1) - \log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$\ln(1+x) - \ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)/(x^4-5*x^2+4),x)
```

```
[Out] ln(1+x)-ln(2+x)
```

Maxima [A] time = 0.963834, size = 15, normalized size = 1.36

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Fricas [A] time = 1.51612, size = 35, normalized size = 3.18

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A] time = 0.097253, size = 8, normalized size = 0.73

$$\log(x+1)-\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

Giac [A] time = 1.08595, size = 18, normalized size = 1.64

$$-\log(|x+2|)+\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -log(abs(x + 2)) + log(abs(x + 1))
```

3.74 $\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4} dx$

Optimal. Leaf size=22

$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

```
[Out] (d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Rubi [A] time = 0.0206828, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1586, 632, 31}

$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(d+ex)\left(2-3x+x^2\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex}{2+3x+x^2}\,dx$$
$$= -\left((d-2e)\int \frac{1}{2+x}\,dx\right) + (d-e)\int \frac{1}{1+x}\,dx$$
$$= (d-e)\log(1+x) - (d-2e)\log(2+x)$$

Mathematica [A] time = 0.0073826, size = 23, normalized size = 1.05

$$(d-e)\log(x+1) + (2e-d)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (d - e)*Log[1 + x] + (-d + 2*e)*Log[2 + x]
```

Maple [A] time = 0.004, size = 29, normalized size = 1.3

$$-\ln(2+x)\,d + 2\,\ln(2+x)\,e + \ln(1+x)\,d - \ln(1+x)\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x)
```

```
[Out] -ln(2+x)*d+2*ln(2+x)*e+ln(1+x)*d-ln(1+x)*e
```

Maxima [A] time = 0.9715, size = 30, normalized size = 1.36

$$-(d-2\,e)\log(x+2) + (d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Fricas [A] time = 1.52891, size = 59, normalized size = 2.68

$$-(d-2e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Sympy [A] time = 0.252077, size = 29, normalized size = 1.32

$$(-d+2e)\log\left(x+\frac{4d-6e}{2d-3e}\right)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] (-d + 2*e)*log(x + (4*d - 6*e)/(2*d - 3*e)) + (d - e)*log(x + 1)
```

Giac [A] time = 1.06985, size = 35, normalized size = 1.59

$$-(d-2e)\log(|x+2|)+(d-e)\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -(d - 2*e)*log(abs(x + 2)) + (d - e)*log(abs(x + 1))
```

3.75 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=29

$$\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f)+fx$$

```
[Out] f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Rubi [A] time = 0.0500694, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1586, 1657, 632, 31}

$$\log(x+1)(d-e+f)-\log(x+2)(d-2e+4f)+fx$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2}{2+3x+x^2}\,dx \\
&= \int \left(f+\frac{d-2f+(e-3f)x}{2+3x+x^2}\right)dx \\
&= fx+\int \frac{d-2f+(e-3f)x}{2+3x+x^2}\,dx \\
&= fx+(d-e+f)\int \frac{1}{1+x}\,dx-(d-2e+4f)\int \frac{1}{2+x}\,dx \\
&= fx+(d-e+f)\log(1+x)-(d-2e+4f)\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.013349, size = 30, normalized size = 1.03

$$\log(x+1)(d-e+f)+\log(x+2)(-d+2e-4f)+fx$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (d - e + f)*Log[1 + x] + (-d + 2*e - 4*f)*Log[2 + x]
```

Maple [A] time = 0.005, size = 45, normalized size = 1.6

$$fx-\ln(2+x)\,d+2\,\ln(2+x)\,e-4\,\ln(2+x)\,f+\ln(1+x)\,d-\ln(1+x)\,e+\ln(1+x)\,f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] f*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+ln(1+x)*d-ln(1+x)*e+ln(1+x)*f
```

Maxima [A] time = 0.968562, size = 39, normalized size = 1.34

$$fx - \left(d - 2\,e + 4\,f\right)\log\left(x + 2\right) + \left(d - e + f\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Fricas [A] time = 1.48516, size = 80, normalized size = 2.76

$$fx - \left(d - 2\,e + 4\,f\right)\log\left(x + 2\right) + \left(d - e + f\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Sympy [A] time = 0.594252, size = 44, normalized size = 1.52

$$fx + \left(-d + 2e - 4f\right)\log\left(x + \frac{4d - 6e + 10f}{2d - 3e + 5f}\right) + \left(d - e + f\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] f*x + (-d + 2*e - 4*f)*log(x + (4*d - 6*e + 10*f)/(2*d - 3*e + 5*f)) + (d -
 e + f)*log(x + 1)
```

Giac [A] time = 1.06533, size = 45, normalized size = 1.55

$$fx - \left(d + 4\,f - 2\,e\right)\log\left(|x + 2|\right) + \left(d + f - e\right)\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")

[Out] f*x - (d + 4*f - 2*e)*log(abs(x + 2)) + (d + f - e)*log(abs(x + 1))
```

3.76 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=47

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

```
[Out] (f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g
)*Log[2 + x]
```

Rubi [A] time = 0.0677106, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1586, 1657, 632, 31}

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g
)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
```

```
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3}{2+3x+x^2}\,dx \\ &= \int \left(f-3g+gx+\frac{d-2f+6g+(e-3f+7g)x}{2+3x+x^2}\right)dx \\ &= (f-3g)x+\frac{gx^2}{2}+\int \frac{d-2f+6g+(e-3f+7g)x}{2+3x+x^2}\,dx \\ &= (f-3g)x+\frac{gx^2}{2}-(d-2e+4f-8g)\int \frac{1}{2+x}\,dx+(d-e+f-g)\int \frac{1}{1+x} \\ &= (f-3g)x+\frac{gx^2}{2}+(d-e+f-g)\log(1+x)-(d-2e+4f-8g)\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.020238, size = 44, normalized size = 0.94

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+fx+\frac{1}{2}g(x-6)x$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (g*(-6 + x)*x)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)
*Log[2 + x]
```

Maple [A] time = 0.006, size = 69, normalized size = 1.5

$$\frac{gx^2}{2}+fx-3\,gx-\ln(2+x)\,d+2\,\ln(2+x)\,e-4\,\ln(2+x)\,f+8\,\ln(2+x)\,g+\ln(1+x)\,d-\ln(1+x)\,e+\ln(1+x)\,f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*g*x^2+f*x-3*g*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+8*ln(2+x)*g+ln(1+x)*d
-ln(1+x)*e+ln(1+x)*f-ln(1+x)*g
```

Maxima [A] time = 0.962906, size = 61, normalized size = 1.3

$$\frac{1}{2}\,gx^2 + \left(f - 3\,g\right)x - \left(d - 2\,e + 4\,f - 8\,g\right)\log\left(x + 2\right) + \left(d - e + f - g\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d - e + f - g
)*log(x + 1)
```

Fricas [A] time = 1.50892, size = 120, normalized size = 2.55

$$\frac{1}{2}\,gx^2 + \left(f - 3\,g\right)x - \left(d - 2\,e + 4\,f - 8\,g\right)\log\left(x + 2\right) + \left(d - e + f - g\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d - e + f - g
)*log(x + 1)
```

Sympy [A] time = 0.885272, size = 66, normalized size = 1.4

$$\frac{gx^2}{2} + x\left(f - 3g\right) + \left(-d + 2e - 4f + 8g\right)\log\left(x + \frac{4d - 6e + 10f - 18g}{2d - 3e + 5f - 9g}\right) + \left(d - e + f - g\right)\log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**2/2 + x*(f - 3*g) + (-d + 2*e - 4*f + 8*g)*log(x + (4*d - 6*e + 10*f -
 18*g)/(2*d - 3*e + 5*f - 9*g)) + (d - e + f - g)*log(x + 1)
```

Giac [A] time = 1.11015, size = 66, normalized size = 1.4

$$\frac{1}{2}gx^2 + fx - 3gx - (d + 4f - 8g - 2e)\log(|x + 2|) + (d + f - g - e)\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] 1/2*g*x^2 + f*x - 3*g*x - (d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + (d + f -
g - e)*log(abs(x + 1))
```

3.77 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=66

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Log
[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rubi [A] time = 0.0853165, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1586, 1657, 632, 31}

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x
]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Log
[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
```

```
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{2+3x+x^2}\,dx$$
$$= \int \left(f-3g+7h+(g-3h)x+hx^2+\frac{d-2f+6g-14h+(e-3f+7\ldots}{2+3x+x^2}\right.$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+\int \frac{d-2f+6g-14h+(e-3\ldots}{2+3x+x^2}$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\int \frac{1}{1+x}\,dx\ldots$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\log(1+x)\ldots$$

Mathematica [A] time = 0.024024, size = 67, normalized size = 1.02

$$\log(x+1)(d-e+f-g+h)+\log(x+2)(-d+2e-4f+8g-16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Log
[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h)*Log[2 + x]
```

Maple [A] time = 0.006, size = 98, normalized size = 1.5

$$\frac{hx^3}{3}+\frac{gx^2}{2}-\frac{3hx^2}{2}+fx-3gx+7hx-\ln(2+x)d+2\ln(2+x)e-4\ln(2+x)f+8\ln(2+x)g-16\ln(2+x)h+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*h*x^3+1/2*g*x^2-3/2*h*x^2+f*x-3*g*x+7*h*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+
x)*f+8*ln(2+x)*g-16*ln(2+x)*h+ln(1+x)*d-ln(1+x)*e+ln(1+x)*f-ln(1+x)*g+ln(1+
x)*h
```

Maxima [A] time = 0.973296, size = 84, normalized size = 1.27

$$\frac{1}{3}hx^3 + \frac{1}{2}(g - 3h)x^2 + (f - 3g + 7h)x - (d - 2e + 4f - 8g + 16h)\log(x + 2) + (d - e + f - g + h)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"maxima")
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*f - 8*g +
16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Fricas [A] time = 1.51943, size = 170, normalized size = 2.58

$$\frac{1}{3}hx^3 + \frac{1}{2}(g - 3h)x^2 + (f - 3g + 7h)x - (d - 2e + 4f - 8g + 16h)\log(x + 2) + (d - e + f - g + h)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"fricas")
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*f - 8*g +
16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Sympy [A] time = 1.50584, size = 94, normalized size = 1.42

$$\frac{hx^3}{3} + x^2\left(\frac{g}{2} - \frac{3h}{2}\right) + x(f - 3g + 7h) + (-d + 2e - 4f + 8g - 16h)\log\left(x + \frac{4d - 6e + 10f - 18g + 34h}{2d - 3e + 5f - 9g + 17h}\right) + (d - e + f - g$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**3/3 + x**2*(g/2 - 3*h/2) + x*(f - 3*g + 7*h) + (-d + 2*e - 4*f + 8*g -
 16*h)*log(x + (4*d - 6*e + 10*f - 18*g + 34*h)/(2*d - 3*e + 5*f - 9*g + 17
*h)) + (d - e + f - g + h)*log(x + 1)
```

Giac [A] time = 1.08079, size = 93, normalized size = 1.41

$$\frac{1}{3}hx^3 + \frac{1}{2}gx^2 - \frac{3}{2}hx^2 + fx - 3gx + 7hx - \left(d + 4f - 8g + 16h - 2e\right)\log\left(|x+2|\right) + \left(d + f - g + h - e\right)\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"giac")
```

```
[Out] 1/3*h*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + f*x - 3*g*x + 7*h*x - (d + 4*f - 8*g +
16*h - 2*e)*log(abs(x + 2)) + (d + f - g + h - e)*log(abs(x + 1))
```

3.78 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=90

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+$$

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (i
*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h
- 32*i)*Log[2 + x]
```

Rubi [A] time = 0.107277, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1586, 1657, 632, 31}

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
+ x^4),x]
```


```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (i
*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h
- 32*i)*Log[2 + x]
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(2-3x+x^2\right)\left(d+ex+fx^2+gx^3+hx^4+78x^5\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+78x^5}{2+3x+x^2}\,dx$$
$$= \int \Big(-1170+f-3g+7h+(546+g-3h)x-(234-h)x^2+7$$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-\frac{1}{3}(234-h)x^3$$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-\frac{1}{3}(234-h)x^3$$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-\frac{1}{3}(234-h)x^3$$

Mathematica [A] time = 0.0412408, size = 91, normalized size = 1.01

$$\log(x+1)(d-e+f-g+h-i)+\log(x+2)(-d+2e-4f+8g-16h+32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4),x]
```

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (i
*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h
 + 32*i)*Log[2 + x]
```

Maple [A] time = 0.006, size = 134, normalized size = 1.5

$$\frac{ix^4}{4}+\frac{hx^3}{3}-ix^3+\frac{gx^2}{2}-\frac{3\,hx^2}{2}+\frac{7\,ix^2}{2}+fx-3\,gx+7\,hx-15\,ix-\ln\left(2+x\right)d+2\,\ln\left(2+x\right)e-4\,\ln\left(2+x\right)f+8\,\ln\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/4*i*x^4+1/3*h*x^3-i*x^3+1/2*g*x^2-3/2*h*x^2+7/2*i*x^2+f*x-3*g*x+7*h*x-15*
i*x-ln(2+x)*d+2*ln(2+x)*e-4*ln(2+x)*f+8*ln(2+x)*g-16*ln(2+x)*h+32*ln(2+x)*i
+ln(1+x)*d-ln(1+x)*e+ln(1+x)*f-ln(1+x)*g+ln(1+x)*h-ln(1+x)*i
```

Maxima [A] time = 0.97668, size = 113, normalized size = 1.26

$$\frac{1}{4}\,ix^4+\frac{1}{3}\,(h-3\,i)x^3+\frac{1}{2}\,\left(g-3\,h+7\,i\right)x^2+\left(f-3\,g+7\,h-15\,i\right)x-\left(d-2\,e+4\,f-8\,g+16\,h-32\,i\right)\log\left(x+2\right)+\left(d-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="maxima")
```

```
[Out] 1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*g + 7*h -
15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + (d - e + f - g +
 h - i)*log(x + 1)
```

Fricas [A] time = 1.48033, size = 230, normalized size = 2.56

$$\frac{1}{4}\,ix^4+\frac{1}{3}\,(h-3\,i)x^3+\frac{1}{2}\,\left(g-3\,h+7\,i\right)x^2+\left(f-3\,g+7\,h-15\,i\right)x-\left(d-2\,e+4\,f-8\,g+16\,h-32\,i\right)\log\left(x+2\right)+\left(d-e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="fricas")
```

```
[Out] 1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*g + 7*h -
15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + (d - e + f - g +
```

```
 h - i)*log(x + 1)
```

Sympy [A] time = 2.53455, size = 122, normalized size = 1.36

$$\frac{ix^4}{4} + x^3\left(\frac{h}{3} - i\right) + x^2\left(\frac{g}{2} - \frac{3h}{2} + \frac{7i}{2}\right) + x\left(f - 3g + 7h - 15i\right) + \left(-d + 2e - 4f + 8g - 16h + 32i\right)\log\left(x + \frac{4d - 6e + 10f}{2d - 3e + 5f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),
x)
```

```
[Out] i*x**4/4 + x**3*(h/3 - i) + x**2*(g/2 - 3*h/2 + 7*i/2) + x*(f - 3*g + 7*h -
 15*i) + (-d + 2*e - 4*f + 8*g - 16*h + 32*i)*log(x + (4*d - 6*e + 10*f - 1
8*g + 34*h - 66*i)/(2*d - 3*e + 5*f - 9*g + 17*h - 33*i)) + (d - e + f - g
+ h - i)*log(x + 1)
```

Giac [A] time = 1.07949, size = 131, normalized size = 1.46

$$\frac{1}{4}ix^4 + \frac{1}{3}hx^3 - ix^3 + \frac{1}{2}gx^2 - \frac{3}{2}hx^2 + \frac{7}{2}ix^2 + fx - 3gx + 7hx - 15ix - \left(d + 4f - 8g + 16h - 32i - 2e\right)\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="giac")
```

```
[Out] 1/4*i*x^4 + 1/3*h*x^3 - i*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + 7/2*i*x^2 + f*x - 3
*g*x + 7*h*x - 15*i*x - (d + 4*f - 8*g + 16*h - 32*i - 2*e)*log(abs(x + 2))
 + (d + f - g + h - i - e)*log(abs(x + 1))
```

3.79 $\int \frac{2+x}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

```
[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Rubi [A] time = 0.0207676, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 2058}

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{4-5x^2+x^4} \, dx &= \int \frac{1}{2-x-2x^2+x^3} \, dx\\
&= \int \left(\frac{1}{3(-2+x)}-\frac{1}{2(-1+x)}+\frac{1}{6(1+x)}\right) dx\\
&= -\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.0067574, size = 29, normalized size = 1.

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/3 + Log[1 + x]/6
```

Maple [A] time = 0.006, size = 20, normalized size = 0.7

$$\frac{\ln(1+x)}{6}+\frac{\ln(x-2)}{3}-\frac{\ln(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^4-5*x^2+4),x)
```

```
[Out] 1/6*ln(1+x)+1/3*ln(x-2)-1/2*ln(x-1)
```

Maxima [A] time = 0.975499, size = 26, normalized size = 0.9

$$\frac{1}{6}\log(x+1)-\frac{1}{2}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Fricas [A] time = 1.50185, size = 68, normalized size = 2.34

$$\frac{1}{6}\log(x+1)-\frac{1}{2}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A] time = 0.124361, size = 19, normalized size = 0.66

$$\frac{\log(x-2)}{3} - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/2 + log(x + 1)/6
```

Giac [A] time = 1.08238, size = 30, normalized size = 1.03

$$\frac{1}{6}\log(|x+1|) - \frac{1}{2}\log(|x-1|) + \frac{1}{3}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(x + 1)) - 1/2*log(abs(x - 1)) + 1/3*log(abs(x - 2))
```

3.80 $\int \frac{(2+x)(d+ex)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=42

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

```
[Out] -((d + e)*Log[1 - x])/2 + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log[1 + x])/6
```

Rubi [A] time = 0.0518755, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 2074}

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + e)*Log[1 - x])/2 + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log[1 + x])/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx = \int \frac{d+ex}{2-x-2x^2+x^3}\,dx$$
$$= \int \left(\frac{d+2e}{3(-2+x)} + \frac{-d-e}{2(-1+x)} + \frac{d-e}{6(1+x)}\right) dx$$
$$= -\frac{1}{2}(d+e)\log(1-x) + \frac{1}{3}(d+2e)\log(2-x) + \frac{1}{6}(d-e)\log(1+x)$$

Mathematica [A] time = 0.0187446, size = 39, normalized size = 0.93

$$\frac{1}{6}(-3(d+e)\log(1-x) + 2(d+2e)\log(2-x) + (d-e)\log(x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-3*(d + e)*Log[1 - x] + 2*(d + 2*e)*Log[2 - x] + (d - e)*Log[1 + x])/6
```

Maple [A] time = 0.007, size = 44, normalized size = 1.1

$$\frac{\ln(1+x)\,d}{6} - \frac{\ln(1+x)\,e}{6} + \frac{\ln(x-2)\,d}{3} + \frac{2\ln(x-2)\,e}{3} - \frac{\ln(x-1)\,d}{2} - \frac{\ln(x-1)\,e}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/3*ln(x-2)*d+2/3*ln(x-2)*e-1/2*ln(x-1)*d-1/2*l
n(x-1)*e
```

Maxima [A] time = 0.969047, size = 43, normalized size = 1.02

$$\frac{1}{6}(d-e)\log(x+1) - \frac{1}{2}(d+e)\log(x-1) + \frac{1}{3}(d+2e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Fricas [A] time = 1.50864, size = 103, normalized size = 2.45

$$\frac{1}{6}(d-e)\log(x+1)-\frac{1}{2}(d+e)\log(x-1)+\frac{1}{3}(d+2e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Sympy [B] time = 1.14819, size = 304, normalized size = 7.24

$$\frac{(d-e)\log\left(x+\frac{26d^3+66d^2e-9d^2(d-e)+78de^2-12de(d-e)-7d(d-e)^2+46e^3+3e^2(d-e)-8e(d-e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{6}-\frac{(d+e)\log\left(x+\frac{26d^3+66d^2e+27d^2(d+e)+78de^2+}{10d^3}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e)*log(x + (26*d**3 + 66*d**2*e - 9*d**2*(d - e) + 78*d*e**2 - 12*d*e*
(d - e) - 7*d*(d - e)**2 + 46*e**3 + 3*e**2*(d - e) - 8*e*(d - e)**2)/(10*d
**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/6 - (d + e)*log(x + (26*d**3 + 66*
d**2*e + 27*d**2*(d + e) + 78*d*e**2 + 36*d*e*(d + e) - 63*d*(d + e)**2 + 4
6*e**3 - 9*e**2*(d + e) - 72*e*(d + e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**
2 + 35*e**3))/2 + (d + 2*e)*log(x + (26*d**3 + 66*d**2*e - 18*d**2*(d + 2*e
) + 78*d*e**2 - 24*d*e*(d + 2*e) - 28*d*(d + 2*e)**2 + 46*e**3 + 6*e**2*(d
+ 2*e) - 32*e*(d + 2*e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/3
```

Giac [A] time = 1.07426, size = 51, normalized size = 1.21

$$\frac{1}{6}(d-e)\log(|x+1|)-\frac{1}{2}(d+e)\log(|x-1|)+\frac{1}{3}(d+2e)\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*(d - e)*log(abs(x + 1)) - 1/2*(d + e)*log(abs(x - 1)) + 1/3*(d + 2*e)*l
og(abs(x - 2))
```

3.81 $\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

```
[Out] -((d + e + f)*Log[1 - x])/2 + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((d - e + f)
*Log[1 + x])/6
```

Rubi [A] time = 0.0638697, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1586, 2074}

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -((d + e + f)*Log[1 - x])/2 + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((d - e + f)
*Log[1 + x])/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2}{2-x-2x^2+x^3}\,dx$$
$$= \int \left(\frac{d+2e+4f}{3(-2+x)} + \frac{-d-e-f}{2(-1+x)} + \frac{d-e+f}{6(1+x)}\right)dx$$
$$= -\frac{1}{2}(d+e+f)\log(1-x) + \frac{1}{3}(d+2e+4f)\log(2-x) + \frac{1}{6}(d-e+f)\log(1+x)$$

Mathematica [A] time = 0.0204724, size = 44, normalized size = 0.94

$$\frac{1}{6}(-3\log(1-x)(d+e+f) + 2\log(2-x)(d+2e+4f) + \log(x+1)(d-e+f))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-3*(d + e + f)*Log[1 - x] + 2*(d + 2*e + 4*f)*Log[2 - x] + (d - e + f)*Log
[1 + x])/6
```

Maple [A] time = 0.007, size = 65, normalized size = 1.4

$$\frac{\ln(1+x)d}{6} - \frac{\ln(1+x)e}{6} + \frac{\ln(1+x)f}{6} + \frac{\ln(x-2)d}{3} + \frac{2\ln(x-2)e}{3} + \frac{4\ln(x-2)f}{3} - \frac{\ln(x-1)d}{2} - \frac{\ln(x-1)e}{2} - \ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f+1/3*ln(x-2)*d+2/3*ln(x-2)*e+4/3*l
n(x-2)*f-1/2*ln(x-1)*d-1/2*ln(x-1)*e-1/2*ln(x-1)*f
```

Maxima [A] time = 0.944502, size = 50, normalized size = 1.06

$$\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right) - \frac{1}{2}\left(d+e+f\right)\log\left(x-1\right) + \frac{1}{3}\left(d+2e+4f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d + 2*e + 4*
f)*log(x - 2)
```

Fricas [A] time = 1.62067, size = 122, normalized size = 2.6

$$\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2e+4f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d + 2*e + 4*
f)*log(x - 2)
```

Sympy [B] time = 5.15638, size = 716, normalized size = 15.23

$$\frac{\left(d-e+f\right)\log\left(x+\frac{26d^3+66d^2e+132d^2f-9d^2\left(d-e+f\right)+78de^2+276def-12de\left(d-e+f\right)+222df^2+6df\left(d-e+f\right)-7d\left(d-e+f\right)^2+46e^3+204e^2f+3e^2\left(d-e+f\right)+}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+285e}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f - 9*d**2*(d - e + f)
+ 78*d*e**2 + 276*d*e*f - 12*d*e*(d - e + f) + 222*d*f**2 + 6*d*f*(d - e +
f) - 7*d*(d - e + f)**2 + 46*e**3 + 204*e**2*f + 3*e**2*(d - e + f) + 282*e
*f**2 + 36*e*f*(d - e + f) - 8*e*(d - e + f)**2 + 116*f**3 + 51*f**2*(d - e
 + f) - 13*f*(d - e + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2
 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/
6 - (d + e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 27*d**2*(d + e
+ f) + 78*d*e**2 + 276*d*e*f + 36*d*e*(d + e + f) + 222*d*f**2 - 18*d*f*(d
+ e + f) - 63*d*(d + e + f)**2 + 46*e**3 + 204*e**2*f - 9*e**2*(d + e + f)
+ 282*e*f**2 - 108*e*f*(d + e + f) - 72*e*(d + e + f)**2 + 116*f**3 - 153*f
**2*(d + e + f) - 117*f*(d + e + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f +
 102*d*e**2 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 +
```

```
154*f**3))/2 + (d + 2*e + 4*f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f -
18*d**2*(d + 2*e + 4*f) + 78*d*e**2 + 276*d*e*f - 24*d*e*(d + 2*e + 4*f) +
222*d*f**2 + 12*d*f*(d + 2*e + 4*f) - 28*d*(d + 2*e + 4*f)**2 + 46*e**3 + 2
04*e**2*f + 6*e**2*(d + 2*e + 4*f) + 282*e*f**2 + 72*e*f*(d + 2*e + 4*f) -
32*e*(d + 2*e + 4*f)**2 + 116*f**3 + 102*f**2*(d + 2*e + 4*f) - 52*f*(d + 2
*e + 4*f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2 + 318*d*e*f +
246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/3
```

Giac [A] time = 1.0817, size = 58, normalized size = 1.23

$$\frac{1}{6}\left(d+f-e\right)\log\left(|x+1|\right)-\frac{1}{2}\left(d+f+e\right)\log\left(|x-1|\right)+\frac{1}{3}\left(d+4f+2e\right)\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*(d + f - e)*log(abs(x + 1)) - 1/2*(d + f + e)*log(abs(x - 1)) + 1/3*(d
+ 4*f + 2*e)*log(abs(x - 2))
```

3.82 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

```
[Out] g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/3
 + ((d - e + f - g)*Log[1 + x])/6
```

Rubi [A] time = 0.0793655, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 2074}

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/3
 + ((d - e + f - g)*Log[1 + x])/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3}{2-x-2x^2+x^3}\,dx$$
$$= \int \left(g+\frac{d+2e+4f+8g}{3(-2+x)}+\frac{-d-e-f-g}{2(-1+x)}+\frac{d-e+f-g}{6(1+x)}\right)dx$$
$$= gx-\frac{1}{2}(d+e+f+g)\log(1-x)+\frac{1}{3}(d+2e+4f+8g)\log(2-x)+\frac{1}{6}(d-e+f-$$

Mathematica [A] time = 0.025557, size = 55, normalized size = 0.96

$$\frac{1}{6}(-3\log(1-x)(d+e+f+g)+2\log(2-x)(d+2e+4f+8g)+\log(x+1)(d-e+f-g)+6gx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*g*x - 3*(d + e + f + g)*Log[1 - x] + 2*(d + 2*e + 4*f + 8*g)*Log[2 - x]
+ (d - e + f - g)*Log[1 + x])/6
```

Maple [A] time = 0.007, size = 89, normalized size = 1.6

$$gx+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\frac{4\ln(x-2)f}{3}+\frac{8\ln(x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] g*x+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g+1/3*ln(x-2)*d+2
/3*ln(x-2)*e+4/3*ln(x-2)*f+8/3*ln(x-2)*g-1/2*ln(x-1)*d-1/2*ln(x-1)*e-1/2*ln
(x-1)*f-1/2*ln(x-1)*g
```

Maxima [A] time = 0.959427, size = 63, normalized size = 1.11

$$gx+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2e+4f+8g\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x - 1) + 1/3
*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Fricas [A] time = 1.60767, size = 149, normalized size = 2.61

$$gx + \frac{1}{6}\left(d - e + f - g\right)\log\left(x + 1\right) - \frac{1}{2}\left(d + e + f + g\right)\log\left(x - 1\right) + \frac{1}{3}\left(d + 2e + 4f + 8g\right)\log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x - 1) + 1/3
*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [B] time = 26.0929, size = 1389, normalized size = 24.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x + (d - e + f - g)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*
g - 9*d**2*(d - e + f - g) + 78*d*e**2 + 276*d*e*f + 444*d*e*g - 12*d*e*(d
- e + f - g) + 222*d*f**2 + 636*d*f*g + 6*d*f*(d - e + f - g) + 510*d*g**2
+ 36*d*g*(d - e + f - g) - 7*d*(d - e + f - g)**2 + 46*e**3 + 204*e**2*f +
390*e**2*g + 3*e**2*(d - e + f - g) + 282*e*f**2 + 984*e*f*g + 36*e*f*(d -
e + f - g) + 930*e*g**2 + 102*e*g*(d - e + f - g) - 8*e*(d - e + f - g)**2
+ 116*f**3 + 534*f**2*g + 51*f**2*(d - e + f - g) + 924*f*g**2 + 228*f*g*(d
 - e + f - g) - 13*f*(d - e + f - g)**2 + 586*g**3 + 243*g**2*(d - e + f -
g) - 20*g*(d - e + f - g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*
g + 102*d*e**2 + 318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**
2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2
 + 154*f**3 + 717*f**2*g + 966*f*g**2 + 323*g**3))/6 - (d + e + f + g)*log(
x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*g + 27*d**2*(d + e + f + g
) + 78*d*e**2 + 276*d*e*f + 444*d*e*g + 36*d*e*(d + e + f + g) + 222*d*f**2
```

```
 + 636*d*f*g - 18*d*f*(d + e + f + g) + 510*d*g**2 - 108*d*g*(d + e + f + g
) - 63*d*(d + e + f + g)**2 + 46*e**3 + 204*e**2*f + 390*e**2*g - 9*e**2*(d
 + e + f + g) + 282*e*f**2 + 984*e*f*g - 108*e*f*(d + e + f + g) + 930*e*g*
*2 - 306*e*g*(d + e + f + g) - 72*e*(d + e + f + g)**2 + 116*f**3 + 534*f**
2*g - 153*f**2*(d + e + f + g) + 924*f*g**2 - 684*f*g*(d + e + f + g) - 117
*f*(d + e + f + g)**2 + 586*g**3 - 729*g**2*(d + e + f + g) - 180*g*(d + e
+ f + g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g + 102*d*e**2 +
318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**2 + 35*e**3 + 174
*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2 + 154*f**3 + 717
*f**2*g + 966*f*g**2 + 323*g**3))/2 + (d + 2*e + 4*f + 8*g)*log(x + (26*d**
3 + 66*d**2*e + 132*d**2*f + 174*d**2*g - 18*d**2*(d + 2*e + 4*f + 8*g) + 7
8*d*e**2 + 276*d*e*f + 444*d*e*g - 24*d*e*(d + 2*e + 4*f + 8*g) + 222*d*f**
2 + 636*d*f*g + 12*d*f*(d + 2*e + 4*f + 8*g) + 510*d*g**2 + 72*d*g*(d + 2*e
 + 4*f + 8*g) - 28*d*(d + 2*e + 4*f + 8*g)**2 + 46*e**3 + 204*e**2*f + 390*
e**2*g + 6*e**2*(d + 2*e + 4*f + 8*g) + 282*e*f**2 + 984*e*f*g + 72*e*f*(d
+ 2*e + 4*f + 8*g) + 930*e*g**2 + 204*e*g*(d + 2*e + 4*f + 8*g) - 32*e*(d +
 2*e + 4*f + 8*g)**2 + 116*f**3 + 534*f**2*g + 102*f**2*(d + 2*e + 4*f + 8*
g) + 924*f*g**2 + 456*f*g*(d + 2*e + 4*f + 8*g) - 52*f*(d + 2*e + 4*f + 8*g
)**2 + 586*g**3 + 486*g**2*(d + 2*e + 4*f + 8*g) - 80*g*(d + 2*e + 4*f + 8*
g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g + 102*d*e**2 + 318*d*
e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**2 + 35*e**3 + 174*e**2*
f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2 + 154*f**3 + 717*f**2*
g + 966*f*g**2 + 323*g**3))/3
```

Giac [A] time = 1.08989, size = 72, normalized size = 1.26

$$gx + \frac{1}{6}\left(d + f - g - e\right)\log\left(|x + 1|\right) - \frac{1}{2}\left(d + f + g + e\right)\log\left(|x - 1|\right) + \frac{1}{3}\left(d + 4f + 8g + 2e\right)\log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] g*x + 1/6*(d + f - g - e)*log(abs(x + 1)) - 1/2*(d + f + g + e)*log(abs(x -
 1)) + 1/3*(d + 4*f + 8*g + 2*e)*log(abs(x - 2))
```

3.83 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)$$

```
[Out] (g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((d + 2*e +
4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*Log[1 + x])/6
```

Rubi [A] time = 0.106973, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1586, 2074}

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((d + 2*e +
4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*Log[1 + x])/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{2-x-2x^2+x^3}\,dx$$
$$= \int \left(g\left(1+\frac{2h}{g}\right)+\frac{d+2e+4f+8g+16h}{3(-2+x)}+\frac{-d-e-f-g-h}{2(-1+x)}+hx+\frac{d-e\cdots}{6\cdots}\right.$$
$$= (g+2h)x+\frac{hx^2}{2}-\frac{1}{2}(d+e+f+g+h)\log(1-x)+\frac{1}{3}(d+2e+4f+8g+16\cdots$$

Mathematica [A] time = 0.0360083, size = 71, normalized size = 0.96

$$\frac{1}{6}\left(-3\log(1-x)(d+e+f+g+h)+2\log(2-x)(d+2(e+2f+4g+8h))+\log(x+1)(d-e+f-g+h)+6x(g+2h)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*(g + 2*h)*x + 3*h*x^2 - 3*(d + e + f + g + h)*Log[1 - x] + 2*(d + 2*(e +
 2*f + 4*g + 8*h))*Log[2 - x] + (d - e + f - g + h)*Log[1 + x])/6
```

Maple [A] time = 0.007, size = 120, normalized size = 1.6

$$\frac{hx^2}{2}+gx+2hx+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(1+x)h}{6}+\frac{\ln(x-2)d}{3}+\frac{2\ln(x-2)e}{3}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*h*x^2+g*x+2*h*x+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f-1/6*ln(1+x)*g
+1/6*ln(1+x)*h+1/3*ln(x-2)*d+2/3*ln(x-2)*e+4/3*ln(x-2)*f+8/3*ln(x-2)*g+16/3
*ln(x-2)*h-1/2*ln(x-1)*d-1/2*ln(x-1)*e-1/2*ln(x-1)*f-1/2*ln(x-1)*g-1/2*ln(x
-1)*h
```

Maxima [A] time = 0.961344, size = 84, normalized size = 1.14

$$\frac{1}{2}hx^2+\left(g+2h\right)x+\frac{1}{6}\left(d-e+f-g+h\right)\log(x+1)-\frac{1}{2}\left(d+e+f+g+h\right)\log(x-1)+\frac{1}{3}\left(d+2e+4f+8g+16h\right)\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/2*(d + e +
 f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Fricas [A] time = 1.87172, size = 196, normalized size = 2.65

$$\frac{1}{2}hx^2 + \left(g + 2h\right)x + \frac{1}{6}\left(d - e + f - g + h\right)\log\left(x + 1\right) - \frac{1}{2}\left(d + e + f + g + h\right)\log\left(x - 1\right) + \frac{1}{3}\left(d + 2e + 4f + 8g + 16h$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/2*(d + e +
 f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Sympy [B] time = 137.208, size = 2388, normalized size = 32.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**2/2 + x*(g + 2*h) + (d - e + f - g + h)*log(x + (26*d**3 + 66*d**2*e +
 132*d**2*f + 174*d**2*g + 348*d**2*h - 9*d**2*(d - e + f - g + h) + 78*d*e
**2 + 276*d*e*f + 444*d*e*g + 852*d*e*h - 12*d*e*(d - e + f - g + h) + 222*
d*f**2 + 636*d*f*g + 1164*d*f*h + 6*d*f*(d - e + f - g + h) + 510*d*g**2 +
1788*d*g*h + 36*d*g*(d - e + f - g + h) + 1518*d*h**2 + 102*d*h*(d - e + f
- g + h) - 7*d*(d - e + f - g + h)**2 + 46*e**3 + 204*e**2*f + 390*e**2*g +
 708*e**2*h + 3*e**2*(d - e + f - g + h) + 282*e*f**2 + 984*e*f*g + 1716*e*
f*h + 36*e*f*(d - e + f - g + h) + 930*e*g**2 + 3144*e*g*h + 102*e*g*(d - e
 + f - g + h) + 2586*e*h**2 + 228*e*h*(d - e + f - g + h) - 8*e*(d - e + f
- g + h)**2 + 116*f**3 + 534*f**2*g + 852*f**2*h + 51*f**2*(d - e + f - g +
```

```
 h) + 924*f*g**2 + 2796*f*g*h + 228*f*g*(d - e + f - g + h) + 1932*f*h**2 +
 486*f*h*(d - e + f - g + h) - 13*f*(d - e + f - g + h)**2 + 586*g**3 + 258
0*g**2*h + 243*g**2*(d - e + f - g + h) + 3414*g*h**2 + 996*g*h*(d - e + f
- g + h) - 20*g*(d - e + f - g + h)**2 + 1196*h**3 + 1011*h**2*(d - e + f -
 g + h) - 37*h*(d - e + f - g + h)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f +
213*d**2*g + 390*d**2*h + 102*d*e**2 + 318*d*e*f + 564*d*e*g + 1038*d*e*h +
 246*d*f**2 + 894*d*f*g + 1644*d*f*h + 750*d*g**2 + 2766*d*g*h + 2550*d*h**
2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 462*e**2*h + 285*e*f**2 + 852*e*f*g
 + 1578*e*f*h + 537*e*g**2 + 2004*e*g*h + 1869*e*h**2 + 154*f**3 + 717*f**2
*g + 1326*f**2*h + 966*f*g**2 + 3594*f*g*h + 3342*f*h**2 + 323*g**3 + 1830*
g**2*h + 3453*g*h**2 + 2170*h**3))/6 - (d + e + f + g + h)*log(x + (26*d**3
 + 66*d**2*e + 132*d**2*f + 174*d**2*g + 348*d**2*h + 27*d**2*(d + e + f +
g + h) + 78*d*e**2 + 276*d*e*f + 444*d*e*g + 852*d*e*h + 36*d*e*(d + e + f
+ g + h) + 222*d*f**2 + 636*d*f*g + 1164*d*f*h - 18*d*f*(d + e + f + g + h)
 + 510*d*g**2 + 1788*d*g*h - 108*d*g*(d + e + f + g + h) + 1518*d*h**2 - 30
6*d*h*(d + e + f + g + h) - 63*d*(d + e + f + g + h)**2 + 46*e**3 + 204*e**
2*f + 390*e**2*g + 708*e**2*h - 9*e**2*(d + e + f + g + h) + 282*e*f**2 + 9
84*e*f*g + 1716*e*f*h - 108*e*f*(d + e + f + g + h) + 930*e*g**2 + 3144*e*g
*h - 306*e*g*(d + e + f + g + h) + 2586*e*h**2 - 684*e*h*(d + e + f + g + h
) - 72*e*(d + e + f + g + h)**2 + 116*f**3 + 534*f**2*g + 852*f**2*h - 153*
f**2*(d + e + f + g + h) + 924*f*g**2 + 2796*f*g*h - 684*f*g*(d + e + f + g
 + h) + 1932*f*h**2 - 1458*f*h*(d + e + f + g + h) - 117*f*(d + e + f + g +
 h)**2 + 586*g**3 + 2580*g**2*h - 729*g**2*(d + e + f + g + h) + 3414*g*h**
2 - 2988*g*h*(d + e + f + g + h) - 180*g*(d + e + f + g + h)**2 + 1196*h**3
 - 3033*h**2*(d + e + f + g + h) - 333*h*(d + e + f + g + h)**2)/(10*d**3 +
 69*d**2*e + 102*d**2*f + 213*d**2*g + 390*d**2*h + 102*d*e**2 + 318*d*e*f
+ 564*d*e*g + 1038*d*e*h + 246*d*f**2 + 894*d*f*g + 1644*d*f*h + 750*d*g**2
 + 2766*d*g*h + 2550*d*h**2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 462*e**2*
h + 285*e*f**2 + 852*e*f*g + 1578*e*f*h + 537*e*g**2 + 2004*e*g*h + 1869*e*
h**2 + 154*f**3 + 717*f**2*g + 1326*f**2*h + 966*f*g**2 + 3594*f*g*h + 3342
*f*h**2 + 323*g**3 + 1830*g**2*h + 3453*g*h**2 + 2170*h**3))/2 + (d + 2*e +
 4*f + 8*g + 16*h)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*g +
 348*d**2*h - 18*d**2*(d + 2*e + 4*f + 8*g + 16*h) + 78*d*e**2 + 276*d*e*f
+ 444*d*e*g + 852*d*e*h - 24*d*e*(d + 2*e + 4*f + 8*g + 16*h) + 222*d*f**2
+ 636*d*f*g + 1164*d*f*h + 12*d*f*(d + 2*e + 4*f + 8*g + 16*h) + 510*d*g**2
 + 1788*d*g*h + 72*d*g*(d + 2*e + 4*f + 8*g + 16*h) + 1518*d*h**2 + 204*d*h
*(d + 2*e + 4*f + 8*g + 16*h) - 28*d*(d + 2*e + 4*f + 8*g + 16*h)**2 + 46*e
**3 + 204*e**2*f + 390*e**2*g + 708*e**2*h + 6*e**2*(d + 2*e + 4*f + 8*g +
16*h) + 282*e*f**2 + 984*e*f*g + 1716*e*f*h + 72*e*f*(d + 2*e + 4*f + 8*g +
 16*h) + 930*e*g**2 + 3144*e*g*h + 204*e*g*(d + 2*e + 4*f + 8*g + 16*h) + 2
586*e*h**2 + 456*e*h*(d + 2*e + 4*f + 8*g + 16*h) - 32*e*(d + 2*e + 4*f + 8
*g + 16*h)**2 + 116*f**3 + 534*f**2*g + 852*f**2*h + 102*f**2*(d + 2*e + 4*
f + 8*g + 16*h) + 924*f*g**2 + 2796*f*g*h + 456*f*g*(d + 2*e + 4*f + 8*g +
16*h) + 1932*f*h**2 + 972*f*h*(d + 2*e + 4*f + 8*g + 16*h) - 52*f*(d + 2*e
+ 4*f + 8*g + 16*h)**2 + 586*g**3 + 2580*g**2*h + 486*g**2*(d + 2*e + 4*f +
```

```
 8*g + 16*h) + 3414*g*h**2 + 1992*g*h*(d + 2*e + 4*f + 8*g + 16*h) - 80*g*(
d + 2*e + 4*f + 8*g + 16*h)**2 + 1196*h**3 + 2022*h**2*(d + 2*e + 4*f + 8*g
 + 16*h) - 148*h*(d + 2*e + 4*f + 8*g + 16*h)**2)/(10*d**3 + 69*d**2*e + 10
2*d**2*f + 213*d**2*g + 390*d**2*h + 102*d*e**2 + 318*d*e*f + 564*d*e*g + 1
038*d*e*h + 246*d*f**2 + 894*d*f*g + 1644*d*f*h + 750*d*g**2 + 2766*d*g*h +
 2550*d*h**2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 462*e**2*h + 285*e*f**2
+ 852*e*f*g + 1578*e*f*h + 537*e*g**2 + 2004*e*g*h + 1869*e*h**2 + 154*f**3
 + 717*f**2*g + 1326*f**2*h + 966*f*g**2 + 3594*f*g*h + 3342*f*h**2 + 323*g
**3 + 1830*g**2*h + 3453*g*h**2 + 2170*h**3))/3
```

Giac [A] time = 1.07699, size = 92, normalized size = 1.24

$$\frac{1}{2}hx^2 + gx + 2hx + \frac{1}{6}\left(d + f - g + h - e\right)\log\left(|x+1|\right) - \frac{1}{2}\left(d + f + g + h + e\right)\log\left(|x-1|\right) + \frac{1}{3}\left(d + 4f + 8g + 16h + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] 1/2*h*x^2 + g*x + 2*h*x + 1/6*(d + f - g + h - e)*log(abs(x + 1)) - 1/2*(d
+ f + g + h + e)*log(abs(x - 1)) + 1/3*(d + 4*f + 8*g + 16*h + 2*e)*log(abs
(x - 2))
```

3.84 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+$$

```
[Out] (g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f + g + h + i
)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/3 + ((d
- e + f - g + h - i)*Log[1 + x])/6
```

Rubi [A] time = 0.136906, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {1586, 2074}

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),x
]
```

```
[Out] (g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f + g + h + i
)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/3 + ((d
- e + f - g + h - i)*Log[1 + x])/6
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+84x^5\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+84x^5}{2-x-2x^2+x^3}\,dx$$
$$= \int \left(420\left(1+\frac{1}{420}(g+2h)\right)+\frac{2688+d+2e+4f+8g+16h}{3(-2+x)}+\frac{-84$$
$$= (420+g+2h)x+\frac{1}{2}(168+h)x^2+28x^3-\frac{1}{2}(84+d+e+f+g+h)$$

Mathematica [A] time = 0.0492444, size = 91, normalized size = 0.95

$$\frac{1}{6}\left(-3\log(1-x)(d+e+f+g+h+i)+2\log(2-x)(d+2e+4(f+2g+4h+8i))+\log(x+1)(d-e+f-g+h-i)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (6*(g + 2*h + 5*i)*x + 3*(h + 2*i)*x^2 + 2*i*x^3 - 3*(d + e + f + g + h + i
)*Log[1 - x] + 2*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + (d - e +
f - g + h - i)*Log[1 + x])/6
```

Maple [A] time = 0.009, size = 156, normalized size = 1.6

$$\frac{ix^3}{3}+\frac{hx^2}{2}+ix^2+gx+2hx+5ix+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(1+x)h}{6}-\frac{\ln(1+x)i}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x)
```

```
[Out] 1/3*i*x^3+1/2*h*x^2+i*x^2+g*x+2*h*x+5*i*x+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*l
n(1+x)*f-1/6*ln(1+x)*g+1/6*ln(1+x)*h-1/6*ln(1+x)*i+1/3*ln(x-2)*d+2/3*ln(x-2
)*e+4/3*ln(x-2)*f+8/3*ln(x-2)*g+16/3*ln(x-2)*h+32/3*ln(x-2)*i-1/2*ln(x-1)*d
-1/2*ln(x-1)*e-1/2*ln(x-1)*f-1/2*ln(x-1)*g-1/2*ln(x-1)*h-1/2*ln(x-1)*i
```

Maxima [A] time = 0.978034, size = 111, normalized size = 1.16

$$\frac{1}{3}ix^3+\frac{1}{2}(h+2i)x^2+\left(g+2h+5i\right)x+\frac{1}{6}\left(d-e+f-g+h-i\right)\log(x+1)-\frac{1}{2}\left(d+e+f+g+h+i\right)\log(x-1)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"maxima")
```

```
[Out] 1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e + f - g + h
- i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1) + 1/3*(d + 2*e + 4
*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Fricas [A] time = 2.15066, size = 251, normalized size = 2.61

$$\frac{1}{3}ix^3 + \frac{1}{2}(h+2i)x^2 + \left(g+2h+5i\right)x + \frac{1}{6}\left(d-e+f-g+h-i\right)\log(x+1) - \frac{1}{2}\left(d+e+f+g+h+i\right)\log(x-1) + \frac{1}{3}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"fricas")
```

```
[Out] 1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e + f - g + h
- i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1) + 1/3*(d + 2*e + 4
*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09888, size = 122, normalized size = 1.27

$$\frac{1}{3}ix^3 + \frac{1}{2}hx^2 + ix^2 + gx + 2hx + 5ix + \frac{1}{6}\left(d+f-g+h-i-e\right)\log(|x+1|) - \frac{1}{2}\left(d+f+g+h+i+e\right)\log(|x-1|) + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"giac")
```

```
[Out] 1/3*i*x^3 + 1/2*h*x^2 + i*x^2 + g*x + 2*h*x + 5*i*x + 1/6*(d + f - g + h -
i - e)*log(abs(x + 1)) - 1/2*(d + f + g + h + i + e)*log(abs(x - 1)) + 1/3*
(d + 4*f + 8*g + 16*h + 32*i + 2*e)*log(abs(x - 2))
```

3.85 $\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=46

$$\frac{1}{12(x+2)} - \frac{1}{18}\log(1-x) + \frac{1}{48}\log(2-x) + \frac{1}{6}\log(x+1) - \frac{19}{144}\log(x+2)$$

```
[Out] 1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (19*Log[2 +
 x])/144
```

Rubi [A] time = 0.0508525, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1586, 2074}

$$\frac{1}{12(x+2)} - \frac{1}{18}\log(1-x) + \frac{1}{48}\log(2-x) + \frac{1}{6}\log(x+1) - \frac{19}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (19*Log[2 +
 x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{2-x-2x^2+x^3}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{1}{(2+x)^2\left(2-x-2x^2+x^3\right)}\,dx$$
$$= \int \left(\frac{1}{48(-2+x)} - \frac{1}{18(-1+x)} + \frac{1}{6(1+x)} - \frac{1}{12(2+x)^2} - \frac{19}{144(2+x)}\right) dx$$
$$= \frac{1}{12(2+x)} - \frac{1}{18}\log(1-x) + \frac{1}{48}\log(2-x) + \frac{1}{6}\log(1+x) - \frac{19}{144}\log(2+x)$$

Mathematica [A] time = 0.0219459, size = 42, normalized size = 0.91

$$\frac{1}{144}\left(\frac{12}{x+2} + 24\log(-x-1) - 8\log(1-x) + 3\log(2-x) - 19\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (12/(2 + x) + 24*Log[-1 - x] - 8*Log[1 - x] + 3*Log[2 - x] - 19*Log[2 + x])
/144
```

Maple [A] time = 0.009, size = 33, normalized size = 0.7

$$\frac{1}{24+12\,x} - \frac{19\,\ln(2+x)}{144} + \frac{\ln(1+x)}{6} + \frac{\ln(x-2)}{48} - \frac{\ln(x-1)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/12/(2+x)-19/144*ln(2+x)+1/6*ln(1+x)+1/48*ln(x-2)-1/18*ln(x-1)
```

Maxima [A] time = 0.96261, size = 43, normalized size = 0.93

$$\frac{1}{12\,(x+2)} - \frac{19}{144}\,\log(x+2) + \frac{1}{6}\,\log(x+1) - \frac{1}{18}\,\log(x-1) + \frac{1}{48}\,\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/12/(x + 2) - 19/144*log(x + 2) + 1/6*log(x + 1) - 1/18*log(x - 1) + 1/48*
log(x - 2)
```

Fricas [A] time = 1.51187, size = 155, normalized size = 3.37

$$-\frac{19\,(x+2)\log\left(x+2\right)-24\,(x+2)\log\left(x+1\right)+8\,(x+2)\log\left(x-1\right)-3\,(x+2)\log\left(x-2\right)-12}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(19*(x + 2)*log(x + 2) - 24*(x + 2)*log(x + 1) + 8*(x + 2)*log(x - 1
) - 3*(x + 2)*log(x - 2) - 12)/(x + 2)
```

Sympy [A] time = 0.235098, size = 34, normalized size = 0.74

$$\frac{\log\left(x-2\right)}{48}-\frac{\log\left(x-1\right)}{18}+\frac{\log\left(x+1\right)}{6}-\frac{19\log\left(x+2\right)}{144}+\frac{1}{12x+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] log(x - 2)/48 - log(x - 1)/18 + log(x + 1)/6 - 19*log(x + 2)/144 + 1/(12*x
+ 24)
```

Giac [A] time = 1.0805, size = 49, normalized size = 1.07

$$\frac{1}{12\,(x+2)}-\frac{19}{144}\,\log\left(|x+2|\right)+\frac{1}{6}\,\log\left(|x+1|\right)-\frac{1}{18}\,\log\left(|x-1|\right)+\frac{1}{48}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/12/(x + 2) - 19/144*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/18*log(abs(
x - 1)) + 1/48*log(abs(x - 2))
```

3.86 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=71

$$\frac{d-2e}{12(x+2)} - \frac{1}{18}(d+e)\log(1-x) + \frac{1}{48}(d+2e)\log(2-x) + \frac{1}{6}(d-e)\log(x+1) - \frac{1}{144}(19d-26e)\log(x+2)$$

```
[Out] (d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log[2 - x])/4
8 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])/144
```

Rubi [A] time = 0.174384, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 6742}

$$\frac{d-2e}{12(x+2)} - \frac{1}{18}(d+e)\log(1-x) + \frac{1}{48}(d+2e)\log(2-x) + \frac{1}{6}(d-e)\log(x+1) - \frac{1}{144}(19d-26e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log[2 - x])/4
8 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex}{(2+x)^2\left(2-x-2x^2+x^3\right)}\,dx$$
$$= \int \left(\frac{d+2e}{48(-2+x)} + \frac{-d-e}{18(-1+x)} + \frac{d-e}{6(1+x)} + \frac{-d+2e}{12(2+x)^2} + \frac{-19d+26e}{144(2+x)}\right) dx$$
$$= \frac{d-2e}{12(2+x)} - \frac{1}{18}(d+e)\log(1-x) + \frac{1}{48}(d+2e)\log(2-x) + \frac{1}{6}(d-e)\log(1+x) - \frac{1}{14}$$

Mathematica [A] time = 0.0390137, size = 66, normalized size = 0.93

$$\frac{1}{144}\left(\frac{12(d-2e)}{x+2} + 24(d-e)\log(-x-1) - 8(d+e)\log(1-x) + 3(d+2e)\log(2-x) + (26e-19d)\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e))/(2 + x) + 24*(d - e)*Log[-1 - x] - 8*(d + e)*Log[1 - x] + 3
*(d + 2*e)*Log[2 - x] + (-19*d + 26*e)*Log[2 + x])/144
```

Maple [A] time = 0.01, size = 74, normalized size = 1.

$$-\frac{19\ln(2+x)d}{144} + \frac{13\ln(2+x)e}{72} + \frac{d}{24+12x} - \frac{e}{12+6x} + \frac{\ln(1+x)d}{6} - \frac{\ln(1+x)e}{6} + \frac{\ln(x-2)d}{48} + \frac{\ln(x-2)e}{24} - \frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -19/144*ln(2+x)*d+13/72*ln(2+x)*e+1/12/(2+x)*d-1/6/(2+x)*e+1/6*ln(1+x)*d-1/
6*ln(1+x)*e+1/48*ln(x-2)*d+1/24*ln(x-2)*e-1/18*ln(x-1)*d-1/18*ln(x-1)*e
```

Maxima [A] time = 0.986229, size = 77, normalized size = 1.08

$$-\frac{1}{144}(19d-26e)\log(x+2) + \frac{1}{6}(d-e)\log(x+1) - \frac{1}{18}(d+e)\log(x-1) + \frac{1}{48}(d+2e)\log(x-2) + \frac{d-2e}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/144*(19*d - 26*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/18*(d + e)*log
(x - 1) + 1/48*(d + 2*e)*log(x - 2) + 1/12*(d - 2*e)/(x + 2)
```

Fricas [A] time = 1.6831, size = 263, normalized size = 3.7

$$-\frac{((19\,d-26\,e)x+38\,d-52\,e)\log(x+2)-24\,((d-e)x+2\,d-2\,e)\log(x+1)+8\,((d+e)x+2\,d+2\,e)\log(x-1)-3\,((d}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(((19*d - 26*e)*x + 38*d - 52*e)*log(x + 2) - 24*((d - e)*x + 2*d -
2*e)*log(x + 1) + 8*((d + e)*x + 2*d + 2*e)*log(x - 1) - 3*((d + 2*e)*x + 2
*d + 4*e)*log(x - 2) - 12*d + 24*e)/(x + 2)
```

Sympy [B] time = 5.55325, size = 1188, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e)/(12*x + 24) + (d - e)*log(x + (-1534775*d**6 + 8032360*d**5*e - 9
84027*d**5*(d - e) - 12991180*d**4*e**2 + 11797266*d**4*e*(d - e) + 3567168
*d**4*(d - e)**2 + 1075200*d**3*e**3 - 32721528*d**3*e**2*(d - e) - 8725248
*d**3*e*(d - e)**2 - 247104*d**3*(d - e)**3 + 16959280*d**2*e**4 + 38977296
*d**2*e**3*(d - e) - 2820096*d**2*e**2*(d - e)**2 - 10357632*d**2*e*(d - e)
**3 - 15836800*d*e**5 - 21294960*d*e**4*(d - e) + 15436800*d*e**3*(d - e)**
2 + 16277760*d*e**2*(d - e)**3 + 4283840*e**6 + 3876000*e**5*(d - e) - 6865
920*e**4*(d - e)**2 - 4078080*e**3*(d - e)**3)/(801262*d**6 - 4662251*d**5*
e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990800*d*
e**5 - 2380000*e**6))/6 - (d + e)*log(x + (-1534775*d**6 + 8032360*d**5*e +
 328009*d**5*(d + e) - 12991180*d**4*e**2 - 3932422*d**4*e*(d + e) + 396352
```

```
*d**4*(d + e)**2 + 1075200*d**3*e**3 + 10907176*d**3*e**2*(d + e) - 969472*
d**3*e*(d + e)**2 + 9152*d**3*(d + e)**3 + 16959280*d**2*e**4 - 12992432*d*
*2*e**3*(d + e) - 313344*d**2*e**2*(d + e)**2 + 383616*d**2*e*(d + e)**3 -
15836800*d*e**5 + 7098320*d*e**4*(d + e) + 1715200*d*e**3*(d + e)**2 - 6028
80*d*e**2*(d + e)**3 + 4283840*e**6 - 1292000*e**5*(d + e) - 762880*e**4*(d
 + e)**2 + 151040*e**3*(d + e)**3)/(801262*d**6 - 4662251*d**5*e + 7296938*
d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990800*d*e**5 - 23800
00*e**6))/18 + (d + 2*e)*log(x + (-1534775*d**6 + 8032360*d**5*e - 984027*d
**5*(d + 2*e)/8 - 12991180*d**4*e**2 + 5898633*d**4*e*(d + 2*e)/4 + 55737*d
**4*(d + 2*e)**2 + 1075200*d**3*e**3 - 4090191*d**3*e**2*(d + 2*e) - 136332
*d**3*e*(d + 2*e)**2 - 3861*d**3*(d + 2*e)**3/8 + 16959280*d**2*e**4 + 4872
162*d**2*e**3*(d + 2*e) - 44064*d**2*e**2*(d + 2*e)**2 - 80919*d**2*e*(d +
2*e)**3/4 - 15836800*d*e**5 - 2661870*d*e**4*(d + 2*e) + 241200*d*e**3*(d +
 2*e)**2 + 63585*d*e**2*(d + 2*e)**3/2 + 4283840*e**6 + 484500*e**5*(d + 2*
e) - 107280*e**4*(d + 2*e)**2 - 7965*e**3*(d + 2*e)**3)/(801262*d**6 - 4662
251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9
990800*d*e**5 - 2380000*e**6))/48 - (19*d - 26*e)*log(x + (-1534775*d**6 +
8032360*d**5*e + 328009*d**5*(19*d - 26*e)/8 - 12991180*d**4*e**2 - 1966211
*d**4*e*(19*d - 26*e)/4 + 6193*d**4*(19*d - 26*e)**2 + 1075200*d**3*e**3 +
1363397*d**3*e**2*(19*d - 26*e) - 15148*d**3*e*(19*d - 26*e)**2 + 143*d**3*
(19*d - 26*e)**3/8 + 16959280*d**2*e**4 - 1624054*d**2*e**3*(19*d - 26*e) -
 4896*d**2*e**2*(19*d - 26*e)**2 + 2997*d**2*e*(19*d - 26*e)**3/4 - 1583680
0*d*e**5 + 887290*d*e**4*(19*d - 26*e) + 26800*d*e**3*(19*d - 26*e)**2 - 23
55*d*e**2*(19*d - 26*e)**3/2 + 4283840*e**6 - 161500*e**5*(19*d - 26*e) - 1
1920*e**4*(19*d - 26*e)**2 + 295*e**3*(19*d - 26*e)**3)/(801262*d**6 - 4662
251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9
990800*d*e**5 - 2380000*e**6))/144
```

Giac [A] time = 1.09707, size = 89, normalized size = 1.25

$$-\frac{1}{144}\,(19\,d-26\,e)\log\left(|x+2|\right)+\frac{1}{6}\,(d-e)\log\left(|x+1|\right)-\frac{1}{18}\,(d+e)\log\left(|x-1|\right)+\frac{1}{48}\,(d+2\,e)\log\left(|x-2|\right)+\frac{d-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/144*(19*d - 26*e)*log(abs(x + 2)) + 1/6*(d - e)*log(abs(x + 1)) - 1/18*(
d + e)*log(abs(x - 1)) + 1/48*(d + 2*e)*log(abs(x - 2)) + 1/12*(d - 2*e)/(x
 + 2)
```

3.87 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=82

$$\frac{d-2e+4f}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f)+\frac{1}{48}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)-\frac{1}{144}\log(x+2)(19d-26$$

```
[Out] (d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d + 2*e + 4*
f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d - 26*e + 28*f)*Log[
2 + x])/144
```

Rubi [A] time = 0.198761, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1586, 6742}

$$\frac{d-2e+4f}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f)+\frac{1}{48}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)-\frac{1}{144}\log(x+2)(19d-26$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```



```
[Out] (d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d + 2*e + 4*
f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d - 26*e + 28*f)*Log[
2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2}{(2+x)^2(2-x-2x^2+x^3)}\,dx$$
$$= \int \left(\frac{d+2e+4f}{48(-2+x)} + \frac{-d-e-f}{18(-1+x)} + \frac{d-e+f}{6(1+x)} + \frac{-d+2e-4f}{12(2+x)^2} + \frac{-19d+26e-2}{144(2+x)}\right.$$
$$= \frac{d-2e+4f}{12(2+x)} - \frac{1}{18}(d+e+f)\log(1-x) + \frac{1}{48}(d+2e+4f)\log(2-x) + \frac{1}{6}(d-$$

Mathematica [A] time = 0.0521707, size = 77, normalized size = 0.94

$$\frac{1}{144}\left(\frac{12(d-2e+4f)}{x+2} + 24\log(-x-1)(d-e+f) - 8\log(1-x)(d+e+f) + 3\log(2-x)(d+2e+4f) + \log(x+2)(-1\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f))/(2 + x) + 24*(d - e + f)*Log[-1 - x] - 8*(d + e + f)*
Log[1 - x] + 3*(d + 2*e + 4*f)*Log[2 - x] + (-19*d + 26*e - 28*f)*Log[2 + x
])/144
```

Maple [A] time = 0.01, size = 110, normalized size = 1.3

$$\frac{13\ln(2+x)e}{72} - \frac{7\ln(2+x)f}{36} - \frac{19\ln(2+x)d}{144} + \frac{d}{24+12x} - \frac{e}{12+6x} + \frac{f}{6+3x} + \frac{\ln(1+x)d}{6} - \frac{\ln(1+x)e}{6} + \frac{\ln(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 13/72*ln(2+x)*e-7/36*ln(2+x)*f-19/144*ln(2+x)*d+1/12/(2+x)*d-1/6/(2+x)*e+1/
3/(2+x)*f+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x)*f+1/48*ln(x-2)*d+1/24*ln(
x-2)*e+1/12*ln(x-2)*f-1/18*ln(x-1)*d-1/18*ln(x-1)*e-1/18*ln(x-1)*f
```

Maxima [A] time = 0.983578, size = 92, normalized size = 1.12

$$-\frac{1}{144}(19d-26e+28f)\log(x+2) + \frac{1}{6}(d-e+f)\log(x+1) - \frac{1}{18}(d+e+f)\log(x-1) + \frac{1}{48}(d+2e+4f)\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxim
a")
```

```
[Out] -1/144*(19*d - 26*e + 28*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/18*
(d + e + f)*log(x - 1) + 1/48*(d + 2*e + 4*f)*log(x - 2) + 1/12*(d - 2*e +
4*f)/(x + 2)
```

Fricas [A] time = 2.18009, size = 335, normalized size = 4.09

$$-\frac{\left(\left(19\,d-26\,e+28\,f\right)x+38\,d-52\,e+56\,f\right)\log\left(x+2\right)-24\left(\left(d-e+f\right)x+2\,d-2\,e+2\,f\right)\log\left(x+1\right)+8\left(\left(d+e+f\right)x}{144\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="frica
s")
```

```
[Out] -1/144*(((19*d - 26*e + 28*f)*x + 38*d - 52*e + 56*f)*log(x + 2) - 24*((d -
 e + f)*x + 2*d - 2*e + 2*f)*log(x + 1) + 8*((d + e + f)*x + 2*d + 2*e + 2*
f)*log(x - 1) - 3*((d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) - 12*d +
 24*e - 48*f)/(x + 2)
```

Sympy [B] time = 105.348, size = 4767, normalized size = 58.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - e + f)*log(x + (-1534775*d**6 + 8032360*d**5*e - 12464043*d**5*f - 984
027*d**5*(d - e + f) - 12991180*d**4*e**2 + 53064770*d**4*e*f + 11797266*d*
*4*e*(d - e + f) - 41194260*d**4*f**2 - 7912356*d**4*f*(d - e + f) + 356716
8*d**4*(d - e + f)**2 + 1075200*d**3*e**3 - 67598200*d**3*e**2*f - 32721528
*d**3*e**2*(d - e + f) + 138738400*d**3*e*f**2 + 59411424*d**3*e*f*(d - e +
 f) - 8725248*d**3*e*(d - e + f)**2 - 70926880*d**3*f**3 - 23846304*d**3*f*
*2*(d - e + f) + 19430208*d**3*f*(d - e + f)**2 - 247104*d**3*(d - e + f)**
```

```
3 + 16959280*d**2*e**4 + 8077200*d**2*e**3*f + 38977296*d**2*e**3*(d - e +
f) - 132649440*d**2*e**2*f**2 - 117189216*d**2*e**2*f*(d - e + f) - 2820096
*d**2*e**2*(d - e + f)**2 + 178427200*d**2*e*f**3 + 114278976*d**2*e*f**2*(
d - e + f) - 42339456*d**2*e*f*(d - e + f)**2 - 10357632*d**2*e*(d - e + f)
**3 - 67113600*d**2*f**4 - 34460544*d**2*f**3*(d - e + f) + 38893824*d**2*f
**2*(d - e + f)**2 + 2135808*d**2*f*(d - e + f)**3 - 15836800*d*e**5 + 3627
5600*d*e**4*f - 21294960*d*e**4*(d - e + f) + 17097600*d*e**3*f**2 + 893879
04*d*e**3*f*(d - e + f) + 15436800*d*e**3*(d - e + f)**2 - 114789760*d*e**2
*f**3 - 143630208*d*e**2*f**2*(d - e + f) + 6780672*d*e**2*f*(d - e + f)**2
 + 16277760*d*e**2*(d - e + f)**3 + 112386560*d*e*f**4 + 98370048*d*e*f**3*
(d - e + f) - 63728640*d*e*f**2*(d - e + f)**2 - 22643712*d*e*f*(d - e + f)
**3 - 33092352*d*f**5 - 24472320*d*f**4*(d - e + f) + 33905664*d*f**3*(d -
e + f)**2 + 8045568*d*f**2*(d - e + f)**3 + 4283840*e**6 - 17164384*e**5*f
+ 3876000*e**5*(d - e + f) + 18093760*e**4*f**2 - 22632000*e**4*f*(d - e +
f) - 6865920*e**4*(d - e + f)**2 + 10387200*e**3*f**3 + 52267776*e**3*f**2*
(d - e + f) + 13957632*e**3*f*(d - e + f)**2 - 4078080*e**3*(d - e + f)**3
- 36528640*e**2*f**4 - 59284992*e**2*f**3*(d - e + f) + 12026880*e**2*f**2*
(d - e + f)**2 + 13851648*e**2*f*(d - e + f)**3 + 27625984*e*f**5 + 3203328
0*e*f**4*(d - e + f) - 30394368*e*f**3*(d - e + f)**2 - 14432256*e*f**2*(d
- e + f)**3 - 6640640*f**6 - 6988800*f**5*(d - e + f) + 10874880*f**4*(d -
e + f)**2 + 6082560*f**3*(d - e + f)**3)/(801262*d**6 - 4662251*d**5*e + 65
98614*d**5*f + 7296938*d**4*e**2 - 32240296*d**4*e*f + 22080168*d**4*f**2 +
 1388616*d**3*e**3 + 44207696*d**3*e**2*f - 87152288*d**3*e*f**2 + 38269376
*d**3*f**3 - 12447440*d**2*e**4 - 4393344*d**2*e**3*f + 97214592*d**2*e**2*
f**2 - 114767360*d**2*e*f**3 + 36053760*d**2*f**4 + 9990800*d*e**5 - 252788
80*d*e**4*f - 19059072*d*e**3*f**2 + 91652864*d*e**2*f**3 - 73388800*d*e*f*
*4 + 17395200*d*f**5 - 2380000*e**6 + 10947200*e**5*f - 11338880*e**4*f**2
- 13562880*e**3*f**3 + 31239680*e**2*f**4 - 18176000*e*f**5 + 3328000*f**6)
)/6 - (d + e + f)*log(x + (-1534775*d**6 + 8032360*d**5*e - 12464043*d**5*f
 + 328009*d**5*(d + e + f) - 12991180*d**4*e**2 + 53064770*d**4*e*f - 39324
22*d**4*e*(d + e + f) - 41194260*d**4*f**2 + 2637452*d**4*f*(d + e + f) + 3
96352*d**4*(d + e + f)**2 + 1075200*d**3*e**3 - 67598200*d**3*e**2*f + 1090
7176*d**3*e**2*(d + e + f) + 138738400*d**3*e*f**2 - 19803808*d**3*e*f*(d +
 e + f) - 969472*d**3*e*(d + e + f)**2 - 70926880*d**3*f**3 + 7948768*d**3*
f**2*(d + e + f) + 2158912*d**3*f*(d + e + f)**2 + 9152*d**3*(d + e + f)**3
 + 16959280*d**2*e**4 + 8077200*d**2*e**3*f - 12992432*d**2*e**3*(d + e + f
) - 132649440*d**2*e**2*f**2 + 39063072*d**2*e**2*f*(d + e + f) - 313344*d*
*2*e**2*(d + e + f)**2 + 178427200*d**2*e*f**3 - 38092992*d**2*e*f**2*(d +
e + f) - 4704384*d**2*e*f*(d + e + f)**2 + 383616*d**2*e*(d + e + f)**3 - 6
7113600*d**2*f**4 + 11486848*d**2*f**3*(d + e + f) + 4321536*d**2*f**2*(d +
 e + f)**2 - 79104*d**2*f*(d + e + f)**3 - 15836800*d*e**5 + 36275600*d*e**
4*f + 7098320*d*e**4*(d + e + f) + 17097600*d*e**3*f**2 - 29795968*d*e**3*f
*(d + e + f) + 1715200*d*e**3*(d + e + f)**2 - 114789760*d*e**2*f**3 + 4787
6736*d*e**2*f**2*(d + e + f) + 753408*d*e**2*f*(d + e + f)**2 - 602880*d*e*
*2*(d + e + f)**3 + 112386560*d*e*f**4 - 32790016*d*e*f**3*(d + e + f) - 70
```

```
80960*d*e*f**2*(d + e + f)**2 + 838656*d*e*f*(d + e + f)**3 - 33092352*d*f*
*5 + 8157440*d*f**4*(d + e + f) + 3767296*d*f**3*(d + e + f)**2 - 297984*d*
f**2*(d + e + f)**3 + 4283840*e**6 - 17164384*e**5*f - 1292000*e**5*(d + e
+ f) + 18093760*e**4*f**2 + 7544000*e**4*f*(d + e + f) - 762880*e**4*(d + e
 + f)**2 + 10387200*e**3*f**3 - 17422592*e**3*f**2*(d + e + f) + 1550848*e*
*3*f*(d + e + f)**2 + 151040*e**3*(d + e + f)**3 - 36528640*e**2*f**4 + 197
61664*e**2*f**3*(d + e + f) + 1336320*e**2*f**2*(d + e + f)**2 - 513024*e**
2*f*(d + e + f)**3 + 27625984*e*f**5 - 10677760*e*f**4*(d + e + f) - 337715
2*e*f**3*(d + e + f)**2 + 534528*e*f**2*(d + e + f)**3 - 6640640*f**6 + 232
9600*f**5*(d + e + f) + 1208320*f**4*(d + e + f)**2 - 225280*f**3*(d + e +
f)**3)/(801262*d**6 - 4662251*d**5*e + 6598614*d**5*f + 7296938*d**4*e**2 -
 32240296*d**4*e*f + 22080168*d**4*f**2 + 1388616*d**3*e**3 + 44207696*d**3
*e**2*f - 87152288*d**3*e*f**2 + 38269376*d**3*f**3 - 12447440*d**2*e**4 -
4393344*d**2*e**3*f + 97214592*d**2*e**2*f**2 - 114767360*d**2*e*f**3 + 360
53760*d**2*f**4 + 9990800*d*e**5 - 25278880*d*e**4*f - 19059072*d*e**3*f**2
 + 91652864*d*e**2*f**3 - 73388800*d*e*f**4 + 17395200*d*f**5 - 2380000*e**
6 + 10947200*e**5*f - 11338880*e**4*f**2 - 13562880*e**3*f**3 + 31239680*e*
*2*f**4 - 18176000*e*f**5 + 3328000*f**6))/18 + (d + 2*e + 4*f)*log(x + (-1
534775*d**6 + 8032360*d**5*e - 12464043*d**5*f - 984027*d**5*(d + 2*e + 4*f
)/8 - 12991180*d**4*e**2 + 53064770*d**4*e*f + 5898633*d**4*e*(d + 2*e + 4*
f)/4 - 41194260*d**4*f**2 - 1978089*d**4*f*(d + 2*e + 4*f)/2 + 55737*d**4*(
d + 2*e + 4*f)**2 + 1075200*d**3*e**3 - 67598200*d**3*e**2*f - 4090191*d**3
*e**2*(d + 2*e + 4*f) + 138738400*d**3*e*f**2 + 7426428*d**3*e*f*(d + 2*e +
 4*f) - 136332*d**3*e*(d + 2*e + 4*f)**2 - 70926880*d**3*f**3 - 2980788*d**
3*f**2*(d + 2*e + 4*f) + 303597*d**3*f*(d + 2*e + 4*f)**2 - 3861*d**3*(d +
2*e + 4*f)**3/8 + 16959280*d**2*e**4 + 8077200*d**2*e**3*f + 4872162*d**2*e
**3*(d + 2*e + 4*f) - 132649440*d**2*e**2*f**2 - 14648652*d**2*e**2*f*(d +
2*e + 4*f) - 44064*d**2*e**2*(d + 2*e + 4*f)**2 + 178427200*d**2*e*f**3 + 1
4284872*d**2*e*f**2*(d + 2*e + 4*f) - 661554*d**2*e*f*(d + 2*e + 4*f)**2 -
80919*d**2*e*(d + 2*e + 4*f)**3/4 - 67113600*d**2*f**4 - 4307568*d**2*f**3*
(d + 2*e + 4*f) + 607716*d**2*f**2*(d + 2*e + 4*f)**2 + 8343*d**2*f*(d + 2*
e + 4*f)**3/2 - 15836800*d*e**5 + 36275600*d*e**4*f - 2661870*d*e**4*(d + 2
*e + 4*f) + 17097600*d*e**3*f**2 + 11173488*d*e**3*f*(d + 2*e + 4*f) + 2412
00*d*e**3*(d + 2*e + 4*f)**2 - 114789760*d*e**2*f**3 - 17953776*d*e**2*f**2
*(d + 2*e + 4*f) + 105948*d*e**2*f*(d + 2*e + 4*f)**2 + 63585*d*e**2*(d + 2
*e + 4*f)**3/2 + 112386560*d*e*f**4 + 12296256*d*e*f**3*(d + 2*e + 4*f) - 9
95760*d*e*f**2*(d + 2*e + 4*f)**2 - 44226*d*e*f*(d + 2*e + 4*f)**3 - 330923
52*d*f**5 - 3059040*d*f**4*(d + 2*e + 4*f) + 529776*d*f**3*(d + 2*e + 4*f)*
*2 + 15714*d*f**2*(d + 2*e + 4*f)**3 + 4283840*e**6 - 17164384*e**5*f + 484
500*e**5*(d + 2*e + 4*f) + 18093760*e**4*f**2 - 2829000*e**4*f*(d + 2*e + 4
*f) - 107280*e**4*(d + 2*e + 4*f)**2 + 10387200*e**3*f**3 + 6533472*e**3*f*
*2*(d + 2*e + 4*f) + 218088*e**3*f*(d + 2*e + 4*f)**2 - 7965*e**3*(d + 2*e
+ 4*f)**3 - 36528640*e**2*f**4 - 7410624*e**2*f**3*(d + 2*e + 4*f) + 187920
*e**2*f**2*(d + 2*e + 4*f)**2 + 27054*e**2*f*(d + 2*e + 4*f)**3 + 27625984*
e*f**5 + 4004160*e*f**4*(d + 2*e + 4*f) - 474912*e*f**3*(d + 2*e + 4*f)**2
```

```
- 28188*e*f**2*(d + 2*e + 4*f)**3 - 6640640*f**6 - 873600*f**5*(d + 2*e + 4
*f) + 169920*f**4*(d + 2*e + 4*f)**2 + 11880*f**3*(d + 2*e + 4*f)**3)/(8012
62*d**6 - 4662251*d**5*e + 6598614*d**5*f + 7296938*d**4*e**2 - 32240296*d*
*4*e*f + 22080168*d**4*f**2 + 1388616*d**3*e**3 + 44207696*d**3*e**2*f - 87
152288*d**3*e*f**2 + 38269376*d**3*f**3 - 12447440*d**2*e**4 - 4393344*d**2
*e**3*f + 97214592*d**2*e**2*f**2 - 114767360*d**2*e*f**3 + 36053760*d**2*f
**4 + 9990800*d*e**5 - 25278880*d*e**4*f - 19059072*d*e**3*f**2 + 91652864*
d*e**2*f**3 - 73388800*d*e*f**4 + 17395200*d*f**5 - 2380000*e**6 + 10947200
*e**5*f - 11338880*e**4*f**2 - 13562880*e**3*f**3 + 31239680*e**2*f**4 - 18
176000*e*f**5 + 3328000*f**6))/48 - (19*d - 26*e + 28*f)*log(x + (-1534775*
d**6 + 8032360*d**5*e - 12464043*d**5*f + 328009*d**5*(19*d - 26*e + 28*f)/
8 - 12991180*d**4*e**2 + 53064770*d**4*e*f - 1966211*d**4*e*(19*d - 26*e +
28*f)/4 - 41194260*d**4*f**2 + 659363*d**4*f*(19*d - 26*e + 28*f)/2 + 6193*
d**4*(19*d - 26*e + 28*f)**2 + 1075200*d**3*e**3 - 67598200*d**3*e**2*f + 1
363397*d**3*e**2*(19*d - 26*e + 28*f) + 138738400*d**3*e*f**2 - 2475476*d**
3*e*f*(19*d - 26*e + 28*f) - 15148*d**3*e*(19*d - 26*e + 28*f)**2 - 7092688
0*d**3*f**3 + 993596*d**3*f**2*(19*d - 26*e + 28*f) + 33733*d**3*f*(19*d -
26*e + 28*f)**2 + 143*d**3*(19*d - 26*e + 28*f)**3/8 + 16959280*d**2*e**4 +
 8077200*d**2*e**3*f - 1624054*d**2*e**3*(19*d - 26*e + 28*f) - 132649440*d
**2*e**2*f**2 + 4882884*d**2*e**2*f*(19*d - 26*e + 28*f) - 4896*d**2*e**2*(
19*d - 26*e + 28*f)**2 + 178427200*d**2*e*f**3 - 4761624*d**2*e*f**2*(19*d
- 26*e + 28*f) - 73506*d**2*e*f*(19*d - 26*e + 28*f)**2 + 2997*d**2*e*(19*d
 - 26*e + 28*f)**3/4 - 67113600*d**2*f**4 + 1435856*d**2*f**3*(19*d - 26*e
+ 28*f) + 67524*d**2*f**2*(19*d - 26*e + 28*f)**2 - 309*d**2*f*(19*d - 26*e
 + 28*f)**3/2 - 15836800*d*e**5 + 36275600*d*e**4*f + 887290*d*e**4*(19*d -
 26*e + 28*f) + 17097600*d*e**3*f**2 - 3724496*d*e**3*f*(19*d - 26*e + 28*f
) + 26800*d*e**3*(19*d - 26*e + 28*f)**2 - 114789760*d*e**2*f**3 + 5984592*
d*e**2*f**2*(19*d - 26*e + 28*f) + 11772*d*e**2*f*(19*d - 26*e + 28*f)**2 -
 2355*d*e**2*(19*d - 26*e + 28*f)**3/2 + 112386560*d*e*f**4 - 4098752*d*e*f
**3*(19*d - 26*e + 28*f) - 110640*d*e*f**2*(19*d - 26*e + 28*f)**2 + 1638*d
*e*f*(19*d - 26*e + 28*f)**3 - 33092352*d*f**5 + 1019680*d*f**4*(19*d - 26*
e + 28*f) + 58864*d*f**3*(19*d - 26*e + 28*f)**2 - 582*d*f**2*(19*d - 26*e
+ 28*f)**3 + 4283840*e**6 - 17164384*e**5*f - 161500*e**5*(19*d - 26*e + 28
*f) + 18093760*e**4*f**2 + 943000*e**4*f*(19*d - 26*e + 28*f) - 11920*e**4*
(19*d - 26*e + 28*f)**2 + 10387200*e**3*f**3 - 2177824*e**3*f**2*(19*d - 26
*e + 28*f) + 24232*e**3*f*(19*d - 26*e + 28*f)**2 + 295*e**3*(19*d - 26*e +
 28*f)**3 - 36528640*e**2*f**4 + 2470208*e**2*f**3*(19*d - 26*e + 28*f) + 2
0880*e**2*f**2*(19*d - 26*e + 28*f)**2 - 1002*e**2*f*(19*d - 26*e + 28*f)**
3 + 27625984*e*f**5 - 1334720*e*f**4*(19*d - 26*e + 28*f) - 52768*e*f**3*(1
9*d - 26*e + 28*f)**2 + 1044*e*f**2*(19*d - 26*e + 28*f)**3 - 6640640*f**6
+ 291200*f**5*(19*d - 26*e + 28*f) + 18880*f**4*(19*d - 26*e + 28*f)**2 - 4
40*f**3*(19*d - 26*e + 28*f)**3)/(801262*d**6 - 4662251*d**5*e + 6598614*d*
*5*f + 7296938*d**4*e**2 - 32240296*d**4*e*f + 22080168*d**4*f**2 + 1388616
*d**3*e**3 + 44207696*d**3*e**2*f - 87152288*d**3*e*f**2 + 38269376*d**3*f*
*3 - 12447440*d**2*e**4 - 4393344*d**2*e**3*f + 97214592*d**2*e**2*f**2 - 1
```

```
14767360*d**2*e*f**3 + 36053760*d**2*f**4 + 9990800*d*e**5 - 25278880*d*e**
4*f - 19059072*d*e**3*f**2 + 91652864*d*e**2*f**3 - 73388800*d*e*f**4 + 173
95200*d*f**5 - 2380000*e**6 + 10947200*e**5*f - 11338880*e**4*f**2 - 135628
80*e**3*f**3 + 31239680*e**2*f**4 - 18176000*e*f**5 + 3328000*f**6))/144 +
(d - 2*e + 4*f)/(12*x + 24)
```

Giac [A] time = 1.08921, size = 104, normalized size = 1.27

$$-\frac{1}{144}\left(19\,d + 28\,f - 26\,e\right)\log\left(|x + 2|\right) + \frac{1}{6}\left(d + f - e\right)\log\left(|x + 1|\right) - \frac{1}{18}\left(d + f + e\right)\log\left(|x - 1|\right) + \frac{1}{48}\left(d + 4\,f + 2\,e\right)\log\left(|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac"
)
```

```
[Out] -1/144*(19*d + 28*f - 26*e)*log(abs(x + 2)) + 1/6*(d + f - e)*log(abs(x + 1
)) - 1/18*(d + f + e)*log(abs(x - 1)) + 1/48*(d + 4*f + 2*e)*log(abs(x - 2)
) + 1/12*(d + 4*f - 2*e)/(x + 2)
```

3.88 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=95

$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}$$

```
[Out] (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])/18 + ((d
+ 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Log[1 + x])/6 - ((19*d
 - 26*e + 28*f - 8*g)*Log[2 + x])/144
```

Rubi [A] time = 0.221323, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {1586, 6742}

$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x
]
```



```
[Out] (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])/18 + ((d
+ 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Log[1 + x])/6 - ((19*d
 - 26*e + 28*f - 8*g)*Log[2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3}{(2+x)^2(2-x-2x^2+x^3)}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g}{48(-2+x)} + \frac{-d-e-f-g}{18(-1+x)} + \frac{d-e+f-g}{6(1+x)} + \frac{-d+2e-4f}{12(2+x)}\right.$$
$$= \frac{d-2e+4f-8g}{12(2+x)} - \frac{1}{18}(d+e+f+g)\log(1-x) + \frac{1}{48}(d+2e+4f+8g)$$

Mathematica [A] time = 0.0466772, size = 90, normalized size = 0.95

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g)}{x+2} + 24\log(-x-1)(d-e+f-g) - 8\log(1-x)(d+e+f+g) + 3\log(2-x)(d+2e+4f+8g)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^
4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g))/(2 + x) + 24*(d - e + f - g)*Log[-1 - x] - 8*(d
 + e + f + g)*Log[1 - x] + 3*(d + 2*e + 4*f + 8*g)*Log[2 - x] + (-19*d + 26
*e - 28*f + 8*g)*Log[2 + x])/144
```

Maple [A] time = 0.011, size = 146, normalized size = 1.5

$$\frac{13\ln(2+x)e}{72} - \frac{7\ln(2+x)f}{36} + \frac{\ln(2+x)g}{18} - \frac{19\ln(2+x)d}{144} + \frac{d}{24+12x} - \frac{e}{12+6x} + \frac{f}{6+3x} - \frac{2g}{6+3x} + \frac{\ln(1+x)d}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 13/72*ln(2+x)*e-7/36*ln(2+x)*f+1/18*ln(2+x)*g-19/144*ln(2+x)*d+1/12/(2+x)*d
-1/6/(2+x)*e+1/3/(2+x)*f-2/3/(2+x)*g+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/6*ln(1+x
)*f-1/6*ln(1+x)*g+1/48*ln(x-2)*d+1/24*ln(x-2)*e+1/12*ln(x-2)*f+1/6*ln(x-2)*
g-1/18*ln(x-1)*d-1/18*ln(x-1)*e-1/18*ln(x-1)*f-1/18*ln(x-1)*g
```

Maxima [A] time = 0.988366, size = 109, normalized size = 1.15

$$-\frac{1}{144}\left(19\,d-26\,e+28\,f-8\,g\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{18}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{48}\left(d+2\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"maxima")
```

```
[Out] -1/144*(19*d - 26*e + 28*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(x +
1) - 1/18*(d + e + f + g)*log(x - 1) + 1/48*(d + 2*e + 4*f + 8*g)*log(x - 2
) + 1/12*(d - 2*e + 4*f - 8*g)/(x + 2)
```

Fricas [A] time = 6.17876, size = 406, normalized size = 4.27

$$-\frac{\left(\left(19\,d-26\,e+28\,f-8\,g\right)x+38\,d-52\,e+56\,f-16\,g\right)\log\left(x+2\right)-24\left(\left(d-e+f-g\right)x+2\,d-2\,e+2\,f-2\,g\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"fricas")
```

```
[Out] -1/144*(((19*d - 26*e + 28*f - 8*g)*x + 38*d - 52*e + 56*f - 16*g)*log(x +
2) - 24*((d - e + f - g)*x + 2*d - 2*e + 2*f - 2*g)*log(x + 1) + 8*((d + e
+ f + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - 3*((d + 2*e + 4*f + 8*g)*x
 + 2*d + 4*e + 8*f + 16*g)*log(x - 2) - 12*d + 24*e - 48*f + 96*g)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08241, size = 122, normalized size = 1.28

$$-\frac{1}{144}\left(19\,d+28\,f-8\,g-26\,e\right)\log\left(|x+2|\right)+\frac{1}{6}\left(d+f-g-e\right)\log\left(|x+1|\right)-\frac{1}{18}\left(d+f+g+e\right)\log\left(|x-1|\right)+\frac{1}{48}\left(d+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"giac")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 26*e)*log(abs(x + 2)) + 1/6*(d + f - g - e)*log
(abs(x + 1)) - 1/18*(d + f + g + e)*log(abs(x - 1)) + 1/48*(d + 4*f + 8*g +
 2*e)*log(abs(x - 2)) + 1/12*(d + 4*f - 8*g - 2*e)/(x + 2)
```

3.89 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=106

$$\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-$$

```
[Out] (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x]
)/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + ((d - e + f - g + h)*
Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80*h)*Log[2 + x])/144
```

Rubi [A] time = 0.265996, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1586, 6742}

$$\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x]
)/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + ((d - e + f - g + h)*
Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80*h)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{(2+x)^2(2-x-2x^2+x^3)}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g+16h}{48(-2+x)} + \frac{-d-e-f-g-h}{18(-1+x)} + \frac{d-e+f-g}{6(1+x)}\right.$$
$$= \frac{d-2e+4f-8g+16h}{12(2+x)} - \frac{1}{18}(d+e+f+g+h)\log(1-x) + \frac{1}{48}(d$$

Mathematica [A] time = 0.0644904, size = 102, normalized size = 0.96

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g+16h)}{x+2} + 24\log(-x-1)(d-e+f-g+h) - 8\log(1-x)(d+e+f+g+h) + 3\log(2-x)(d+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*
x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g + 16*h))/(2 + x) + 24*(d - e + f - g + h)*Log[-1
- x] - 8*(d + e + f + g + h)*Log[1 - x] + 3*(d + 2*(e + 2*f + 4*g + 8*h))*L
og[2 - x] + (-19*d + 26*e - 28*f + 8*g + 80*h)*Log[2 + x])/144
```

Maple [A] time = 0.012, size = 182, normalized size = 1.7

$$-\frac{19\ln(2+x)d}{144} + \frac{13\ln(2+x)e}{72} + \frac{\ln(1+x)d}{6} - \frac{\ln(1+x)e}{6} + \frac{\ln(x-2)d}{48} + \frac{\ln(x-2)e}{24} - \frac{\ln(x-1)d}{18} - \frac{\ln(x-1)e}{18} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -19/144*ln(2+x)*d+13/72*ln(2+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/48*ln(x-2)*
d+1/24*ln(x-2)*e-1/18*ln(x-1)*d-1/18*ln(x-1)*e+4/3/(2+x)*h-2/3/(2+x)*g+1/12
/(2+x)*d-1/6/(2+x)*e+1/3/(2+x)*f+1/18*ln(2+x)*g-1/6*ln(1+x)*g+1/6*ln(x-2)*g
-1/18*ln(x-1)*g+5/9*ln(2+x)*h+1/6*ln(1+x)*h+1/3*ln(x-2)*h-1/18*ln(x-1)*h+1/
12*ln(x-2)*f-1/18*ln(x-1)*f-7/36*ln(2+x)*f+1/6*ln(1+x)*f
```

Maxima [A] time = 0.980009, size = 124, normalized size = 1.17

$$-\frac{1}{144}\left(19\,d-26\,e+28\,f-8\,g-80\,h\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g+h\right)\log\left(x+1\right)-\frac{1}{18}\left(d+e+f+g+h\right)\log\left(x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="maxima")
```

```
[Out] -1/144*(19*d - 26*e + 28*f - 8*g - 80*h)*log(x + 2) + 1/6*(d - e + f - g +
h)*log(x + 1) - 1/18*(d + e + f + g + h)*log(x - 1) + 1/48*(d + 2*e + 4*f +
 8*g + 16*h)*log(x - 2) + 1/12*(d - 2*e + 4*f - 8*g + 16*h)/(x + 2)
```

Fricas [A] time = 37.1706, size = 483, normalized size = 4.56

$$-\frac{\left(\left(19\,d-26\,e+28\,f-8\,g-80\,h\right)x+38\,d-52\,e+56\,f-16\,g-160\,h\right)\log\left(x+2\right)-24\left(\left(d-e+f-g+h\right)x+2\,d-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="fricas")
```

```
[Out] -1/144*(((19*d - 26*e + 28*f - 8*g - 80*h)*x + 38*d - 52*e + 56*f - 16*g -
160*h)*log(x + 2) - 24*((d - e + f - g + h)*x + 2*d - 2*e + 2*f - 2*g + 2*h
)*log(x + 1) + 8*((d + e + f + g + h)*x + 2*d + 2*e + 2*f + 2*g + 2*h)*log(
x - 1) - 3*((d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)
*log(x - 2) - 12*d + 24*e - 48*f + 96*g - 192*h)/(x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2
,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08835, size = 136, normalized size = 1.28

$$-\frac{1}{144}\left(19\,d+28\,f-8\,g-80\,h-26\,e\right)\log\left(|x+2|\right)+\frac{1}{6}\left(d+f-g+h-e\right)\log\left(|x+1|\right)-\frac{1}{18}\left(d+f+g+h+e\right)\log\left(|x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="giac")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e)*log(abs(x + 2)) + 1/6*(d + f - g +
 h - e)*log(abs(x + 1)) - 1/18*(d + f + g + h + e)*log(abs(x - 1)) + 1/48*(
d + 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2)) + 1/12*(d + 4*f - 8*g + 16*h -
2*e)/(x + 2)
```

3.90 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=122

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h+i)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}$$

```
[Out] i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e + f + g +
h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/48
 + ((d - e + f - g + h - i)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80
*h + 352*i)*Log[2 + x])/144
```

Rubi [A] time = 0.314953, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {1586, 6742}

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h+i)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4)^2,x]
```


```
[Out] i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e + f + g +
h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/48
 + ((d - e + f - g + h - i)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80
*h + 352*i)*Log[2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(2-x-2x^2+x^3\right)\left(d+ex+fx^2+gx^3+hx^4+90x^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+90x^5}{(2+x)^2\left(2-x-2x^2+x^3\right)}\,dx$$

$$= \int \left(90 + \frac{2880+d+2e+4f+8g+16h}{48(-2+x)} + \frac{-90-d-e-}{18(-1+}\right.$$

$$= 90x - \frac{2880-d+2e-4f+8g-16h}{12(2+x)} - \frac{1}{18}(90+d+e+f$$

Mathematica [A] time = 0.0703025, size = 118, normalized size = 0.97

$$\frac{1}{144}\left(\frac{12(d-2(e-2f+4g-8h+16i))}{x+2} - 8\log(1-x)(d+e+f+g+h+i) + 3\log(2-x)(d+2e+4(f+2g+4h+8i))\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))
/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (144*i*x + (12*(d - 2*(e - 2*f + 4*g - 8*h + 16*i)))/(2 + x) - 8*(d + e + f
 + g + h + i)*Log[1 - x] + 3*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x]
 + 24*(d - e + f - g + h - i)*Log[1 + x] + (-19*d + 26*e - 28*f + 8*g + 80*
h - 352*i)*Log[2 + x])/144
```

Maple [A] time = 0.011, size = 221, normalized size = 1.8

$$-\frac{19\ln(2+x)d}{144} + \frac{13\ln(2+x)e}{72} + \frac{\ln(1+x)d}{6} - \frac{\ln(1+x)e}{6} + \frac{\ln(x-2)d}{48} + \frac{\ln(x-2)e}{24} - \frac{\ln(x-1)d}{18} - \frac{\ln(x-1)e}{18} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -19/144*ln(2+x)*d+13/72*ln(2+x)*e+1/6*ln(1+x)*d-1/6*ln(1+x)*e+1/48*ln(x-2)*
d+1/24*ln(x-2)*e-1/18*ln(x-1)*d-1/18*ln(x-1)*e-8/3/(2+x)*i+4/3/(2+x)*h-2/3/
(2+x)*g+1/12/(2+x)*d-1/6/(2+x)*e+1/3/(2+x)*f+2/3*ln(x-2)*i-1/18*ln(x-1)*i-2
2/9*ln(2+x)*i-1/6*ln(1+x)*i+1/18*ln(2+x)*g-1/6*ln(1+x)*g+1/6*ln(x-2)*g-1/18
*ln(x-1)*g+5/9*ln(2+x)*h+1/6*ln(1+x)*h+1/3*ln(x-2)*h-1/18*ln(x-1)*h+1/12*ln
```

```
(x-2)*f-1/18*ln(x-1)*f-7/36*ln(2+x)*f+1/6*ln(1+x)*f+i*x
```

Maxima [A] time = 0.952286, size = 146, normalized size = 1.2

$$ix - \frac{1}{144}\left(19\,d - 26\,e + 28\,f - 8\,g - 80\,h + 352\,i\right)\log\left(x+2\right) + \frac{1}{6}\left(d - e + f - g + h - i\right)\log\left(x+1\right) - \frac{1}{18}\left(d + e + f + g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="maxima")
```

```
[Out] i*x - 1/144*(19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*log(x + 2) + 1/6*(d -
 e + f - g + h - i)*log(x + 1) - 1/18*(d + e + f + g + h + i)*log(x - 1) +
1/48*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2) + 1/12*(d - 2*e + 4*f -
 8*g + 16*h - 32*i)/(x + 2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**
2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08429, size = 158, normalized size = 1.3

$$ix - \frac{1}{144}\left(19\,d + 28\,f - 8\,g - 80\,h + 352\,i - 26\,e\right)\log\left(|x+2|\right) + \frac{1}{6}\left(d + f - g + h - i - e\right)\log\left(|x+1|\right) - \frac{1}{18}\left(d + f + g + h\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="giac")
```

```
[Out] i*x - 1/144*(19*d + 28*f - 8*g - 80*h + 352*i - 26*e)*log(abs(x + 2)) + 1/6
*(d + f - g + h - i - e)*log(abs(x + 1)) - 1/18*(d + f + g + h + i + e)*log
(abs(x - 1)) + 1/48*(d + 4*f + 8*g + 16*h + 32*i + 2*e)*log(abs(x - 2)) + 1
/12*(d + 4*f - 8*g + 16*h - 32*i - 2*e)/(x + 2)
```

3.91 $\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3x+5}{12\left(x^2+3x+2\right)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

```
[Out] -(5 + 3*x)/(12*(2 + 3*x + x^2)) - Log[1 - x]/36 + Log[2 - x]/144 - (7*Log[1
 + x])/36 + (31*Log[2 + x])/144
```

Rubi [A] time = 0.0565911, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1586, 974, 1072, 632, 31}

$$-\frac{3x+5}{12\left(x^2+3x+2\right)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5 + 3*x)/(12*(2 + 3*x + x^2)) - Log[1 - x]/36 + Log[2 - x]/144 - (7*Log[1
 + x])/36 + (31*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
```

```
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2}\,dx &= \int \frac{1}{\left(2-3x+x^2\right)\left(2+3x+x^2\right)^2}\,dx \\
&= -\frac{5+3x}{12\left(2+3x+x^2\right)} + \frac{1}{72}\int \frac{-18+48x-18x^2}{\left(2-3x+x^2\right)\left(2+3x+x^2\right)}\,dx \\
&= -\frac{5+3x}{12\left(2+3x+x^2\right)} + \frac{\int \frac{252-108x}{2-3x+x^2}\,dx}{5184} + \frac{\int \frac{-900+108x}{2+3x+x^2}\,dx}{5184} \\
&= -\frac{5+3x}{12\left(2+3x+x^2\right)} + \frac{1}{144}\int \frac{1}{-2+x}\,dx - \frac{1}{36}\int \frac{1}{-1+x}\,dx - \frac{7}{36}\int \frac{1}{1+x}\,dx + \frac{31}{144}\int \frac{1}{2+x}\,dx \\
&= -\frac{5+3x}{12\left(2+3x+x^2\right)} - \frac{1}{36}\log(1-x) + \frac{1}{144}\log(2-x) - \frac{7}{36}\log(1+x) + \frac{31}{144}\log(2+x)
\end{aligned}
$$

Mathematica [A] time = 0.0248618, size = 48, normalized size = 0.86

$$
\frac{1}{144}\left(-\frac{12(3x+5)}{x^2+3x+2} - 4\log(1-x) + \log(2-x) - 28\log(x+1) + 31\log(x+2)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(5 + 3*x))/(2 + 3*x + x^2) - 4*Log[1 - x] + Log[2 - x] - 28*Log[1 + x
] + 31*Log[2 + x])/144
```

Maple [A] time = 0.013, size = 40, normalized size = 0.7

$$
-\frac{1}{24+12x} + \frac{31\ln(2+x)}{144} - \frac{1}{6+6x} - \frac{7\ln(1+x)}{36} + \frac{\ln(x-2)}{144} - \frac{\ln(x-1)}{36}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)+31/144*ln(2+x)-1/6/(1+x)-7/36*ln(1+x)+1/144*ln(x-2)-1/36*ln(x-1
)
```

Maxima [A] time = 0.950417, size = 57, normalized size = 1.02

$$-\frac{3\,x+5}{12\left(x^2+3\,x+2\right)}+\frac{31}{144}\,\log\left(x+2\right)-\frac{7}{36}\,\log\left(x+1\right)-\frac{1}{36}\,\log\left(x-1\right)+\frac{1}{144}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*x + 5)/(x^2 + 3*x + 2) + 31/144*log(x + 2) - 7/36*log(x + 1) - 1/3
6*log(x - 1) + 1/144*log(x - 2)
```

Fricas [A] time = 1.82589, size = 215, normalized size = 3.84

$$\frac{31\left(x^2+3\,x+2\right)\log\left(x+2\right)-28\left(x^2+3\,x+2\right)\log\left(x+1\right)-4\left(x^2+3\,x+2\right)\log\left(x-1\right)+\left(x^2+3\,x+2\right)\log\left(x-2\right)-36}{144\left(x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(31*(x^2 + 3*x + 2)*log(x + 2) - 28*(x^2 + 3*x + 2)*log(x + 1) - 4*(x
^2 + 3*x + 2)*log(x - 1) + (x^2 + 3*x + 2)*log(x - 2) - 36*x - 60)/(x^2 + 3
*x + 2)
```

Sympy [A] time = 0.266986, size = 44, normalized size = 0.79

$$-\frac{3x+5}{12x^2+36x+24}+\frac{\log\left(x-2\right)}{144}-\frac{\log\left(x-1\right)}{36}-\frac{7\log\left(x+1\right)}{36}+\frac{31\log\left(x+2\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(3*x + 5)/(12*x**2 + 36*x + 24) + log(x - 2)/144 - log(x - 1)/36 - 7*log(x
 + 1)/36 + 31*log(x + 2)/144
```

Giac [A] time = 1.08075, size = 62, normalized size = 1.11

$$-\frac{3\,x+5}{12\,(x+2)(x+1)}+\frac{31}{144}\,\log\left(|x+2|\right)-\frac{7}{36}\,\log\left(|x+1|\right)-\frac{1}{36}\,\log\left(|x-1|\right)+\frac{1}{144}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*x + 5)/((x + 2)*(x + 1)) + 31/144*log(abs(x + 2)) - 7/36*log(abs(x
 + 1)) - 1/36*log(abs(x - 1)) + 1/144*log(abs(x - 2))
```

3.92 $\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=89

$$-\frac{x(3d-4e)+5d-6e}{12(x^2+3x+2)} - \frac{1}{36}(d+e)\log(1-x) + \frac{1}{144}(d+2e)\log(2-x) - \frac{1}{36}(7d-13e)\log(x+1) + \frac{1}{144}(31d-50e)\log(x$$

```
[Out] -(5*d - 6*e + (3*d - 4*e)*x)/(12*(2 + 3*x + x^2)) - ((d + e)*Log[1 - x])/36
 + ((d + 2*e)*Log[2 - x])/144 - ((7*d - 13*e)*Log[1 + x])/36 + ((31*d - 50*
e)*Log[2 + x])/144
```

Rubi [A] time = 0.260162, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1586, 1016, 1072, 632, 31}

$$-\frac{x(3d-4e)+5d-6e}{12(x^2+3x+2)} - \frac{1}{36}(d+e)\log(1-x) + \frac{1}{144}(d+2e)\log(2-x) - \frac{1}{36}(7d-13e)\log(x+1) + \frac{1}{144}(31d-50e)\log(x$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5*d - 6*e + (3*d - 4*e)*x)/(12*(2 + 3*x + x^2)) - ((d + e)*Log[1 - x])/36
 + ((d + 2*e)*Log[2 - x])/144 - ((7*d - 13*e)*Log[1 + x])/36 + ((31*d - 50*
e)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(d+ex)\left(2-3x+x^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex}{\left(2-3x+x^2\right)\left(2+3x+x^2\right)^2}\,dx$$
$$= -\frac{5d-6e+(3d-4e)x}{12\left(2+3x+x^2\right)} - \frac{1}{72}\int \frac{6(3d-10e)-24(2d-3e)x+6(3d-4e)x^2}{\left(2-3x+x^2\right)\left(2+3x+x^2\right)}\,dx$$
$$= -\frac{5d-6e+(3d-4e)x}{12\left(2+3x+x^2\right)} - \frac{\int \frac{108(3d-10e)-288(2d-3e)+(-36(3d-10e)+72(3d-4e))x}{2-3x+x^2}\,dx}{5184} - \frac{\int \frac{108(3d-10e)+28}{}}{}$$
$$= -\frac{5d-6e+(3d-4e)x}{12\left(2+3x+x^2\right)} - \frac{1}{36}(7d-13e)\int \frac{1}{1+x}\,dx - \frac{1}{144}(-d-2e)\int \frac{1}{-2+x}\,dx - \frac{1}{36}(d+$$
$$= -\frac{5d-6e+(3d-4e)x}{12\left(2+3x+x^2\right)} - \frac{1}{36}(d+e)\log(1-x) + \frac{1}{144}(d+2e)\log(2-x) - \frac{1}{36}(7d-13e)\log$$

Mathematica [A] time = 0.0507321, size = 80, normalized size = 0.9

$$\frac{1}{144}\left(\frac{12(-3dx-5d+4ex+6e)}{x^2+3x+2} - 4(d+e)\log(1-x) + (d+2e)\log(2-x) + 4(13e-7d)\log(x+1) + (31d-50e)\log(x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-5*d + 6*e - 3*d*x + 4*e*x))/(2 + 3*x + x^2) - 4*(d + e)*Log[1 - x] +
 (d + 2*e)*Log[2 - x] + 4*(-7*d + 13*e)*Log[1 + x] + (31*d - 50*e)*Log[2 +
x])/144
```

Maple [A] time = 0.011, size = 90, normalized size = 1.

$$-\frac{d}{24+12x} + \frac{e}{12+6x} + \frac{31\ln(2+x)d}{144} - \frac{25\ln(2+x)e}{72} - \frac{7\ln(1+x)d}{36} + \frac{13\ln(1+x)e}{36} - \frac{d}{6+6x} + \frac{e}{6+6x} + \frac{\ln(x-}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e+31/144*ln(2+x)*d-25/72*ln(2+x)*e-7/36*ln(1+x)*d+1
3/36*ln(1+x)*e-1/6/(1+x)*d+1/6/(1+x)*e+1/144*ln(x-2)*d+1/72*ln(x-2)*e-1/36*
```

```
ln(x-1)*d-1/36*ln(x-1)*e
```

Maxima [A] time = 0.958643, size = 101, normalized size = 1.13

$$\frac{1}{144}\,(31\,d-50\,e)\log\left(x+2\right)-\frac{1}{36}\,(7\,d-13\,e)\log\left(x+1\right)-\frac{1}{36}\,(d+e)\log\left(x-1\right)+\frac{1}{144}\,(d+2\,e)\log\left(x-2\right)-\frac{(3\,d-4\,e)x}{12\,\left(x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(31*d - 50*e)*log(x + 2) - 1/36*(7*d - 13*e)*log(x + 1) - 1/36*(d + e
)*log(x - 1) + 1/144*(d + 2*e)*log(x - 2) - 1/12*((3*d - 4*e)*x + 5*d - 6*e
)/(x^2 + 3*x + 2)
```

Fricas [A] time = 1.94788, size = 410, normalized size = 4.61

$$-\frac{12\,(3\,d-4\,e)x-\left((31\,d-50\,e)x^2+3\,(31\,d-50\,e)x+62\,d-100\,e\right)\log\left(x+2\right)+4\left((7\,d-13\,e)x^2+3\,(7\,d-13\,e)x+14\,d\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e)*x - ((31*d - 50*e)*x^2 + 3*(31*d - 50*e)*x + 62*d -
100*e)*log(x + 2) + 4*((7*d - 13*e)*x^2 + 3*(7*d - 13*e)*x + 14*d - 26*e)*l
og(x + 1) + 4*((d + e)*x^2 + 3*(d + e)*x + 2*d + 2*e)*log(x - 1) - ((d + 2*
e)*x^2 + 3*(d + 2*e)*x + 2*d + 4*e)*log(x - 2) + 60*d - 72*e)/(x^2 + 3*x +
2)
```

Sympy [B] time = 5.75008, size = 1255, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d + e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 10439775*d**5*(d + e)
 - 511591980*d**4*e**2 - 94132290*d**4*e*(d + e) + 667200*d**4*(d + e)**2 +
 469491120*d**3*e**3 + 333672552*d**3*e**2*(d + e) - 2703328*d**3*e*(d + e)
**2 - 198000*d**3*(d + e)**3 + 322778400*d**2*e**4 - 582497712*d**2*e**3*(d
 + e) + 1752768*d**2*e**2*(d + e)**2 + 1107552*d**2*e*(d + e)**3 - 86349385
6*d*e**5 + 500776560*d*e**4*(d + e) + 4226944*d*e**3*(d + e)**2 - 1880640*d
*e**2*(d + e)**3 + 429000000*e**6 - 169242912*e**5*(d + e) - 4538112*e**4*(
d + e)**2 + 964224*e**3*(d + e)**3)/(13474125*d**6 - 102860175*d**5*e + 274
190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*
e**5 - 256183200*e**6))/36 + (d + 2*e)*log(x + (-24383100*d**6 + 187408066*
d**5*e - 10439775*d**5*(d + 2*e)/4 - 511591980*d**4*e**2 + 47066145*d**4*e*
(d + 2*e)/2 + 41700*d**4*(d + 2*e)**2 + 469491120*d**3*e**3 - 83418138*d**3
*e**2*(d + 2*e) - 168958*d**3*e*(d + 2*e)**2 + 12375*d**3*(d + 2*e)**3/4 +
322778400*d**2*e**4 + 145624428*d**2*e**3*(d + 2*e) + 109548*d**2*e**2*(d +
 2*e)**2 - 34611*d**2*e*(d + 2*e)**3/2 - 863493856*d*e**5 - 125194140*d*e**
4*(d + 2*e) + 264184*d*e**3*(d + 2*e)**2 + 29385*d*e**2*(d + 2*e)**3 + 4290
00000*e**6 + 42310728*e**5*(d + 2*e) - 283632*e**4*(d + 2*e)**2 - 15066*e**
3*(d + 2*e)**3)/(13474125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2 - 2
24142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*e**
6))/144 - (7*d - 13*e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 1043977
5*d**5*(7*d - 13*e) - 511591980*d**4*e**2 - 94132290*d**4*e*(7*d - 13*e) +
667200*d**4*(7*d - 13*e)**2 + 469491120*d**3*e**3 + 333672552*d**3*e**2*(7*
d - 13*e) - 2703328*d**3*e*(7*d - 13*e)**2 - 198000*d**3*(7*d - 13*e)**3 +
322778400*d**2*e**4 - 582497712*d**2*e**3*(7*d - 13*e) + 1752768*d**2*e**2*
(7*d - 13*e)**2 + 1107552*d**2*e*(7*d - 13*e)**3 - 863493856*d*e**5 + 50077
6560*d*e**4*(7*d - 13*e) + 4226944*d*e**3*(7*d - 13*e)**2 - 1880640*d*e**2*
(7*d - 13*e)**3 + 429000000*e**6 - 169242912*e**5*(7*d - 13*e) - 4538112*e*
*4*(7*d - 13*e)**2 + 964224*e**3*(7*d - 13*e)**3)/(13474125*d**6 - 10286017
5*d**5*e + 274190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4
+ 535797456*d*e**5 - 256183200*e**6))/36 + (31*d - 50*e)*log(x + (-24383100
*d**6 + 187408066*d**5*e - 10439775*d**5*(31*d - 50*e)/4 - 511591980*d**4*e
**2 + 47066145*d**4*e*(31*d - 50*e)/2 + 41700*d**4*(31*d - 50*e)**2 + 46949
1120*d**3*e**3 - 83418138*d**3*e**2*(31*d - 50*e) - 168958*d**3*e*(31*d - 5
0*e)**2 + 12375*d**3*(31*d - 50*e)**3/4 + 322778400*d**2*e**4 + 145624428*d
**2*e**3*(31*d - 50*e) + 109548*d**2*e**2*(31*d - 50*e)**2 - 34611*d**2*e*(
31*d - 50*e)**3/2 - 863493856*d*e**5 - 125194140*d*e**4*(31*d - 50*e) + 264
184*d*e**3*(31*d - 50*e)**2 + 29385*d*e**2*(31*d - 50*e)**3 + 429000000*e**
6 + 42310728*e**5*(31*d - 50*e) - 283632*e**4*(31*d - 50*e)**2 - 15066*e**3
*(31*d - 50*e)**3)/(13474125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2
- 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*
e**6))/144 - (5*d - 6*e + x*(3*d - 4*e))/(12*x**2 + 36*x + 24)
```

Giac [A] time = 1.07827, size = 115, normalized size = 1.29

$$\frac{1}{144}\left(31\,d-50\,e\right)\log\left(|x+2|\right)-\frac{1}{36}\left(7\,d-13\,e\right)\log\left(|x+1|\right)-\frac{1}{36}\left(d+e\right)\log\left(|x-1|\right)+\frac{1}{144}\left(d+2\,e\right)\log\left(|x-2|\right)-\frac{(3\,d-}{12(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(31*d - 50*e)*log(abs(x + 2)) - 1/36*(7*d - 13*e)*log(abs(x + 1)) - 1
/36*(d + e)*log(abs(x - 1)) + 1/144*(d + 2*e)*log(abs(x - 2)) - 1/12*((3*d
- 4*e)*x + 5*d - 6*e)/((x + 2)*(x + 1))
```

3.93 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=105

$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12\left(x^2+3x+2\right)}-\frac{1}{36}\log(1-x)(d+e+f)+\frac{1}{144}\log(2-x)(d+2e+4f)-\frac{1}{36}\log(x+1)(7d-13e+19$$

```
[Out] -(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(12*(2 + 3*x + x^2)) - ((d + e + f
)*Log[1 - x])/36 + ((d + 2*e + 4*f)*Log[2 - x])/144 - ((7*d - 13*e + 19*f)*
Log[1 + x])/36 + ((31*d - 50*e + 76*f)*Log[2 + x])/144
```

Rubi [A] time = 0.319504, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1586, 1060, 1072, 632, 31}

$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12\left(x^2+3x+2\right)}-\frac{1}{36}\log(1-x)(d+e+f)+\frac{1}{144}\log(2-x)(d+2e+4f)-\frac{1}{36}\log(x+1)(7d-13e+19$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(12*(2 + 3*x + x^2)) - ((d + e + f
)*Log[1 - x])/36 + ((d + 2*e + 4*f)*Log[2 - x])/144 - ((7*d - 13*e + 19*f)*
Log[1 + x])/36 + ((31*d - 50*e + 76*f)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
```

```
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{72}\int \frac{6(3d-10e+12f)-24(2d-3e+5f)x+}{(2-3x+x^2)(2+3x+}$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{\int \frac{-288(2d-3e+5f)+108(3d-10e+12f)+(72(3d-4e+6f)-36(3}{2-3x+x^2}}{5184}$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{144}(-31d+50e-76f)\int \frac{1}{2+x}\,dx - \frac{1}{144}(-d$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{36}(d+e+f)\log(1-x) + \frac{1}{144}(d+2e+4f)\,\mathrm{lo}$$

Mathematica [A] time = 0.0743086, size = 97, normalized size = 0.92

$$\frac{1}{144}\left(-\frac{12(d(3x+5)-4ex-6e+6fx+8f)}{x^2+3x+2} - 4\log(1-x)(d+e+f) + \log(2-x)(d+2e+4f) - 4\log(x+1)(7d-13e+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(-6*e + 8*f - 4*e*x + 6*f*x + d*(5 + 3*x)))/(2 + 3*x + x^2) - 4*(d +
e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] - 4*(7*d - 13*e + 19*f)*Log[
1 + x] + (31*d - 50*e + 76*f)*Log[2 + x])/144
```

Maple [A] time = 0.013, size = 134, normalized size = 1.3

$$-\frac{d}{24+12x} + \frac{e}{12+6x} - \frac{f}{6+3x} + \frac{31\ln(2+x)d}{144} - \frac{25\ln(2+x)e}{72} + \frac{19\ln(2+x)f}{36} - \frac{7\ln(1+x)d}{36} + \frac{13\ln(1+x)e}{36} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e-1/3/(2+x)*f+31/144*ln(2+x)*d-25/72*ln(2+x)*e+19/3
6*ln(2+x)*f-7/36*ln(1+x)*d+13/36*ln(1+x)*e-19/36*ln(1+x)*f-1/6/(1+x)*d+1/6/
(1+x)*e-1/6/(1+x)*f+1/144*ln(x-2)*d+1/72*ln(x-2)*e+1/36*ln(x-2)*f-1/36*ln(x
```

```
-1)*d-1/36*ln(x-1)*e-1/36*ln(x-1)*f
```

Maxima [A] time = 0.976987, size = 123, normalized size = 1.17

$$\frac{1}{144}\left(31\,d-50\,e+76\,f\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d-13\,e+19\,f\right)\log\left(x+1\right)-\frac{1}{36}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{144}\left(d+2\,e+4\,f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(31*d - 50*e + 76*f)*log(x + 2) - 1/36*(7*d - 13*e + 19*f)*log(x + 1)
 - 1/36*(d + e + f)*log(x - 1) + 1/144*(d + 2*e + 4*f)*log(x - 2) - 1/12*((
3*d - 4*e + 6*f)*x + 5*d - 6*e + 8*f)/(x^2 + 3*x + 2)
```

Fricas [B] time = 2.56752, size = 529, normalized size = 5.04

$$-\frac{12\left(3\,d-4\,e+6\,f\right)x-\left(\left(31\,d-50\,e+76\,f\right)x^{2}+3\left(31\,d-50\,e+76\,f\right)x+62\,d-100\,e+152\,f\right)\log\left(x+2\right)+4\left(\left(7\,d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f)*x - ((31*d - 50*e + 76*f)*x^2 + 3*(31*d - 50*e
 + 76*f)*x + 62*d - 100*e + 152*f)*log(x + 2) + 4*((7*d - 13*e + 19*f)*x^2
+ 3*(7*d - 13*e + 19*f)*x + 14*d - 26*e + 38*f)*log(x + 1) + 4*((d + e + f)
*x^2 + 3*(d + e + f)*x + 2*d + 2*e + 2*f)*log(x - 1) - ((d + 2*e + 4*f)*x^2
 + 3*(d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) + 60*d - 72*e + 96*f)/
(x^2 + 3*x + 2)
```

Sympy [B] time = 117.971, size = 5015, normalized size = 47.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d + e + f)*log(x + (-24383100*d**6 + 187408066*d**5*e - 350082438*d**5*f
+ 10439775*d**5*(d + e + f) - 511591980*d**4*e**2 + 2159032460*d**4*e*f - 9
4132290*d**4*e*(d + e + f) - 2073816840*d**4*f**2 + 137095380*d**4*f*(d + e
 + f) + 667200*d**4*(d + e + f)**2 + 469491120*d**3*e**3 - 4339975440*d**3*
e**2*f + 333672552*d**3*e**2*(d + e + f) + 9775163680*d**3*e*f**2 - 9808078
08*d**3*e*f*(d + e + f) - 2703328*d**3*e*(d + e + f)**2 - 6478462560*d**3*f
**3 + 717713712*d**3*f**2*(d + e + f) + 6408864*d**3*f*(d + e + f)**2 - 198
000*d**3*(d + e + f)**3 + 322778400*d**2*e**4 + 1983785760*d**2*e**3*f - 58
2497712*d**2*e**3*(d + e + f) - 13135991040*d**2*e**2*f**2 + 2589108192*d**
2*e**2*f*(d + e + f) + 1752768*d**2*e**2*(d + e + f)**2 + 21641166400*d**2*
e*f**3 - 3821636448*d**2*e*f**2*(d + e + f) - 18174144*d**2*e*f*(d + e + f)
**2 + 1107552*d**2*e*(d + e + f)**3 - 11234578560*d**2*f**4 + 1872896832*d*
*2*f**3*(d + e + f) + 22905216*d**2*f**2*(d + e + f)**2 - 1601856*d**2*f*(d
 + e + f)**3 - 863493856*d*e**5 + 3068196000*d*e**4*f + 500776560*d*e**4*(d
 + e + f) + 901203840*d*e**3*f**2 - 2992140288*d*e**3*f*(d + e + f) + 42269
44*d*e**3*(d + e + f)**2 - 16302840960*d*e**2*f**3 + 6678358848*d*e**2*f**2
*(d + e + f) + 3970944*d*e**2*f*(d + e + f)**2 - 1880640*d*e**2*(d + e + f)
**3 + 23260904960*d*e*f**4 - 6599923968*d*e*f**3*(d + e + f) - 39645696*d*e
*f**2*(d + e + f)**2 + 5713152*d*e*f*(d + e + f)**3 - 10226813952*d*f**5 +
2436622848*d*f**4*(d + e + f) + 36037632*d*f**3*(d + e + f)**2 - 4243968*d*
f**2*(d + e + f)**3 + 429000000*e**6 - 2689492288*e**5*f - 169242912*e**5*(
d + e + f) + 5722012800*e**4*f**2 + 1275930432*e**4*f*(d + e + f) - 4538112
*e**4*(d + e + f)**2 - 2792459520*e**3*f**3 - 3830276736*e**3*f**2*(d + e +
 f) + 13917952*e**3*f*(d + e + f)**2 + 964224*e**3*(d + e + f)**3 - 6487841
280*e**2*f**4 + 5725304064*e**2*f**3*(d + e + f) - 1516032*e**2*f**2*(d + e
 + f)**2 - 4637952*e**2*f*(d + e + f)**3 + 9596336128*e*f**5 - 4262274048*e
*f**4*(d + e + f) - 27731968*e*f**3*(d + e + f)**2 + 7246848*e*f**2*(d + e
+ f)**3 - 3803258880*f**6 + 1264435200*f**5*(d + e + f) + 21012480*f**4*(d
+ e + f)**2 - 3686400*f**3*(d + e + f)**3)/(13474125*d**6 - 102860175*d**5*
e + 196999875*d**5*f + 274190390*d**4*e**2 - 1214801310*d**4*e*f + 11922611
40*d**4*f**2 - 224142072*d**3*e**3 + 2417766776*d**3*e**2*f - 5675135904*d*
*3*e*f**2 + 3821016960*d**3*f**3 - 245084096*d**2*e**4 - 1010456304*d**2*e*
*3*f + 7752467424*d**2*e**2*f**2 - 13083849984*d**2*e*f**3 + 6834458880*d**
2*f**4 + 535797456*d*e**5 - 1875889936*d*e**4*f - 758531712*d*e**3*f**2 + 1
0575845888*d*e**2*f**3 - 14846923776*d*e*f**4 + 6462996480*d*f**5 - 2561832
00*e**6 + 1579708320*e**5*f - 3185673920*e**4*f**2 + 909173760*e**3*f**3 +
5054735360*e**2*f**4 - 6608977920*e*f**5 + 2521497600*f**6))/36 + (d + 2*e
+ 4*f)*log(x + (-24383100*d**6 + 187408066*d**5*e - 350082438*d**5*f - 1043
9775*d**5*(d + 2*e + 4*f)/4 - 511591980*d**4*e**2 + 2159032460*d**4*e*f + 4
7066145*d**4*e*(d + 2*e + 4*f)/2 - 2073816840*d**4*f**2 - 34273845*d**4*f*(
d + 2*e + 4*f) + 41700*d**4*(d + 2*e + 4*f)**2 + 469491120*d**3*e**3 - 4339
975440*d**3*e**2*f - 83418138*d**3*e**2*(d + 2*e + 4*f) + 9775163680*d**3*e
```

```
*f**2 + 245201952*d**3*e*f*(d + 2*e + 4*f) - 168958*d**3*e*(d + 2*e + 4*f)*
*2 - 6478462560*d**3*f**3 - 179428428*d**3*f**2*(d + 2*e + 4*f) + 400554*d*
*3*f*(d + 2*e + 4*f)**2 + 12375*d**3*(d + 2*e + 4*f)**3/4 + 322778400*d**2*
e**4 + 1983785760*d**2*e**3*f + 145624428*d**2*e**3*(d + 2*e + 4*f) - 13135
991040*d**2*e**2*f**2 - 647277048*d**2*e**2*f*(d + 2*e + 4*f) + 109548*d**2
*e**2*(d + 2*e + 4*f)**2 + 21641166400*d**2*e*f**3 + 955409112*d**2*e*f**2*
(d + 2*e + 4*f) - 1135884*d**2*e*f*(d + 2*e + 4*f)**2 - 34611*d**2*e*(d + 2
*e + 4*f)**3/2 - 11234578560*d**2*f**4 - 468224208*d**2*f**3*(d + 2*e + 4*f
) + 1431576*d**2*f**2*(d + 2*e + 4*f)**2 + 25029*d**2*f*(d + 2*e + 4*f)**3
- 863493856*d*e**5 + 3068196000*d*e**4*f - 125194140*d*e**4*(d + 2*e + 4*f)
 + 901203840*d*e**3*f**2 + 748035072*d*e**3*f*(d + 2*e + 4*f) + 264184*d*e*
*3*(d + 2*e + 4*f)**2 - 16302840960*d*e**2*f**3 - 1669589712*d*e**2*f**2*(d
 + 2*e + 4*f) + 248184*d*e**2*f*(d + 2*e + 4*f)**2 + 29385*d*e**2*(d + 2*e
+ 4*f)**3 + 23260904960*d*e*f**4 + 1649980992*d*e*f**3*(d + 2*e + 4*f) - 24
77856*d*e*f**2*(d + 2*e + 4*f)**2 - 89268*d*e*f*(d + 2*e + 4*f)**3 - 102268
13952*d*f**5 - 609155712*d*f**4*(d + 2*e + 4*f) + 2252352*d*f**3*(d + 2*e +
 4*f)**2 + 66312*d*f**2*(d + 2*e + 4*f)**3 + 429000000*e**6 - 2689492288*e*
*5*f + 42310728*e**5*(d + 2*e + 4*f) + 5722012800*e**4*f**2 - 318982608*e**
4*f*(d + 2*e + 4*f) - 283632*e**4*(d + 2*e + 4*f)**2 - 2792459520*e**3*f**3
 + 957569184*e**3*f**2*(d + 2*e + 4*f) + 869872*e**3*f*(d + 2*e + 4*f)**2 -
 15066*e**3*(d + 2*e + 4*f)**3 - 6487841280*e**2*f**4 - 1431326016*e**2*f**
3*(d + 2*e + 4*f) - 94752*e**2*f**2*(d + 2*e + 4*f)**2 + 72468*e**2*f*(d +
2*e + 4*f)**3 + 9596336128*e*f**5 + 1065568512*e*f**4*(d + 2*e + 4*f) - 173
3248*e*f**3*(d + 2*e + 4*f)**2 - 113232*e*f**2*(d + 2*e + 4*f)**3 - 3803258
880*f**6 - 316108800*f**5*(d + 2*e + 4*f) + 1313280*f**4*(d + 2*e + 4*f)**2
 + 57600*f**3*(d + 2*e + 4*f)**3)/(13474125*d**6 - 102860175*d**5*e + 19699
9875*d**5*f + 274190390*d**4*e**2 - 1214801310*d**4*e*f + 1192261140*d**4*f
**2 - 224142072*d**3*e**3 + 2417766776*d**3*e**2*f - 5675135904*d**3*e*f**2
 + 3821016960*d**3*f**3 - 245084096*d**2*e**4 - 1010456304*d**2*e**3*f + 77
52467424*d**2*e**2*f**2 - 13083849984*d**2*e*f**3 + 6834458880*d**2*f**4 +
535797456*d*e**5 - 1875889936*d*e**4*f - 758531712*d*e**3*f**2 + 1057584588
8*d*e**2*f**3 - 14846923776*d*e*f**4 + 6462996480*d*f**5 - 256183200*e**6 +
 1579708320*e**5*f - 3185673920*e**4*f**2 + 909173760*e**3*f**3 + 505473536
0*e**2*f**4 - 6608977920*e*f**5 + 2521497600*f**6))/144 - (7*d - 13*e + 19*
f)*log(x + (-24383100*d**6 + 187408066*d**5*e - 350082438*d**5*f + 10439775
*d**5*(7*d - 13*e + 19*f) - 511591980*d**4*e**2 + 2159032460*d**4*e*f - 941
32290*d**4*e*(7*d - 13*e + 19*f) - 2073816840*d**4*f**2 + 137095380*d**4*f*
(7*d - 13*e + 19*f) + 667200*d**4*(7*d - 13*e + 19*f)**2 + 469491120*d**3*e
**3 - 4339975440*d**3*e**2*f + 333672552*d**3*e**2*(7*d - 13*e + 19*f) + 97
75163680*d**3*e*f**2 - 980807808*d**3*e*f*(7*d - 13*e + 19*f) - 2703328*d**
3*e*(7*d - 13*e + 19*f)**2 - 6478462560*d**3*f**3 + 717713712*d**3*f**2*(7*
d - 13*e + 19*f) + 6408864*d**3*f*(7*d - 13*e + 19*f)**2 - 198000*d**3*(7*d
 - 13*e + 19*f)**3 + 322778400*d**2*e**4 + 1983785760*d**2*e**3*f - 5824977
12*d**2*e**3*(7*d - 13*e + 19*f) - 13135991040*d**2*e**2*f**2 + 2589108192*
d**2*e**2*f*(7*d - 13*e + 19*f) + 1752768*d**2*e**2*(7*d - 13*e + 19*f)**2
```

+ 21641166400*d**2*e*f**3 - 3821636448*d**2*e*f**2*(7*d - 13*e + 19*f) - 18
174144*d**2*e*f*(7*d - 13*e + 19*f)**2 + 1107552*d**2*e*(7*d - 13*e + 19*f)
3 - 11234578560*d2*f**4 + 1872896832*d**2*f**3*(7*d - 13*e + 19*f) + 22
905216*d**2*f**2*(7*d - 13*e + 19*f)**2 - 1601856*d**2*f*(7*d - 13*e + 19*f
)**3 - 863493856*d*e**5 + 3068196000*d*e**4*f + 500776560*d*e**4*(7*d - 13*
e + 19*f) + 901203840*d*e**3*f**2 - 2992140288*d*e**3*f*(7*d - 13*e + 19*f)
 + 4226944*d*e**3*(7*d - 13*e + 19*f)**2 - 16302840960*d*e**2*f**3 + 667835
8848*d*e**2*f**2*(7*d - 13*e + 19*f) + 3970944*d*e**2*f*(7*d - 13*e + 19*f)
2 - 1880640*d*e2*(7*d - 13*e + 19*f)**3 + 23260904960*d*e*f**4 - 659992
3968*d*e*f**3*(7*d - 13*e + 19*f) - 39645696*d*e*f**2*(7*d - 13*e + 19*f)**
2 + 5713152*d*e*f*(7*d - 13*e + 19*f)**3 - 10226813952*d*f**5 + 2436622848*
d*f**4*(7*d - 13*e + 19*f) + 36037632*d*f**3*(7*d - 13*e + 19*f)**2 - 42439
68*d*f**2*(7*d - 13*e + 19*f)**3 + 429000000*e**6 - 2689492288*e**5*f - 169
242912*e**5*(7*d - 13*e + 19*f) + 5722012800*e**4*f**2 + 1275930432*e**4*f*
(7*d - 13*e + 19*f) - 4538112*e**4*(7*d - 13*e + 19*f)**2 - 2792459520*e**3
*f**3 - 3830276736*e**3*f**2*(7*d - 13*e + 19*f) + 13917952*e**3*f*(7*d - 1
3*e + 19*f)**2 + 964224*e**3*(7*d - 13*e + 19*f)**3 - 6487841280*e**2*f**4
 + 5725304064*e**2*f**3*(7*d - 13*e + 19*f) - 1516032*e**2*f**2*(7*d - 13*e
 + 19*f)**2 - 4637952*e**2*f*(7*d - 13*e + 19*f)**3 + 9596336128*e*f**5 - 42
62274048*e*f**4*(7*d - 13*e + 19*f) - 27731968*e*f**3*(7*d - 13*e + 19*f)**
2 + 7246848*e*f**2*(7*d - 13*e + 19*f)**3 - 3803258880*f**6 + 1264435200*f*
5(7*d - 13*e + 19*f) + 21012480*f**4*(7*d - 13*e + 19*f)**2 - 3686400*f**
3*(7*d - 13*e + 19*f)**3)/(13474125*d**6 - 102860175*d**5*e + 196999875*d**
5*f + 274190390*d**4*e**2 - 1214801310*d**4*e*f + 1192261140*d**4*f**2 - 22
4142072*d**3*e**3 + 2417766776*d**3*e**2*f - 5675135904*d**3*e*f**2 + 38210
16960*d**3*f**3 - 245084096*d**2*e**4 - 1010456304*d**2*e**3*f + 7752467424
*d**2*e**2*f**2 - 13083849984*d**2*e*f**3 + 6834458880*d**2*f**4 + 53579745
6*d*e**5 - 1875889936*d*e**4*f - 758531712*d*e**3*f**2 + 10575845888*d*e**2
*f**3 - 14846923776*d*e*f**4 + 6462996480*d*f**5 - 256183200*e**6 + 1579708
320*e**5*f - 3185673920*e**4*f**2 + 909173760*e**3*f**3 + 5054735360*e**2*f
4 - 6608977920*e*f5 + 2521497600*f**6))/36 + (31*d - 50*e + 76*f)*log(x
 + (-24383100*d**6 + 187408066*d**5*e - 350082438*d**5*f - 10439775*d**5*(3
1*d - 50*e + 76*f)/4 - 511591980*d**4*e**2 + 2159032460*d**4*e*f + 47066145
*d**4*e*(31*d - 50*e + 76*f)/2 - 2073816840*d**4*f**2 - 34273845*d**4*f*(31
*d - 50*e + 76*f) + 41700*d**4*(31*d - 50*e + 76*f)**2 + 469491120*d**3*e**
3 - 4339975440*d**3*e**2*f - 83418138*d**3*e**2*(31*d - 50*e + 76*f) + 9775
163680*d**3*e*f**2 + 245201952*d**3*e*f*(31*d - 50*e + 76*f) - 168958*d**3*
e*(31*d - 50*e + 76*f)**2 - 6478462560*d**3*f**3 - 179428428*d**3*f**2*(31*
d - 50*e + 76*f) + 400554*d**3*f*(31*d - 50*e + 76*f)**2 + 12375*d**3*(31*d
 - 50*e + 76*f)**3/4 + 322778400*d**2*e**4 + 1983785760*d**2*e**3*f + 14562
4428*d**2*e**3*(31*d - 50*e + 76*f) - 13135991040*d**2*e**2*f**2 - 64727704
8*d**2*e**2*f*(31*d - 50*e + 76*f) + 109548*d**2*e**2*(31*d - 50*e + 76*f)*
*2 + 21641166400*d**2*e*f**3 + 955409112*d**2*e*f**2*(31*d - 50*e + 76*f) -
 1135884*d**2*e*f*(31*d - 50*e + 76*f)**2 - 34611*d**2*e*(31*d - 50*e + 76*
f)**3/2 - 11234578560*d**2*f**4 - 468224208*d**2*f**3*(31*d - 50*e + 76*f)

```
+ 1431576*d**2*f**2*(31*d - 50*e + 76*f)**2 + 25029*d**2*f*(31*d - 50*e + 7
6*f)**3 - 863493856*d*e**5 + 3068196000*d*e**4*f - 125194140*d*e**4*(31*d -
 50*e + 76*f) + 901203840*d*e**3*f**2 + 748035072*d*e**3*f*(31*d - 50*e + 7
6*f) + 264184*d*e**3*(31*d - 50*e + 76*f)**2 - 16302840960*d*e**2*f**3 - 16
69589712*d*e**2*f**2*(31*d - 50*e + 76*f) + 248184*d*e**2*f*(31*d - 50*e +
76*f)**2 + 29385*d*e**2*(31*d - 50*e + 76*f)**3 + 23260904960*d*e*f**4 + 16
49980992*d*e*f**3*(31*d - 50*e + 76*f) - 2477856*d*e*f**2*(31*d - 50*e + 76
*f)**2 - 89268*d*e*f*(31*d - 50*e + 76*f)**3 - 10226813952*d*f**5 - 6091557
12*d*f**4*(31*d - 50*e + 76*f) + 2252352*d*f**3*(31*d - 50*e + 76*f)**2 + 6
6312*d*f**2*(31*d - 50*e + 76*f)**3 + 429000000*e**6 - 2689492288*e**5*f +
42310728*e**5*(31*d - 50*e + 76*f) + 5722012800*e**4*f**2 - 318982608*e**4*
f*(31*d - 50*e + 76*f) - 283632*e**4*(31*d - 50*e + 76*f)**2 - 2792459520*e
**3*f**3 + 957569184*e**3*f**2*(31*d - 50*e + 76*f) + 869872*e**3*f*(31*d -
 50*e + 76*f)**2 - 15066*e**3*(31*d - 50*e + 76*f)**3 - 6487841280*e**2*f**
4 - 1431326016*e**2*f**3*(31*d - 50*e + 76*f) - 94752*e**2*f**2*(31*d - 50*
e + 76*f)**2 + 72468*e**2*f*(31*d - 50*e + 76*f)**3 + 9596336128*e*f**5 + 1
065568512*e*f**4*(31*d - 50*e + 76*f) - 1733248*e*f**3*(31*d - 50*e + 76*f)
**2 - 113232*e*f**2*(31*d - 50*e + 76*f)**3 - 3803258880*f**6 - 316108800*f
**5*(31*d - 50*e + 76*f) + 1313280*f**4*(31*d - 50*e + 76*f)**2 + 57600*f**
3*(31*d - 50*e + 76*f)**3)/(13474125*d**6 - 102860175*d**5*e + 196999875*d*
*5*f + 274190390*d**4*e**2 - 1214801310*d**4*e*f + 1192261140*d**4*f**2 - 2
24142072*d**3*e**3 + 2417766776*d**3*e**2*f - 5675135904*d**3*e*f**2 + 3821
016960*d**3*f**3 - 245084096*d**2*e**4 - 1010456304*d**2*e**3*f + 775246742
4*d**2*e**2*f**2 - 13083849984*d**2*e*f**3 + 6834458880*d**2*f**4 + 5357974
56*d*e**5 - 1875889936*d*e**4*f - 758531712*d*e**3*f**2 + 10575845888*d*e**
2*f**3 - 14846923776*d*e*f**4 + 6462996480*d*f**5 - 256183200*e**6 + 157970
8320*e**5*f - 3185673920*e**4*f**2 + 909173760*e**3*f**3 + 5054735360*e**2*
f**4 - 6608977920*e*f**5 + 2521497600*f**6))/144 - (5*d - 6*e + 8*f + x*(3*
d - 4*e + 6*f))/(12*x**2 + 36*x + 24)
```

Giac [A] time = 1.09447, size = 136, normalized size = 1.3

$$\frac{1}{144}\left(31\,d+76\,f-50\,e\right)\log\left(|x+2|\right)-\frac{1}{36}\left(7\,d+19\,f-13\,e\right)\log\left(|x+1|\right)-\frac{1}{36}\left(d+f+e\right)\log\left(|x-1|\right)+\frac{1}{144}\left(d+4\,f+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(31*d + 76*f - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19*f - 13*e)*log(a
bs(x + 1)) - 1/36*(d + f + e)*log(abs(x - 1)) + 1/144*(d + 4*f + 2*e)*log(a
bs(x - 2)) - 1/12*((3*d + 6*f - 4*e)*x + 5*d + 8*f - 6*e)/((x + 2)*(x + 1))
```

3.94 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=117

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d$$

```
[Out] -(d - e + f - g)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e
 + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/144 - ((7*d -
 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g)*Log[2 +
 x])/144
```

Rubi [A] time = 0.245746, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1586, 6728}

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] -(d - e + f - g)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e
 + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/144 - ((7*d -
 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g)*Log[2 +
 x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g}{144(-2+x)} + \frac{-d-e-f-g}{36(-1+x)} + \frac{d-e+f-g}{6(1+x)^2} + \frac{-7d+13e-19f}{36(1+x)}\right.$$
$$= -\frac{d-e+f-g}{6(1+x)} - \frac{d-2e+4f-8g}{12(2+x)} - \frac{1}{36}(d+e+f+g)\log(1-x) + \frac{1}{144}(d$$

Mathematica [A] time = 0.0599166, size = 114, normalized size = 0.97

$$\frac{1}{144}\left(\frac{12(-3dx-5d+4ex+6e-6fx-8f+10gx+12g)}{x^2+3x+2} - 4\log(1-x)(d+e+f+g) + \log(2-x)(d+2e+4f+8g)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x
]
```

```
[Out] ((12*(-5*d + 6*e - 8*f + 12*g - 3*d*x + 4*e*x - 6*f*x + 10*g*x))/(2 + 3*x +
 x^2) - 4*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x] + 4
*(-7*d + 13*e - 19*f + 25*g)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g)*Log[
2 + x])/144
```

Maple [A] time = 0.013, size = 178, normalized size = 1.5

$$-\frac{d}{24+12x} + \frac{e}{12+6x} - \frac{f}{6+3x} + \frac{2g}{6+3x} + \frac{31\ln(2+x)d}{144} - \frac{25\ln(2+x)e}{72} + \frac{19\ln(2+x)f}{36} - \frac{13\ln(2+x)g}{18} - \frac{7\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] -1/12/(2+x)*d+1/6/(2+x)*e-1/3/(2+x)*f+2/3/(2+x)*g+31/144*ln(2+x)*d-25/72*ln
(2+x)*e+19/36*ln(2+x)*f-13/18*ln(2+x)*g-7/36*ln(1+x)*d+13/36*ln(1+x)*e-19/3
6*ln(1+x)*f+25/36*ln(1+x)*g-1/6/(1+x)*d+1/6/(1+x)*e-1/6/(1+x)*f+1/6/(1+x)*g
+1/144*ln(x-2)*d+1/72*ln(x-2)*e+1/36*ln(x-2)*f+1/18*ln(x-2)*g-1/36*ln(x-1)*
d-1/36*ln(x-1)*e-1/36*ln(x-1)*f-1/36*ln(x-1)*g
```

Maxima [A] time = 0.94504, size = 144, normalized size = 1.23

$$\frac{1}{144}\left(31\,d-50\,e+76\,f-104\,g\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d-13\,e+19\,f-25\,g\right)\log\left(x+1\right)-\frac{1}{36}\left(d+e+f+g\right)\log\left(x-1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g)*log(x + 2) - 1/36*(7*d - 13*e + 19*f - 2
5*g)*log(x + 1) - 1/36*(d + e + f + g)*log(x - 1) + 1/144*(d + 2*e + 4*f +
8*g)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10*g)*x + 5*d - 6*e + 8*f - 12*g
)/(x^2 + 3*x + 2)
```

Fricas [B] time = 7.25608, size = 655, normalized size = 5.6

$$-\frac{12\left(3\,d-4\,e+6\,f-10\,g\right)x-\left(\left(31\,d-50\,e+76\,f-104\,g\right)x^{2}+3\left(31\,d-50\,e+76\,f-104\,g\right)x+62\,d-100\,e+152\,f-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g)*x - ((31*d - 50*e + 76*f - 104*g)*x^2 +
 3*(31*d - 50*e + 76*f - 104*g)*x + 62*d - 100*e + 152*f - 208*g)*log(x + 2
) + 4*((7*d - 13*e + 19*f - 25*g)*x^2 + 3*(7*d - 13*e + 19*f - 25*g)*x + 14
*d - 26*e + 38*f - 50*g)*log(x + 1) + 4*((d + e + f + g)*x^2 + 3*(d + e + f
 + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - ((d + 2*e + 4*f + 8*g)*x^2 +
3*(d + 2*e + 4*f + 8*g)*x + 2*d + 4*e + 8*f + 16*g)*log(x - 2) + 60*d - 72*
e + 96*f - 144*g)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10134, size = 158, normalized size = 1.35

$$\frac{1}{144}\left(31\,d+76\,f-104\,g-50\,e\right)\log\left(|x+2|\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g-13\,e\right)\log\left(|x+1|\right)-\frac{1}{36}\left(d+f+g+e\right)\log\left(|x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/144*(31*d + 76*f - 104*g - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19*f - 25*
g - 13*e)*log(abs(x + 1)) - 1/36*(d + f + g + e)*log(abs(x - 1)) + 1/144*(d
 + 4*f + 8*g + 2*e)*log(abs(x - 2)) - 1/12*((3*d + 6*f - 10*g - 4*e)*x + 5*
d + 8*f - 12*g - 6*e)/((x + 2)*(x + 1))
```

3.95 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=131

$$-\frac{d-e+f-g+h}{6(x+1)}-\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{36}\log(1-x)(d+e+f+g+h)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h$$

```
[Out] -(d - e + f - g + h)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)
) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g + 16*h)*Log
[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x])/36 + ((31*d -
 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Rubi [A] time = 0.28044, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {1586, 6728}

$$-\frac{d-e+f-g+h}{6(x+1)}-\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{36}\log(1-x)(d+e+f+g+h)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2
,x]
```

```
[Out] -(d - e + f - g + h)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)
) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g + 16*h)*Log
[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x])/36 + ((31*d -
 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g+16h}{144(-2+x)} + \frac{-d-e-f-g-h}{36(-1+x)} + \frac{d-e+f-g+h}{6(1+x)^2}\right.$$
$$= -\frac{d-e+f-g+h}{6(1+x)} - \frac{d-2e+4f-8g+16h}{12(2+x)} - \frac{1}{36}(d+e+f+g+h$$

Mathematica [A] time = 0.0694655, size = 136, normalized size = 1.04

$$\frac{1}{144}\left(-\frac{12(d(3x+5)+2(-e(2x+3)+3fx+4f-5gx-6g+9hx+10h))}{x^2+3x+2} - 4\log(1-x)(d+e+f+g+h) + \log(2-x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] ((-12*(d*(5 + 3*x) + 2*(4*f - 6*g + 10*h + 3*f*x - 5*g*x + 9*h*x - e*(3 + 2
*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2*f
 + 4*g + 8*h))*Log[2 - x] - 4*(7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x]
+ (31*d - 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Maple [A] time = 0.015, size = 222, normalized size = 1.7

$$\frac{31\ln(2+x)d}{144} - \frac{25\ln(2+x)e}{72} - \frac{7\ln(1+x)d}{36} + \frac{13\ln(1+x)e}{36} + \frac{\ln(x-2)d}{144} + \frac{\ln(x-2)e}{72} - \frac{\ln(x-1)d}{36} - \frac{\ln(x-1}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 31/144*ln(2+x)*d-25/72*ln(2+x)*e-7/36*ln(1+x)*d+13/36*ln(1+x)*e+1/144*ln(x-
2)*d+1/72*ln(x-2)*e-1/36*ln(x-1)*d-1/36*ln(x-1)*e-1/6/(1+x)*h-4/3/(2+x)*h+2
/3/(2+x)*g-1/6/(1+x)*d+1/6/(1+x)*e-1/12/(2+x)*d+1/6/(2+x)*e+1/6/(1+x)*g-1/6
/(1+x)*f-1/3/(2+x)*f-13/18*ln(2+x)*g+25/36*ln(1+x)*g+1/18*ln(x-2)*g-1/36*ln
(x-1)*g+7/9*ln(2+x)*h-31/36*ln(1+x)*h+1/9*ln(x-2)*h-1/36*ln(x-1)*h+1/36*ln(
```

```
x-2)*f-1/36*ln(x-1)*f+19/36*ln(2+x)*f-19/36*ln(1+x)*f
```

Maxima [A] time = 0.957816, size = 166, normalized size = 1.27

$$\frac{1}{144}\left(31\,d-50\,e+76\,f-104\,g+112\,h\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d-13\,e+19\,f-25\,g+31\,h\right)\log\left(x+1\right)-\frac{1}{36}\left(d+e+f+g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="maxima")
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g + 112*h)*log(x + 2) - 1/36*(7*d - 13*e +
19*f - 25*g + 31*h)*log(x + 1) - 1/36*(d + e + f + g + h)*log(x - 1) + 1/14
4*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2) - 1/12*((3*d - 4*e + 6*f - 10*g +
 18*h)*x + 5*d - 6*e + 8*f - 12*g + 20*h)/(x^2 + 3*x + 2)
```

Fricas [B] time = 34.4963, size = 783, normalized size = 5.98

$$-\frac{12\left(3\,d-4\,e+6\,f-10\,g+18\,h\right)x-\left(\left(31\,d-50\,e+76\,f-104\,g+112\,h\right)x^{2}+3\left(31\,d-50\,e+76\,f-104\,g+112\,h\right)x+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h)*x - ((31*d - 50*e + 76*f - 104*g
 + 112*h)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h)*x + 62*d - 100*e + 1
52*f - 208*g + 224*h)*log(x + 2) + 4*((7*d - 13*e + 19*f - 25*g + 31*h)*x^2
 + 3*(7*d - 13*e + 19*f - 25*g + 31*h)*x + 14*d - 26*e + 38*f - 50*g + 62*h
)*log(x + 1) + 4*((d + e + f + g + h)*x^2 + 3*(d + e + f + g + h)*x + 2*d +
 2*e + 2*f + 2*g + 2*h)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h)*x^2 + 3*
(d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)*log(x - 2)
+ 60*d - 72*e + 96*f - 144*g + 240*h)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08553, size = 180, normalized size = 1.37

$$\frac{1}{144}\left(31\,d + 76\,f - 104\,g + 112\,h - 50\,e\right)\log\left(|x+2|\right) - \frac{1}{36}\left(7\,d + 19\,f - 25\,g + 31\,h - 13\,e\right)\log\left(|x+1|\right) - \frac{1}{36}\left(d + f + g\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="giac")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19
*f - 25*g + 31*h - 13*e)*log(abs(x + 1)) - 1/36*(d + f + g + h + e)*log(abs
(x - 1)) + 1/144*(d + 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2)) - 1/12*((3*d
+ 6*f - 10*g + 18*h - 4*e)*x + 5*d + 8*f - 12*g + 20*h - 6*e)/((x + 2)*(x +
 1))
```

3.96 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4$$

```
[Out] -(d - e + f - g + h - i)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h - 32*i)/
(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36 + ((d + 2*e + 4*f +
8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h - 37
*i)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*Log[2 + x
])/144
```

Rubi [A] time = 0.329545, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1586, 6728}

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
+ x^4)^2,x]
```


```
[Out] -(d - e + f - g + h - i)/(6*(1 + x)) - (d - 2*e + 4*f - 8*g + 16*h - 32*i)/
(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36 + ((d + 2*e + 4*f +
8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h - 37
*i)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*Log[2 + x
])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(2-3x+x^2\right)\left(d+ex+fx^2+gx^3+hx^4+96x^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+96x^5}{\left(2-3x+x^2\right)\left(2+3x+x^2\right)^2}\,dx$$
$$= \int \left(\frac{3072+d+2e+4f+8g+16h}{144(-2+x)} + \frac{-96-d-e-f-g-h}{36(-1+x)}\right.$$
$$= \frac{96-d+e-f+g-h}{6(1+x)} + \frac{3072-d+2e-4f+8g-16h}{12(2+x)} - \frac{1}{36}$$

Mathematica [A] time = 0.0920893, size = 153, normalized size = 1.04

$$\frac{1}{144}\left(\frac{12(2(e(2x+3)-3fx-4f+5gx+6g-9hx-10h+17ix+18i)-d(3x+5))}{x^2+3x+2} - 4\log(1-x)(d+e+f+g+h+i\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-(d*(5 + 3*x)) + 2*(-4*f + 6*g - 10*h + 18*i - 3*f*x + 5*g*x - 9*h*x
+ 17*i*x + e*(3 + 2*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h + i)*Log[1
 - x] + (d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + 4*(-7*d + 13*e - 1
9*f + 25*g - 31*h + 37*i)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g + 112*h
- 32*i)*Log[2 + x])/144
```

Maple [A] time = 0.013, size = 266, normalized size = 1.8

$$\frac{31\ln(2+x)d}{144} - \frac{25\ln(2+x)e}{72} - \frac{7\ln(1+x)d}{36} + \frac{13\ln(1+x)e}{36} + \frac{\ln(x-2)d}{144} + \frac{\ln(x-2)e}{72} - \frac{\ln(x-1)d}{36} - \frac{\ln(x-1}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 31/144*ln(2+x)*d-25/72*ln(2+x)*e-7/36*ln(1+x)*d+13/36*ln(1+x)*e+1/144*ln(x-
2)*d+1/72*ln(x-2)*e-1/36*ln(x-1)*d-1/36*ln(x-1)*e+1/6/(1+x)*i+8/3/(2+x)*i-1
```

```
/6/(1+x)*h-4/3/(2+x)*h+2/3/(2+x)*g-1/6/(1+x)*d+1/6/(1+x)*e-1/12/(2+x)*d+1/6
/(2+x)*e+1/6/(1+x)*g-1/6/(1+x)*f-1/3/(2+x)*f+2/9*ln(x-2)*i-1/36*ln(x-1)*i-2
/9*ln(2+x)*i+37/36*ln(1+x)*i-13/18*ln(2+x)*g+25/36*ln(1+x)*g+1/18*ln(x-2)*g
-1/36*ln(x-1)*g+7/9*ln(2+x)*h-31/36*ln(1+x)*h+1/9*ln(x-2)*h-1/36*ln(x-1)*h+
1/36*ln(x-2)*f-1/36*ln(x-1)*f+19/36*ln(2+x)*f-19/36*ln(1+x)*f
```

Maxima [A] time = 0.96922, size = 188, normalized size = 1.28

$$\frac{1}{144}\left(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i\right)\log\left(x+1\right)-\frac{1}{36}\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="maxima")
```

```
[Out] 1/144*(31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*log(x + 2) - 1/36*(7*d -
13*e + 19*f - 25*g + 31*h - 37*i)*log(x + 1) - 1/36*(d + e + f + g + h + i)
*log(x - 1) + 1/144*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2) - 1/12*(
(3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x + 5*d - 6*e + 8*f - 12*g + 20*h -
36*i)/(x^2 + 3*x + 2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)*
*2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12725, size = 201, normalized size = 1.37

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-32\,i-50\,e\right)\log\left(|x+2|\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-37\,i-13\,e\right)\log\left(|x+1|\right)-\frac{1}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="giac")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 32*i - 50*e)*log(abs(x + 2)) - 1/36*(7
*d + 19*f - 25*g + 31*h - 37*i - 13*e)*log(abs(x + 1)) - 1/36*(d + f + g +
h + i + e)*log(abs(x - 1)) + 1/144*(d + 4*f + 8*g + 16*h + 32*i + 2*e)*log(
abs(x - 2)) - 1/12*((3*d + 6*f - 10*g + 18*h - 34*i - 4*e)*x + 5*d + 8*f -
12*g + 20*h - 36*i - 6*e)/((x + 2)*(x + 1))
```

3.97 $\int \frac{2+x}{\left(4-5x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=68

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

```
[Out] 1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18 - (35*Log[
2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Rubi [A] time = 0.057872, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1586, 2074}

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18 - (35*Log[
2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{2+x}{\left(4-5x^2+x^4\right)^2} dx = \int \frac{1}{(2+x)\left(2-x-2x^2+x^3\right)^2} dx$$
$$= \int \left(\frac{1}{36(-2+x)^2} - \frac{35}{432(-2+x)} + \frac{1}{12(-1+x)^2} + \frac{1}{18(-1+x)} + \frac{1}{36(1+x)^2} + \frac{1}{54(1+x)} + \frac{1}{144(\right.$$
$$= \frac{1}{12(1-x)} + \frac{1}{36(2-x)} - \frac{1}{36(1+x)} + \frac{1}{18}\log(1-x) - \frac{35}{432}\log(2-x) + \frac{1}{54}\log(1+x) + \frac{1}{144}\log$$

Mathematica [A] time = 0.0288866, size = 60, normalized size = 0.88

$$\frac{1}{432}\left(\frac{12\left(-5x^2+6x+5\right)}{x^3-2x^2-x+2} + 24\log(1-x) - 35\log(2-x) + 8\log(x+1) + 3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(5 + 6*x - 5*x^2))/(2 - x - 2*x^2 + x^3) + 24*Log[1 - x] - 35*Log[2 -
x] + 8*Log[1 + x] + 3*Log[2 + x])/432
```

Maple [A] time = 0.013, size = 47, normalized size = 0.7

$$\frac{\ln(2+x)}{144} - \frac{1}{36+36x} + \frac{\ln(1+x)}{54} - \frac{1}{36x-72} - \frac{35\ln(x-2)}{432} - \frac{1}{12x-12} + \frac{\ln(x-1)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)-1/36/(1+x)+1/54*ln(1+x)-1/36/(x-2)-35/432*ln(x-2)-1/12/(x-1)+
1/18*ln(x-1)
```

Maxima [A] time = 0.95761, size = 70, normalized size = 1.03

$$-\frac{5x^2-6x-5}{36\left(x^3-2x^2-x+2\right)} + \frac{1}{144}\log(x+2) + \frac{1}{54}\log(x+1) + \frac{1}{18}\log(x-1) - \frac{35}{432}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/(x^3 - 2*x^2 - x + 2) + 1/144*log(x + 2) + 1/54*log
(x + 1) + 1/18*log(x - 1) - 35/432*log(x - 2)
```

Fricas [B] time = 1.73935, size = 271, normalized size = 3.99

$$-\frac{60\,x^2 - 3\left(x^3 - 2\,x^2 - x + 2\right)\log\left(x + 2\right) - 8\left(x^3 - 2\,x^2 - x + 2\right)\log\left(x + 1\right) - 24\left(x^3 - 2\,x^2 - x + 2\right)\log\left(x - 1\right) + 35\left(x^3 - \right.}{432\left(x^3 - 2\,x^2 - x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(60*x^2 - 3*(x^3 - 2*x^2 - x + 2)*log(x + 2) - 8*(x^3 - 2*x^2 - x +
2)*log(x + 1) - 24*(x^3 - 2*x^2 - x + 2)*log(x - 1) + 35*(x^3 - 2*x^2 - x +
 2)*log(x - 2) - 72*x - 60)/(x^3 - 2*x^2 - x + 2)
```

Sympy [A] time = 0.279434, size = 53, normalized size = 0.78

$$-\frac{5x^2 - 6x - 5}{36x^3 - 72x^2 - 36x + 72} - \frac{35\log\left(x - 2\right)}{432} + \frac{\log\left(x - 1\right)}{18} + \frac{\log\left(x + 1\right)}{54} + \frac{\log\left(x + 2\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(5*x**2 - 6*x - 5)/(36*x**3 - 72*x**2 - 36*x + 72) - 35*log(x - 2)/432 + l
og(x - 1)/18 + log(x + 1)/54 + log(x + 2)/144
```

Giac [A] time = 1.10012, size = 76, normalized size = 1.12

$$-\frac{5\,x^2 - 6\,x - 5}{36\,(x + 1)(x - 1)(x - 2)} + \frac{1}{144}\,\log\left(|x + 2|\right) + \frac{1}{54}\,\log\left(|x + 1|\right) + \frac{1}{18}\,\log\left(|x - 1|\right) - \frac{35}{432}\,\log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/((x + 1)*(x - 1)*(x - 2)) + 1/144*log(abs(x + 2)) +
 1/54*log(abs(x + 1)) + 1/18*log(abs(x - 1)) - 35/432*log(abs(x - 2))
```

3.98 $\int \frac{(2+x)(d+ex)}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=105

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{14}$$

```
[Out] (d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 + x)) + ((2*
d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/432 + ((2*d + e)*Log[1
 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Rubi [A] time = 0.196274, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1586, 6742}

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{14}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 + x)) + ((2*
d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/432 + ((2*d + e)*Log[1
 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)(d+ex)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e}{36(-2+x)^2} + \frac{-35d-58e}{432(-2+x)} + \frac{d+e}{12(-1+x)^2} + \frac{2d+5e}{36(-1+x)} + \frac{d-e}{36(1+x)^2} + \frac{2d+e}{108(1+x)} + \frac{d}{144}\right.$$
$$= \frac{d+e}{12(1-x)} + \frac{d+2e}{36(2-x)} - \frac{d-e}{36(1+x)} + \frac{1}{36}(2d+5e)\log(1-x) - \frac{1}{432}(35d+58e)\log(2-x) + \frac{1}{10}$$

Mathematica [A] time = 0.0871165, size = 97, normalized size = 0.92

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2e\left(5-2x^2\right)\right)}{x^3-2x^2-x+2} + 12(2d+5e)\log(1-x) - (35d+58e)\log(2-x) + 4(2d+e)\log(x+1) + 3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*e*(5 - 2*x^2)))/(2 - x - 2*x^2 + x^3) + 12*(2
*d + 5*e)*Log[1 - x] - (35*d + 58*e)*Log[2 - x] + 4*(2*d + e)*Log[1 + x] +
3*(d - 2*e)*Log[2 + x])/432
```

Maple [A] time = 0.014, size = 106, normalized size = 1.

$$\frac{\ln(2+x)\,d}{144} - \frac{\ln(2+x)\,e}{72} - \frac{d}{36+36\,x} + \frac{e}{36+36\,x} + \frac{\ln(1+x)\,d}{54} + \frac{\ln(1+x)\,e}{108} - \frac{35\,\ln(x-2)\,d}{432} - \frac{29\,\ln(x-2)\,e}{216} - \frac{}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e-1/36/(1+x)*d+1/36/(1+x)*e+1/54*ln(1+x)*d+1/1
08*ln(1+x)*e-35/432*ln(x-2)*d-29/216*ln(x-2)*e-1/36/(x-2)*d-1/18/(x-2)*e-1/
12/(x-1)*d-1/12/(x-1)*e+1/18*ln(x-1)*d+5/36*ln(x-1)*e
```

Maxima [A] time = 0.956801, size = 119, normalized size = 1.13

$$\frac{1}{144}(d-2e)\log(x+2) + \frac{1}{108}(2d+e)\log(x+1) + \frac{1}{36}(2d+5e)\log(x-1) - \frac{1}{432}(35d+58e)\log(x-2) - \frac{(5d+4}{36(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e)*log(x + 2) + 1/108*(2*d + e)*log(x + 1) + 1/36*(2*d + 5*e)*
log(x - 1) - 1/432*(35*d + 58*e)*log(x - 2) - 1/36*((5*d + 4*e)*x^2 - 6*d*x
 - 5*d - 10*e)/(x^3 - 2*x^2 - x + 2)
```

Fricas [B] time = 1.84778, size = 535, normalized size = 5.1

$$-\frac{12\,(5\,d+4\,e)x^2-72\,dx-3\left((d-2\,e)x^3-2\,(d-2\,e)x^2-(d-2\,e)x+2\,d-4\,e\right)\log\left(x+2\right)-4\left((2\,d+e)x^3-2\,(2\,d+e)x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e)*x^2 - 72*d*x - 3*((d - 2*e)*x^3 - 2*(d - 2*e)*x^2 -
(d - 2*e)*x + 2*d - 4*e)*log(x + 2) - 4*((2*d + e)*x^3 - 2*(2*d + e)*x^2 -
(2*d + e)*x + 4*d + 2*e)*log(x + 1) - 12*((2*d + 5*e)*x^3 - 2*(2*d + 5*e)*x
^2 - (2*d + 5*e)*x + 4*d + 10*e)*log(x - 1) + ((35*d + 58*e)*x^3 - 2*(35*d
+ 58*e)*x^2 - (35*d + 58*e)*x + 70*d + 116*e)*log(x - 2) - 60*d - 120*e)/(x
^3 - 2*x^2 - x + 2)
```

Sympy [B] time = 5.59425, size = 1032, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*(d - 2*e)/
4 + 364910432*d**3*e**2 - 18128055*d**3*e*(d - 2*e) - 83772*d**3*(d - 2*e)*
*2 + 686697536*d**2*e**3 - 60296868*d**2*e**2*(d - 2*e) - 597816*d**2*e*(d
- 2*e)**2 + 65907*d**2*(d - 2*e)**3/4 + 614357568*d*e**4 - 85949220*d*e**3*
(d - 2*e) - 1500048*d*e**2*(d - 2*e)**2 + 105840*d*e*(d - 2*e)**3 + 2084704
00*e**5 - 45136356*e**4*(d - 2*e) - 1196064*e**3*(d - 2*e)**2 + 128277*e**2
*(d - 2*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 3620
```

```
61760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/144 + (2*d + e)*log(x
 + (8710660*d**5 + 91884504*d**4*e - 2526593*d**4*(2*d + e) + 364910432*d**
3*e**2 - 24170740*d**3*e*(2*d + e) - 148928*d**3*(2*d + e)**2 + 686697536*d
**2*e**3 - 80395824*d**2*e**2*(2*d + e) - 1062784*d**2*e*(2*d + e)**2 + 390
56*d**2*(2*d + e)**3 + 614357568*d*e**4 - 114598960*d*e**3*(2*d + e) - 2666
752*d*e**2*(2*d + e)**2 + 250880*d*e*(2*d + e)**3 + 208470400*e**5 - 601818
08*e**4*(2*d + e) - 2126336*e**3*(2*d + e)**2 + 304064*e**2*(2*d + e)**3)/(
3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 362061760*d**2*e**3
+ 370298160*d*e**4 + 146466320*e**5))/108 + (2*d + 5*e)*log(x + (8710660*d*
*5 + 91884504*d**4*e - 7579779*d**4*(2*d + 5*e) + 364910432*d**3*e**2 - 725
12220*d**3*e*(2*d + 5*e) - 1340352*d**3*(2*d + 5*e)**2 + 686697536*d**2*e**
3 - 241187472*d**2*e**2*(2*d + 5*e) - 9565056*d**2*e*(2*d + 5*e)**2 + 10545
12*d**2*(2*d + 5*e)**3 + 614357568*d*e**4 - 343796880*d*e**3*(2*d + 5*e) -
24000768*d*e**2*(2*d + 5*e)**2 + 6773760*d*e*(2*d + 5*e)**3 + 208470400*e**
5 - 180545424*e**4*(2*d + 5*e) - 19137024*e**3*(2*d + 5*e)**2 + 8209728*e**
2*(2*d + 5*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 3
62061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/36 - (35*d + 58*e)
*log(x + (8710660*d**5 + 91884504*d**4*e + 2526593*d**4*(35*d + 58*e)/4 + 3
64910432*d**3*e**2 + 6042685*d**3*e*(35*d + 58*e) - 9308*d**3*(35*d + 58*e)
**2 + 686697536*d**2*e**3 + 20098956*d**2*e**2*(35*d + 58*e) - 66424*d**2*e
*(35*d + 58*e)**2 - 2441*d**2*(35*d + 58*e)**3/4 + 614357568*d*e**4 + 28649
740*d*e**3*(35*d + 58*e) - 166672*d*e**2*(35*d + 58*e)**2 - 3920*d*e*(35*d
+ 58*e)**3 + 208470400*e**5 + 15045452*e**4*(35*d + 58*e) - 132896*e**3*(35
*d + 58*e)**2 - 4751*e**2*(35*d + 58*e)**3)/(3374210*d**5 + 38645295*d**4*e
 + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320
*e**5))/432 - (-6*d*x - 5*d - 10*e + x**2*(5*d + 4*e))/(36*x**3 - 72*x**2 -
 36*x + 72)
```

Giac [A] time = 1.10073, size = 132, normalized size = 1.26

$$\frac{1}{144}(d-2e)\log(|x+2|)+\frac{1}{108}(2d+e)\log(|x+1|)+\frac{1}{36}(2d+5e)\log(|x-1|)-\frac{1}{432}(35d+58e)\log(|x-2|)-\frac{(5d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d - 2*e)*log(abs(x + 2)) + 1/108*(2*d + e)*log(abs(x + 1)) + 1/36*(2
*d + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 58*e)*log(abs(x - 2)) - 1/36*((5*
d + 4*e)*x^2 - 6*d*x - 5*d - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

3.99 $\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{d-e+f}{36(x+1)}+\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f)-\frac{1}{432}\log(2-x)(35d+58e+92f)+\frac{1}{108}\log(x+$$

```
[Out] (d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e + f)/(36*(
1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d + 58*e + 92*f)*Log[2 -
 x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 + ((d - 2*e + 4*f)*Log[2 + x])/
144
```

Rubi [A] time = 0.221716, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1586, 6742}

$$-\frac{d-e+f}{36(x+1)}+\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f)-\frac{1}{432}\log(2-x)(35d+58e+92f)+\frac{1}{108}\log(x+$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] (d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e + f)/(36*(
1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d + 58*e + 92*f)*Log[2 -
 x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 + ((d - 2*e + 4*f)*Log[2 + x])/
144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f}{36(-2+x)^2} + \frac{-35d-58e-92f}{432(-2+x)} + \frac{d+e+f}{12(-1+x)^2} + \frac{2d+5e+8f}{36(-1+x)} + \frac{d-e+f}{36(1+x)^2} + \frac{2d}{10}\right.$$
$$= \frac{d+e+f}{12(1-x)} + \frac{d+2e+4f}{36(2-x)} - \frac{d-e+f}{36(1+x)} + \frac{1}{36}(2d+5e+8f)\log(1-x) - \frac{1}{432}(35d+58e+$$

Mathematica [A] time = 0.0538168, size = 121, normalized size = 0.99

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+e\left(10-4x^2\right)+2f\left(-4x^2+3x+4\right)\right)}{x^3-2x^2-x+2}+12\log(1-x)(2d+5e+8f)-\log(2-x)(35d+58e\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + e*(10 - 4*x^2) + 2*f*(4 + 3*x - 4*x^2)))/(2 - x
 - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f)*Log[1 - x] - (35*d + 58*e + 92*f)*Lo
g[2 - x] + 4*(2*d + e - 4*f)*Log[1 + x] + 3*(d - 2*e + 4*f)*Log[2 + x])/432
```

Maple [A] time = 0.015, size = 158, normalized size = 1.3

$$\frac{\ln(2+x)\,d}{144} - \frac{\ln(2+x)\,e}{72} + \frac{\ln(2+x)\,f}{36} - \frac{d}{36+36\,x} + \frac{e}{36+36\,x} - \frac{f}{36+36\,x} + \frac{\ln(1+x)\,d}{54} + \frac{\ln(1+x)\,e}{108} - \frac{\ln(1+x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e+1/36*ln(2+x)*f-1/36/(1+x)*d+1/36/(1+x)*e-1/3
6/(1+x)*f+1/54*ln(1+x)*d+1/108*ln(1+x)*e-1/27*ln(1+x)*f-35/432*ln(x-2)*d-29
/216*ln(x-2)*e-23/108*ln(x-2)*f-1/36/(x-2)*d-1/18/(x-2)*e-1/9/(x-2)*f-1/12/
(x-1)*d-1/12/(x-1)*e-1/12/(x-1)*f+1/18*ln(x-1)*d+5/36*ln(x-1)*e+2/9*ln(x-1)
*f
```

Maxima [A] time = 0.965564, size = 146, normalized size = 1.2

$$\frac{1}{144}\left(d-2e+4f\right)\log\left(x+2\right)+\frac{1}{108}\left(2d+e-4f\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+5e+8f\right)\log\left(x-1\right)-\frac{1}{432}\left(35d+58e+92\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e + 4*f)*log(x + 2) + 1/108*(2*d + e - 4*f)*log(x + 1) + 1/36*
(2*d + 5*e + 8*f)*log(x - 1) - 1/432*(35*d + 58*e + 92*f)*log(x - 2) - 1/36
*((5*d + 4*e + 8*f)*x^2 - 6*(d + f)*x - 5*d - 10*e - 8*f)/(x^3 - 2*x^2 - x
+ 2)
```

Fricas [B] time = 2.15583, size = 698, normalized size = 5.72

$$-\frac{12\left(5d+4e+8f\right)x^2-72\left(d+f\right)x-3\left(\left(d-2e+4f\right)x^3-2\left(d-2e+4f\right)x^2-\left(d-2e+4f\right)x+2d-4e+8f\right)\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f)*x^3 -
2*(d - 2*e + 4*f)*x^2 - (d - 2*e + 4*f)*x + 2*d - 4*e + 8*f)*log(x + 2) - 4
*((2*d + e - 4*f)*x^3 - 2*(2*d + e - 4*f)*x^2 - (2*d + e - 4*f)*x + 4*d + 2
*e - 8*f)*log(x + 1) - 12*((2*d + 5*e + 8*f)*x^3 - 2*(2*d + 5*e + 8*f)*x^2
- (2*d + 5*e + 8*f)*x + 4*d + 10*e + 16*f)*log(x - 1) + ((35*d + 58*e + 92*
f)*x^3 - 2*(35*d + 58*e + 92*f)*x^2 - (35*d + 58*e + 92*f)*x + 70*d + 116*e
 + 184*f)*log(x - 2) - 60*d - 120*e - 96*f)/(x^3 - 2*x^2 - x + 2)
```

Sympy [B] time = 104.122, size = 5192, normalized size = 42.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e + 4*f)*log(x + (8710660*d**6 + 109305824*d**5*e + 136707258*d**5*f
 - 7579779*d**5*(d - 2*e + 4*f)/4 + 548679440*d**4*e**2 + 1278644860*d**4*e
*f - 43835889*d**4*e*(d - 2*e + 4*f)/2 + 627558840*d**4*f**2 - 40496817*d**
4*f*(d - 2*e + 4*f) - 83772*d**4*(d - 2*e + 4*f)**2 + 1416518400*d**3*e**3
+ 4598750960*d**3*e**2*f - 96552978*d**3*e**2*(d - 2*e + 4*f) + 3756616640*
d**3*e*f**2 - 337816392*d**3*e*f*(d - 2*e + 4*f) - 765360*d**3*e*(d - 2*e +
 4*f)**2 + 42632000*d**3*f**3 - 290155704*d**3*f**2*(d - 2*e + 4*f) - 11482
2*d**3*f*(d - 2*e + 4*f)**2 + 65907*d**3*(d - 2*e + 4*f)**3/4 + 1987752640*
d**2*e**4 + 7924847520*d**2*e**3*f - 206542956*d**2*e**3*(d - 2*e + 4*f) +
7373599680*d**2*e**2*f**2 - 1045445256*d**2*e**2*f*(d - 2*e + 4*f) - 269568
0*d**2*e**2*(d - 2*e + 4*f)**2 - 4656496000*d**2*e*f**3 - 1745170416*d**2*e
*f**2*(d - 2*e + 4*f) - 1702188*d**2*e*f*(d - 2*e + 4*f)**2 + 277587*d**2*e
*(d - 2*e + 4*f)**3/2 - 6902995200*d**2*f**4 - 963557664*d**2*f**3*(d - 2*e
 + 4*f) + 4475304*d**2*f**2*(d - 2*e + 4*f)**2 + 298323*d**2*f*(d - 2*e + 4
*f)**3 + 1437185536*d*e**5 + 6500930720*d*e**4*f - 217034796*d*e**4*(d - 2*
e + 4*f) + 4803912960*d*e**3*f**2 - 1422980448*d*e**3*f*(d - 2*e + 4*f) - 4
196160*d*e**3*(d - 2*e + 4*f)**2 - 17839947520*d*e**2*f**3 - 3475531872*d*e
**2*f**2*(d - 2*e + 4*f) - 5674536*d*e**2*f*(d - 2*e + 4*f)**2 + 339957*d*e
**2*(d - 2*e + 4*f)**3 - 34607006720*d*e*f**4 - 3750241920*d*e*f**3*(d - 2*
e + 4*f) + 15048288*d*e*f**2*(d - 2*e + 4*f)**2 + 1193940*d*e*f*(d - 2*e +
4*f)**3 - 17540771328*d*f**5 - 1508647872*d*f**4*(d - 2*e + 4*f) + 20676960
*d*f**3*(d - 2*e + 4*f)**2 + 1012500*d*f**2*(d - 2*e + 4*f)**3 + 416940800*
e**6 + 2005475776*e**5*f - 90272712*e**5*(d - 2*e + 4*f) + 122654080*e**4*f
**2 - 717642192*e**4*f*(d - 2*e + 4*f) - 2392128*e**4*(d - 2*e + 4*f)**2 -
16377853440*e**3*f**3 - 2270414784*e**3*f**2*(d - 2*e + 4*f) - 5251536*e**3
*f*(d - 2*e + 4*f)**2 + 256554*e**3*(d - 2*e + 4*f)**3 - 39387581440*e**2*f
**4 - 3567808896*e**2*f**3*(d - 2*e + 4*f) + 12299040*e**2*f**2*(d - 2*e +
4*f)**2 + 1124604*e**2*f*(d - 2*e + 4*f)**3 - 37694227456*e*f**5 - 27789446
40*e*f**4*(d - 2*e + 4*f) + 39067200*e*f**3*(d - 2*e + 4*f)**2 + 1575288*e*
f**2*(d - 2*e + 4*f)**3 - 13332408320*f**6 - 856423680*f**5*(d - 2*e + 4*f)
 + 25200000*f**4*(d - 2*e + 4*f)**2 + 704592*f**3*(d - 2*e + 4*f)**3)/(3374
210*d**6 + 45393715*d**5*e + 44170854*d**5*f + 247848970*d**4*e**2 + 464201
768*d**4*e*f + 91507752*d**4*f**2 + 703178520*d**3*e**3 + 1914915472*d**3*e
**2*f + 607100704*d**3*e*f**2 - 999338816*d**3*f**3 + 1094421680*d**2*e**4
+ 3892700544*d**2*e**3*f + 1545770112*d**2*e**2*f**2 - 6739384832*d**2*e*f*
*3 - 5963752704*d**2*f**4 + 887062640*d*e**5 + 3907683424*d*e**4*f + 188954
4576*d*e**3*f**2 - 14086786304*d*e**2*f**3 - 24613469440*d*e*f**4 - 1190036
7360*d*f**5 + 292932640*e**6 + 1550127488*e**5*f + 952305536*e**4*f**2 - 92
35000320*e**3*f**3 - 24236925440*e**2*f**4 - 23421052928*e*f**5 - 815490457
6*f**6))/144 + (2*d + e - 4*f)*log(x + (8710660*d**6 + 109305824*d**5*e + 1
36707258*d**5*f - 2526593*d**5*(2*d + e - 4*f) + 548679440*d**4*e**2 + 1278
644860*d**4*e*f - 29223926*d**4*e*(2*d + e - 4*f) + 627558840*d**4*f**2 - 5
3995756*d**4*f*(2*d + e - 4*f) - 148928*d**4*(2*d + e - 4*f)**2 + 141651840
0*d**3*e**3 + 4598750960*d**3*e**2*f - 128737304*d**3*e**2*(2*d + e - 4*f)
+ 3756616640*d**3*e*f**2 - 450421856*d**3*e*f*(2*d + e - 4*f) - 1360640*d**
```

3*e*(2*d + e - 4*f)**2 + 42632000*d**3*f**3 - 386874272*d**3*f**2*(2*d + e - 4*f) - 204128*d**3*f*(2*d + e - 4*f)**2 + 39056*d**3*(2*d + e - 4*f)**3 + 1987752640*d**2*e**4 + 7924847520*d**2*e**3*f - 275390608*d**2*e**3*(2*d + e - 4*f) + 7373599680*d**2*e**2*f**2 - 1393927008*d**2*e**2*f*(2*d + e - 4*f) - 4792320*d**2*e**2*(2*d + e - 4*f)**2 - 4656496000*d**2*e*f**3 - 2326893888*d**2*e*f**2*(2*d + e - 4*f) - 3026112*d**2*e*f*(2*d + e - 4*f)**2 + 328992*d**2*e*(2*d + e - 4*f)**3 - 6902995200*d**2*f**4 - 1284743552*d**2*f**3*(2*d + e - 4*f) + 7956096*d**2*f**2*(2*d + e - 4*f)**2 + 707136*d**2*f*(2*d + e - 4*f)**3 + 1437185536*d*e**5 + 6500930720*d*e**4*f - 289379728*d*e**4*(2*d + e - 4*f) + 4803912960*d*e**3*f**2 - 1897307264*d*e**3*f*(2*d + e - 4*f) - 7459840*d*e**3*(2*d + e - 4*f)**2 - 17839947520*d*e**2*f**3 - 4634042496*d*e**2*f**2*(2*d + e - 4*f) - 10088064*d*e**2*f*(2*d + e - 4*f)**2 + 805824*d*e**2*(2*d + e - 4*f)**3 - 34607006720*d*e*f**4 - 5000322560*d*e*f**3*(2*d + e - 4*f) + 26752512*d*e*f**2*(2*d + e - 4*f)**2 + 2830080*d*e*f*(2*d + e - 4*f)**3 - 17540771328*d*f**5 - 2011530496*d*f**4*(2*d + e - 4*f) + 36759040*d*f**3*(2*d + e - 4*f)**2 + 2400000*d*f**2*(2*d + e - 4*f)**3 + 416940800*e**6 + 2005475776*e**5*f - 120363616*e**5*(2*d + e - 4*f) + 122654080*e**4*f**2 - 956856256*e**4*f*(2*d + e - 4*f) - 4252672*e**4*(2*d + e - 4*f)**2 - 16377853440*e**3*f**3 - 3027219712*e**3*f**2*(2*d + e - 4*f) - 9336064*e**3*f*(2*d + e - 4*f)**2 + 608128*e**3*(2*d + e - 4*f)**3 - 39387581440*e**2*f**4 - 4757078528*e**2*f**3*(2*d + e - 4*f) + 21864960*e**2*f**2*(2*d + e - 4*f)**2 + 2665728*e**2*f*(2*d + e - 4*f)**3 - 37694227456*e*f**5 - 3705259520*e*f**4*(2*d + e - 4*f) + 69452800*e*f**3*(2*d + e - 4*f)**2 + 3734016*e*f**2*(2*d + e - 4*f)**3 - 13332408320*f**6 - 1141898240*f**5*(2*d + e - 4*f) + 44800000*f**4*(2*d + e - 4*f)**2 + 1670144*f**3*(2*d + e - 4*f)**3)/(3374210*d**6 + 45393715*d**5*e + 44170854*d**5*f + 247848970*d**4*e**2 + 464201768*d**4*e*f + 91507752*d**4*f**2 + 703178520*d**3*e**3 + 1914915472*d**3*e**2*f + 607100704*d**3*e*f**2 - 999338816*d**3*f**3 + 1094421680*d**2*e**4 + 3892700544*d**2*e**3*f + 1545770112*d**2*e**2*f**2 - 6739384832*d**2*e*f**3 - 5963752704*d**2*f**4 + 887062640*d*e**5 + 3907683424*d*e**4*f + 1889544576*d*e**3*f**2 - 14086786304*d*e**2*f**3 - 24613469440*d*e*f**4 - 11900367360*d*f**5 + 292932640*e**6 + 1550127488*e**5*f + 952305536*e**4*f**2 - 9235000320*e**3*f**3 - 24236925440*e**2*f**4 - 23421052928*e*f**5 - 8154904576*f**6))/108 + (2*d + 5*e + 8*f)*log(x + (8710660*d**6 + 109305824*d**5*e + 136707258*d**5*f - 7579779*d**5*(2*d + 5*e + 8*f) + 548679440*d**4*e**2 + 1278644860*d**4*e*f - 87671778*d**4*e*(2*d + 5*e + 8*f) + 627558840*d**4*f**2 - 161987268*d**4*f*(2*d + 5*e + 8*f) - 1340352*d**4*(2*d + 5*e + 8*f)**2 + 1416518400*d**3*e**3 + 4598750960*d**3*e**2*f - 386211912*d**3*e**2*(2*d + 5*e + 8*f) + 3756616640*d**3*e*f**2 - 1351265568*d**3*e*f*(2*d + 5*e + 8*f) - 12245760*d**3*e*(2*d + 5*e + 8*f)**2 + 42632000*d**3*f**3 - 1160622816*d**3*f**2*(2*d + 5*e + 8*f) - 1837152*d**3*f*(2*d + 5*e + 8*f)**2 + 1054512*d**3*(2*d + 5*e + 8*f)**3 + 1987752640*d**2*e**4 + 7924847520*d**2*e**3*f - 826171824*d**2*e**3*(2*d + 5*e + 8*f) + 7373599680*d**2*e**2*f**2 - 4181781024*d**2*e**2*f*(2*d + 5*e + 8*f) - 43130880*d**2*e**2*(2*d + 5*e + 8*f)**2 - 4656496000*d**2*e*f**3 - 6980681664*d**2*e*f**2*(2*d

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+ 5*e + 8*f) - 27235008*d**2*e*f*(2*d + 5*e + 8*f)**2 + 8882784*d**2*e*(2*d
 + 5*e + 8*f)**3 - 6902995200*d**2*f**4 - 3854230656*d**2*f**3*(2*d + 5*e +
 8*f) + 71604864*d**2*f**2*(2*d + 5*e + 8*f)**2 + 19092672*d**2*f*(2*d + 5*
e + 8*f)**3 + 1437185536*d*e**5 + 6500930720*d*e**4*f - 868139184*d*e**4*(2
*d + 5*e + 8*f) + 4803912960*d*e**3*f**2 - 5691921792*d*e**3*f*(2*d + 5*e +
 8*f) - 67138560*d*e**3*(2*d + 5*e + 8*f)**2 - 17839947520*d*e**2*f**3 - 13
902127488*d*e**2*f**2*(2*d + 5*e + 8*f) - 90792576*d*e**2*f*(2*d + 5*e + 8*
f)**2 + 21757248*d*e**2*(2*d + 5*e + 8*f)**3 - 34607006720*d*e*f**4 - 15000
967680*d*e*f**3*(2*d + 5*e + 8*f) + 240772608*d*e*f**2*(2*d + 5*e + 8*f)**2
 + 76412160*d*e*f*(2*d + 5*e + 8*f)**3 - 17540771328*d*f**5 - 6034591488*d*
f**4*(2*d + 5*e + 8*f) + 330831360*d*f**3*(2*d + 5*e + 8*f)**2 + 64800000*d
*f**2*(2*d + 5*e + 8*f)**3 + 416940800*e**6 + 2005475776*e**5*f - 361090848
*e**5*(2*d + 5*e + 8*f) + 122654080*e**4*f**2 - 2870568768*e**4*f*(2*d + 5*
e + 8*f) - 38274048*e**4*(2*d + 5*e + 8*f)**2 - 16377853440*e**3*f**3 - 908
1659136*e**3*f**2*(2*d + 5*e + 8*f) - 84024576*e**3*f*(2*d + 5*e + 8*f)**2
+ 16419456*e**3*(2*d + 5*e + 8*f)**3 - 39387581440*e**2*f**4 - 14271235584*
e**2*f**3*(2*d + 5*e + 8*f) + 196784640*e**2*f**2*(2*d + 5*e + 8*f)**2 + 71
974656*e**2*f*(2*d + 5*e + 8*f)**3 - 37694227456*e*f**5 - 11115778560*e*f**
4*(2*d + 5*e + 8*f) + 625075200*e*f**3*(2*d + 5*e + 8*f)**2 + 100818432*e*f
**2*(2*d + 5*e + 8*f)**3 - 13332408320*f**6 - 3425694720*f**5*(2*d + 5*e +
8*f) + 403200000*f**4*(2*d + 5*e + 8*f)**2 + 45093888*f**3*(2*d + 5*e + 8*f
)**3)/(3374210*d**6 + 45393715*d**5*e + 44170854*d**5*f + 247848970*d**4*e*
*2 + 464201768*d**4*e*f + 91507752*d**4*f**2 + 703178520*d**3*e**3 + 191491
5472*d**3*e**2*f + 607100704*d**3*e*f**2 - 999338816*d**3*f**3 + 1094421680
*d**2*e**4 + 3892700544*d**2*e**3*f + 1545770112*d**2*e**2*f**2 - 673938483
2*d**2*e*f**3 - 5963752704*d**2*f**4 + 887062640*d*e**5 + 3907683424*d*e**4
*f + 1889544576*d*e**3*f**2 - 14086786304*d*e**2*f**3 - 24613469440*d*e*f**
4 - 11900367360*d*f**5 + 292932640*e**6 + 1550127488*e**5*f + 952305536*e**
4*f**2 - 9235000320*e**3*f**3 - 24236925440*e**2*f**4 - 23421052928*e*f**5
- 8154904576*f**6))/36 - (35*d + 58*e + 92*f)*log(x + (8710660*d**6 + 10930
5824*d**5*e + 136707258*d**5*f + 2526593*d**5*(35*d + 58*e + 92*f)/4 + 5486
79440*d**4*e**2 + 1278644860*d**4*e*f + 14611963*d**4*e*(35*d + 58*e + 92*f
)/2 + 627558840*d**4*f**2 + 13498939*d**4*f*(35*d + 58*e + 92*f) - 9308*d**
4*(35*d + 58*e + 92*f)**2 + 1416518400*d**3*e**3 + 4598750960*d**3*e**2*f +
 32184326*d**3*e**2*(35*d + 58*e + 92*f) + 3756616640*d**3*e*f**2 + 1126054
64*d**3*e*f*(35*d + 58*e + 92*f) - 85040*d**3*e*(35*d + 58*e + 92*f)**2 + 4
2632000*d**3*f**3 + 96718568*d**3*f**2*(35*d + 58*e + 92*f) - 12758*d**3*f*
(35*d + 58*e + 92*f)**2 - 2441*d**3*(35*d + 58*e + 92*f)**3/4 + 1987752640*
d**2*e**4 + 7924847520*d**2*e**3*f + 68847652*d**2*e**3*(35*d + 58*e + 92*f
) + 7373599680*d**2*e**2*f**2 + 348481752*d**2*e**2*f*(35*d + 58*e + 92*f)
- 299520*d**2*e**2*(35*d + 58*e + 92*f)**2 - 4656496000*d**2*e*f**3 + 58172
3472*d**2*e*f**2*(35*d + 58*e + 92*f) - 189132*d**2*e*f*(35*d + 58*e + 92*f
)**2 - 10281*d**2*e*(35*d + 58*e + 92*f)**3/2 - 6902995200*d**2*f**4 + 3211
85888*d**2*f**3*(35*d + 58*e + 92*f) + 497256*d**2*f**2*(35*d + 58*e + 92*f
)**2 - 11049*d**2*f*(35*d + 58*e + 92*f)**3 + 1437185536*d*e**5 + 650093072
```

```
0*d*e**4*f + 72344932*d*e**4*(35*d + 58*e + 92*f) + 4803912960*d*e**3*f**2
+ 474326816*d*e**3*f*(35*d + 58*e + 92*f) - 466240*d*e**3*(35*d + 58*e + 92
*f)**2 - 17839947520*d*e**2*f**3 + 1158510624*d*e**2*f**2*(35*d + 58*e + 92
*f) - 630504*d*e**2*f*(35*d + 58*e + 92*f)**2 - 12591*d*e**2*(35*d + 58*e +
 92*f)**3 - 34607006720*d*e*f**4 + 1250080640*d*e*f**3*(35*d + 58*e + 92*f)
 + 1672032*d*e*f**2*(35*d + 58*e + 92*f)**2 - 44220*d*e*f*(35*d + 58*e + 92
*f)**3 - 17540771328*d*f**5 + 502882624*d*f**4*(35*d + 58*e + 92*f) + 22974
40*d*f**3*(35*d + 58*e + 92*f)**2 - 37500*d*f**2*(35*d + 58*e + 92*f)**3 +
416940800*e**6 + 2005475776*e**5*f + 30090904*e**5*(35*d + 58*e + 92*f) + 1
22654080*e**4*f**2 + 239214064*e**4*f*(35*d + 58*e + 92*f) - 265792*e**4*(3
5*d + 58*e + 92*f)**2 - 16377853440*e**3*f**3 + 756804928*e**3*f**2*(35*d +
 58*e + 92*f) - 583504*e**3*f*(35*d + 58*e + 92*f)**2 - 9502*e**3*(35*d + 5
8*e + 92*f)**3 - 39387581440*e**2*f**4 + 1189269632*e**2*f**3*(35*d + 58*e
+ 92*f) + 1366560*e**2*f**2*(35*d + 58*e + 92*f)**2 - 41652*e**2*f*(35*d +
58*e + 92*f)**3 - 37694227456*e*f**5 + 926314880*e*f**4*(35*d + 58*e + 92*f
) + 4340800*e*f**3*(35*d + 58*e + 92*f)**2 - 58344*e*f**2*(35*d + 58*e + 92
*f)**3 - 13332408320*f**6 + 285474560*f**5*(35*d + 58*e + 92*f) + 2800000*f
**4*(35*d + 58*e + 92*f)**2 - 26096*f**3*(35*d + 58*e + 92*f)**3)/(3374210*
d**6 + 45393715*d**5*e + 44170854*d**5*f + 247848970*d**4*e**2 + 464201768*
d**4*e*f + 91507752*d**4*f**2 + 703178520*d**3*e**3 + 1914915472*d**3*e**2*
f + 607100704*d**3*e*f**2 - 999338816*d**3*f**3 + 1094421680*d**2*e**4 + 38
92700544*d**2*e**3*f + 1545770112*d**2*e**2*f**2 - 6739384832*d**2*e*f**3 -
 5963752704*d**2*f**4 + 887062640*d*e**5 + 3907683424*d*e**4*f + 1889544576
*d*e**3*f**2 - 14086786304*d*e**2*f**3 - 24613469440*d*e*f**4 - 11900367360
*d*f**5 + 292932640*e**6 + 1550127488*e**5*f + 952305536*e**4*f**2 - 923500
0320*e**3*f**3 - 24236925440*e**2*f**4 - 23421052928*e*f**5 - 8154904576*f*
*6))/432 - (-5*d - 10*e - 8*f + x**2*(5*d + 4*e + 8*f) + x*(-6*d - 6*f))/(3
6*x**3 - 72*x**2 - 36*x + 72)
```

Giac [A] time = 1.07637, size = 159, normalized size = 1.3

$$\frac{1}{144}\left(d+4f-2e\right)\log\left(|x+2|\right)+\frac{1}{108}\left(2d-4f+e\right)\log\left(|x+1|\right)+\frac{1}{36}\left(2d+8f+5e\right)\log\left(|x-1|\right)-\frac{1}{432}\left(35d+92f+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d + 4*f - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + e)*log(abs(x + 1
)) + 1/36*(2*d + 8*f + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 92*f + 58*e)*lo
g(abs(x - 2)) - 1/36*((5*d + 8*f + 4*e)*x^2 - 6*(d + f)*x - 5*d - 8*f - 10*
e)/((x + 1)*(x - 1)*(x - 2))
```

3.100 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{d-e+f-g}{36(x+1)}+\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g)-\frac{1}{432}\log(2-x)(35d+58e+$$

```
[Out] (d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x)) - (d - e
+ f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1 - x])/36 - ((35*d +
 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g)*Log[1 + x])/
108 + ((d - 2*e + 4*f - 8*g)*Log[2 + x])/144
```

Rubi [A] time = 0.252681, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1586, 6742}

$$-\frac{d-e+f-g}{36(x+1)}+\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g)-\frac{1}{432}\log(2-x)(35d+58e+$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```



```
[Out] (d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x)) - (d - e
+ f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1 - x])/36 - ((35*d +
 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g)*Log[1 + x])/
108 + ((d - 2*e + 4*f - 8*g)*Log[2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g}{36(-2+x)^2} + \frac{-35d-58e-92f-136g}{432(-2+x)} + \frac{d+e+f+g}{12(-1+x)^2} + \frac{2d+5e+8f}{36(-1+}\right.$$
$$= \frac{d+e+f+g}{12(1-x)} + \frac{d+2e+4f+8g}{36(2-x)} - \frac{d-e+f-g}{36(1+x)} + \frac{1}{36}(2d+5e+8f+11g)\log(1$$

Mathematica [A] time = 0.0790906, size = 144, normalized size = 1.02

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2\left(e\left(5-2x^2\right)+f\left(-4x^2+3x+4\right)+g\left(8-5x^2\right)\right)\right)}{x^3-2x^2-x+2}+12\log(1-x)(2d+5e+8f+11g)-\mathrm{l}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(g*(8 - 5*x^2) + f*(4 + 3*x - 4*x^2) + e*(5 -
 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f + 11*g)*Log[1 - x] -
 (35*d + 58*e + 92*f + 136*g)*Log[2 - x] + 4*(2*d + e - 4*f + 7*g)*Log[1 +
x] + 3*(d - 2*e + 4*f - 8*g)*Log[2 + x])/432
```

Maple [A] time = 0.014, size = 210, normalized size = 1.5

$$\frac{\ln(2+x)d}{144} - \frac{\ln(2+x)e}{72} + \frac{\ln(1+x)d}{54} + \frac{\ln(1+x)e}{108} - \frac{35\ln(x-2)d}{432} - \frac{29\ln(x-2)e}{216} + \frac{\ln(x-1)d}{18} + \frac{5\ln(x-1)e}{36} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e+1/54*ln(1+x)*d+1/108*ln(1+x)*e-35/432*ln(x-2
)*d-29/216*ln(x-2)*e+1/18*ln(x-1)*d+5/36*ln(x-1)*e-1/36/(1+x)*d+1/36/(1+x)*
e-2/9/(x-2)*g-1/36/(x-2)*d-1/18/(x-2)*e-1/12/(x-1)*g-1/12/(x-1)*d-1/12/(x-1
)*e+1/36/(1+x)*g-1/36/(1+x)*f-1/9/(x-2)*f-1/12/(x-1)*f-1/18*ln(2+x)*g+7/108
*ln(1+x)*g-17/54*ln(x-2)*g+11/36*ln(x-1)*g-23/108*ln(x-2)*f+2/9*ln(x-1)*f+1
/36*ln(2+x)*f-1/27*ln(1+x)*f
```

Maxima [A] time = 0.969686, size = 170, normalized size = 1.21

$$\frac{1}{144}\left(d-2e+4f-8g\right)\log\left(x+2\right)+\frac{1}{108}\left(2d+e-4f+7g\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+5e+8f+11g\right)\log\left(x-1\right)-\frac{1}{43}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/108*(2*d + e - 4*f + 7*g)*log(x
+ 1) + 1/36*(2*d + 5*e + 8*f + 11*g)*log(x - 1) - 1/432*(35*d + 58*e + 92*f
 + 136*g)*log(x - 2) - 1/36*((5*d + 4*e + 8*f + 10*g)*x^2 - 6*(d + f)*x - 5
*d - 10*e - 8*f - 16*g)/(x^3 - 2*x^2 - x + 2)
```

Fricas [B] time = 6.2306, size = 867, normalized size = 6.15

$$-\frac{12\left(5d+4e+8f+10g\right)x^2-72\left(d+f\right)x-3\left(\left(d-2e+4f-8g\right)x^3-2\left(d-2e+4f-8g\right)x^2-\left(d-2e+4f-8g\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f
- 8*g)*x^3 - 2*(d - 2*e + 4*f - 8*g)*x^2 - (d - 2*e + 4*f - 8*g)*x + 2*d -
4*e + 8*f - 16*g)*log(x + 2) - 4*((2*d + e - 4*f + 7*g)*x^3 - 2*(2*d + e -
4*f + 7*g)*x^2 - (2*d + e - 4*f + 7*g)*x + 4*d + 2*e - 8*f + 14*g)*log(x +
1) - 12*((2*d + 5*e + 8*f + 11*g)*x^3 - 2*(2*d + 5*e + 8*f + 11*g)*x^2 - (2
*d + 5*e + 8*f + 11*g)*x + 4*d + 10*e + 16*f + 22*g)*log(x - 1) + ((35*d +
58*e + 92*f + 136*g)*x^3 - 2*(35*d + 58*e + 92*f + 136*g)*x^2 - (35*d + 58*
e + 92*f + 136*g)*x + 70*d + 116*e + 184*f + 272*g)*log(x - 2) - 60*d - 120
*e - 96*f - 192*g)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09334, size = 184, normalized size = 1.3

$$\frac{1}{144}\left(d+4f-8g-2e\right)\log\left(|x+2|\right)+\frac{1}{108}\left(2d-4f+7g+e\right)\log\left(|x+1|\right)+\frac{1}{36}\left(2d+8f+11g+5e\right)\log\left(|x-1|\right)-\frac{\ }{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + 7*g + e)*l
og(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 5*e)*log(abs(x - 1)) - 1/432*(35*
d + 92*f + 136*g + 58*e)*log(abs(x - 2)) - 1/36*((5*d + 8*f + 10*g + 4*e)*x
^2 - 6*(d + f)*x - 5*d - 8*f - 16*g - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

3.101 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=158

$$-\frac{d-e+f-g+h}{36(x+1)}+\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h)-\frac{1}{432}\log$$

```
[Out] (d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(36*(2 - x)
) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g + 14*h)*Log
[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h)*Log[2 - x])/432 + ((2*d
 + e - 4*f + 7*g - 10*h)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h)*Lo
g[2 + x])/144
```

Rubi [A] time = 0.28863, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1586, 6742}

$$-\frac{d-e+f-g+h}{36(x+1)}+\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h)-\frac{1}{432}\log$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```


```
[Out] (d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(36*(2 - x)
) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g + 14*h)*Log
[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h)*Log[2 - x])/432 + ((2*d
 + e - 4*f + 7*g - 10*h)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h)*Lo
g[2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g+16h}{36(-2+x)^2} + \frac{-35d-58e-92f-136g-176h}{432(-2+x)} + \frac{d+e+f}{12(-1}\right.$$
$$= \frac{d+e+f+g+h}{12(1-x)} + \frac{d+2e+4f+8g+16h}{36(2-x)} - \frac{d-e+f-g+h}{36(1+x)} + \frac{1}{36}(2d+5$$

Mathematica [A] time = 0.0998471, size = 169, normalized size = 1.07

$$\frac{1}{432}\left(\frac{12\left(d\left(-5x^2+6x+5\right)+2\left(e\left(5-2x^2\right)+f\left(-4x^2+3x+4\right)-5gx^2+8g-10hx^2+3hx+10h\right)\right)}{x^3-2x^2-x+2}+12\log(1-x)(2d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x
]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(8*g + 10*h + 3*h*x - 5*g*x^2 - 10*h*x^2 + f*
(4 + 3*x - 4*x^2) + e*(5 - 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e
+ 8*f + 11*g + 14*h)*Log[1 - x] - (35*d + 58*e + 92*f + 136*g + 176*h)*Log[
2 - x] + 4*(2*d + e - 4*f + 7*g - 10*h)*Log[1 + x] + 3*(d - 2*e + 4*f - 8*g
 + 16*h)*Log[2 + x])/432
```

Maple [A] time = 0.018, size = 262, normalized size = 1.7

$$\frac{\ln(2+x)d}{144} - \frac{\ln(2+x)e}{72} + \frac{\ln(1+x)d}{54} + \frac{\ln(1+x)e}{108} - \frac{35\ln(x-2)d}{432} - \frac{29\ln(x-2)e}{216} + \frac{\ln(x-1)d}{18} + \frac{5\ln(x-1)e}{36} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e+1/54*ln(1+x)*d+1/108*ln(1+x)*e-35/432*ln(x-2
)*d-29/216*ln(x-2)*e+1/18*ln(x-1)*d+5/36*ln(x-1)*e-4/9/(x-2)*h-1/12/(x-1)*h
-1/36/(1+x)*h-1/36/(1+x)*d+1/36/(1+x)*e-2/9/(x-2)*g-1/36/(x-2)*d-1/18/(x-2)
*e-1/12/(x-1)*g-1/12/(x-1)*d-1/12/(x-1)*e+1/36/(1+x)*g-1/36/(1+x)*f-1/9/(x-
```

```
2)*f-1/12/(x-1)*f-1/18*ln(2+x)*g+7/108*ln(1+x)*g-17/54*ln(x-2)*g+11/36*ln(x
-1)*g+1/9*ln(2+x)*h-5/54*ln(1+x)*h-11/27*ln(x-2)*h+7/18*ln(x-1)*h-23/108*ln
(x-2)*f+2/9*ln(x-1)*f+1/36*ln(2+x)*f-1/27*ln(1+x)*f
```

Maxima [A] time = 0.975518, size = 196, normalized size = 1.24

$$\frac{1}{144}\left(d-2e+4f-8g+16h\right)\log(x+2)+\frac{1}{108}\left(2d+e-4f+7g-10h\right)\log(x+1)+\frac{1}{36}\left(2d+5e+8f+11g+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/108*(2*d + e - 4*f + 7*g
- 10*h)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*h)*log(x - 1) - 1/43
2*(35*d + 58*e + 92*f + 136*g + 176*h)*log(x - 2) - 1/36*((5*d + 4*e + 8*f
+ 10*g + 20*h)*x^2 - 6*(d + f + h)*x - 5*d - 10*e - 8*f - 16*g - 20*h)/(x^3
 - 2*x^2 - x + 2)
```

Fricas [B] time = 34.3086, size = 1049, normalized size = 6.64

$$-\frac{12\left(5d+4e+8f+10g+20h\right)x^2-72\left(d+f+h\right)x-3\left(\left(d-2e+4f-8g+16h\right)x^3-2\left(d-2e+4f-8g+16h\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 72*(d + f + h)*x - 3*((d -
 2*e + 4*f - 8*g + 16*h)*x^3 - 2*(d - 2*e + 4*f - 8*g + 16*h)*x^2 - (d - 2*
e + 4*f - 8*g + 16*h)*x + 2*d - 4*e + 8*f - 16*g + 32*h)*log(x + 2) - 4*((2
*d + e - 4*f + 7*g - 10*h)*x^3 - 2*(2*d + e - 4*f + 7*g - 10*h)*x^2 - (2*d
+ e - 4*f + 7*g - 10*h)*x + 4*d + 2*e - 8*f + 14*g - 20*h)*log(x + 1) - 12*
((2*d + 5*e + 8*f + 11*g + 14*h)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h)*x^
2 - (2*d + 5*e + 8*f + 11*g + 14*h)*x + 4*d + 10*e + 16*f + 22*g + 28*h)*lo
g(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h)*x^3 - 2*(35*d + 58*e + 92*
f + 136*g + 176*h)*x^2 - (35*d + 58*e + 92*f + 136*g + 176*h)*x + 70*d + 11
```

```
6*e + 184*f + 272*g + 352*h)*log(x - 2) - 60*d - 120*e - 96*f - 192*g - 240
*h)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08478, size = 209, normalized size = 1.32

$$\frac{1}{144}\left(d+4f-8g+16h-2e\right)\log\left(|x+2|\right)+\frac{1}{108}\left(2d-4f+7g-10h+e\right)\log\left(|x+1|\right)+\frac{1}{36}\left(2d+8f+11g+14h+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + 7*g
 - 10*h + e)*log(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 14*h + 5*e)*log(abs
(x - 1)) - 1/432*(35*d + 92*f + 136*g + 176*h + 58*e)*log(abs(x - 2)) - 1/3
6*((5*d + 8*f + 10*g + 20*h + 4*e)*x^2 - 6*(d + f + h)*x - 5*d - 8*f - 16*g
 - 20*h - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

3.102 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+1$$

```
[Out] (d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h + 32*i)/
(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 1
1*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h +
160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h + 13*i)*Log[1 + x])/1
08 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x])/144
```

Rubi [A] time = 0.343235, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {1586, 6742}

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+1$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^2
,x]
```


```
[Out] (d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h + 32*i)/
(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 1
1*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h +
160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h + 13*i)*Log[1 + x])/1
08 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x])/144
```


Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+102x^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+102x^5}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{3264+d+2e+4f+8g+16h}{36(-2+x)^2} + \frac{-16320-35d-58e-92f-}{432(-2+x)}\right.$$
$$= \frac{102+d+e+f+g+h}{12(1-x)} + \frac{3264+d+2e+4f+8g+16h}{36(2-x)} + \frac{102-}{}$$

Mathematica [A] time = 0.119405, size = 195, normalized size = 1.1

$$\frac{-5dx^2+6dx+5d-4ex^2+10e-8fx^2+6fx+8f-10gx^2+16g-20hx^2+6hx+20h-34ix^2+40i}{36\left(x^3-2x^2-x+2\right)} + \frac{1}{36}\log(1-x)(2$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] (5*d + 10*e + 8*f + 16*g + 20*h + 40*i + 6*d*x + 6*f*x + 6*h*x - 5*d*x^2 -
4*e*x^2 - 8*f*x^2 - 10*g*x^2 - 20*h*x^2 - 34*i*x^2)/(36*(2 - x - 2*x^2 + x^
3)) + ((2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*Log[1 - x])/36 + ((-35*d - 58
*e - 92*f - 136*g - 176*h - 160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g
- 10*h + 13*i)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2
 + x])/144
```

Maple [A] time = 0.016, size = 314, normalized size = 1.8

$$\frac{\ln(2+x)\,d}{144} - \frac{\ln(2+x)\,e}{72} + \frac{\ln(1+x)\,d}{54} + \frac{\ln(1+x)\,e}{108} - \frac{35\ln(x-2)\,d}{432} - \frac{29\ln(x-2)\,e}{216} + \frac{\ln(x-1)\,d}{18} + \frac{5\ln(x-1)\,e}{36} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x)
```

```
[Out] 1/144*ln(2+x)*d-1/72*ln(2+x)*e+1/54*ln(1+x)*d+1/108*ln(1+x)*e-35/432*ln(x-2
)*d-29/216*ln(x-2)*e+1/18*ln(x-1)*d+5/36*ln(x-1)*e-8/9/(x-2)*i-1/12/(x-1)*i
+1/36/(1+x)*i-4/9/(x-2)*h-1/12/(x-1)*h-1/36/(1+x)*h-1/36/(1+x)*d+1/36/(1+x)
```

```
*e-2/9/(x-2)*g-1/36/(x-2)*d-1/18/(x-2)*e-1/12/(x-1)*g-1/12/(x-1)*d-1/12/(x-
1)*e+1/36/(1+x)*g-1/36/(1+x)*f-1/9/(x-2)*f-1/12/(x-1)*f-10/27*ln(x-2)*i+17/
36*ln(x-1)*i-2/9*ln(2+x)*i+13/108*ln(1+x)*i-1/18*ln(2+x)*g+7/108*ln(1+x)*g-
17/54*ln(x-2)*g+11/36*ln(x-1)*g+1/9*ln(2+x)*h-5/54*ln(1+x)*h-11/27*ln(x-2)*
h+7/18*ln(x-1)*h-23/108*ln(x-2)*f+2/9*ln(x-1)*f+1/36*ln(2+x)*f-1/27*ln(1+x)
*f
```

Maxima [A] time = 0.985017, size = 220, normalized size = 1.24

$$\frac{1}{144}\left(d-2e+4f-8g+16h-32i\right)\log\left(x+2\right)+\frac{1}{108}\left(2d+e-4f+7g-10h+13i\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+5e+8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="maxima")
```

```
[Out] 1/144*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/108*(2*d + e - 4*f
 + 7*g - 10*h + 13*i)*log(x + 1) + 1/36*(2*d + 5*e + 8*f + 11*g + 14*h + 17
*i)*log(x - 1) - 1/432*(35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*log(x -
 2) - 1/36*((5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*x^2 - 6*(d + f + h)*x -
5*d - 10*e - 8*f - 16*g - 20*h - 40*i)/(x^3 - 2*x^2 - x + 2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08367, size = 234, normalized size = 1.32

$$\frac{1}{144}\left(d+4f-8g+16h-32i-2e\right)\log\left(|x+2|\right)+\frac{1}{108}\left(2d-4f+7g-10h+13i+e\right)\log\left(|x+1|\right)+\frac{1}{36}\left(2d+8f+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="giac")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 32*i - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*
f + 7*g - 10*h + 13*i + e)*log(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 14*h
+ 17*i + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 92*f + 136*g + 176*h + 160*i
+ 58*e)*log(abs(x - 2)) - 1/36*((5*d + 8*f + 10*g + 20*h + 34*i + 4*e)*x^2
- 6*(d + f + h)*x - 5*d - 8*f - 16*g - 20*h - 40*i - 10*e)/((x + 1)*(x - 1)
*(x - 2))
```

3.103 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=717

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+\sqrt{a}\sqrt{c}\left(24abcf-180ac^2d+9b^2cd-4b^3f\right)+57ab^2cf-144abc^2d+18b^3cd\cdots\right.}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*x*Sq
rt[a + b*x^2 + c*x^4])/(315*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b^2 - 4*
a*c)*(2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) + (x*(9
*b^2*c*d + 90*a*c^2*d - 4*b^3*f + 9*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14*a
*c*f)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + ((2*c*e - b*g)*(b + 2*c*x^2
)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (x*(3*(3*c*d + b*f) + 7*c*f*x^2)*(a
 + b*x^2 + c*x^4)^(3/2))/(63*c) + (g*(a + b*x^2 + c*x^4)^(5/2))/(10*c) + (3
*(b^2 - 4*a*c)^2*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2)) + (a^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^
4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) - (a
^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f
+ Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f))*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11
/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.595828, antiderivative size = 717, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1673, 1176, 1197, 1103, 1195, 1247, 640, 612, 621, 206}

$$-\frac{x\sqrt{a+bx^2+cx^4}\left(-84a^2c^2f+57ab^2cf-144abc^2d+18b^3cd-8b^4f\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2f+\sqrt{a}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*x*Sq
rt[a + b*x^2 + c*x^4])/(315*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b^2 - 4*
```

```
a*c)*(2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) + (x*(9
*b^2*c*d + 90*a*c^2*d - 4*b^3*f + 9*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14*a
*c*f)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + ((2*c*e - b*g)*(b + 2*c*x^2
)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (x*(3*(3*c*d + b*f) + 7*c*f*x^2)*(a
 + b*x^2 + c*x^4)^(3/2))/(63*c) + (g*(a + b*x^2 + c*x^4)^(5/2))/(10*c) + (3
*(b^2 - 4*a*c)^2*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*
x^2 + c*x^4])])/(512*c^(7/2)) + (a^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^
4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) - (a
^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f
+ Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f))*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11
/4)*Sqrt[a + b*x^2 + c*x^4])
```

<u>Rule 1673</u>

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

<u>Rule 1176</u>

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

<u>Rule 1197</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

<u>Rule 1103</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
```

```
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(d+fx^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx + \int x\left(e+gx^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$$

$$= \frac{x\left(3(3cd+bf)+7cfx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c} + \frac{1}{2}\text{Subst}\left(\int (e+gx)\left(a+b\right.\right.$$

$$= \frac{x\left(9b^2cd+90ac^2d-4b^3f+9abcf+3c\left(9bcd-4b^2f+14acf\right)x^2\right)\sqrt{a+}}{315c^2}$$

$$= \frac{x\left(9b^2cd+90ac^2d-4b^3f+9abcf+3c\left(9bcd-4b^2f+14acf\right)x^2\right)\sqrt{a+}}{315c^2}$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} -$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} -$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} -$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.095, size = 3038, normalized size = 4.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))*a^2-2/15*f*a^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b
)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/
2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))+2/15*f*a^3*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*(
(-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
1/35*d*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-
b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*b^3/c*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/
2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+19/210*f*a^2*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))/c*b^2*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-19/210*f*a^2*2^(1/2)/(
((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))/c*b^2*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-4/315*f*a*2^(1/2)/(((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^4
/c^2*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+4/315*f*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)
/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^4/c^2*Ellip
ticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))+1/35*d*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(
4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^3/c*EllipticF(1/2*x*2^
(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))-1/140*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b
^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*a/c*b^2-8/35*d*a^2*2^(1/2)/(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1
```

```
/2))*b*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+8/35*d*a^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)
-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b*Elliptic
E(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-2/105*f/c*a^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)
*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-
b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b+1/315*f/c^2*a*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))*b^3+1/8*e*c*x^6*(c*x^4+b*x^2+a)^(1/2)+3/16*e*b*x^4*
(c*x^4+b*x^2+a)^(1/2)+5/16*e*a*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128*e*b^3/c^2*(c
*x^4+b*x^2+a)^(1/2)+3/256*e*b^4/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/16*e*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(
1/2)+1/7*d*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*d*b*x^3*(c*x^4+b*x^2+a)^(1/2)+3
/7*d*x*(c*x^4+b*x^2+a)^(1/2)*a+1/10*g*c*x^8*(c*x^4+b*x^2+a)^(1/2)+11/80*g*b
*x^6*(c*x^4+b*x^2+a)^(1/2)+1/5*g*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256*g*b^4/c^
3*(c*x^4+b*x^2+a)^(1/2)-3/512*g*b^5/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))+7/160*g*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)+8/105*f/c*x*(c*x^4
+b*x^2+a)^(1/2)*a*b-1/128*g*b^3/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)-5/64*g*a*b^2/
c^2*(c*x^4+b*x^2+a)^(1/2)+3/64*g*a*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))-3/32*g*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))+1/105*f/c*x^3*(c*x^4+b*x^2+a)^(1/2)*b^2-4/315*f/c^2*x*(c*x^4+
b*x^2+a)^(1/2)*b^3+1/64*e*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*e*a*b/c*(c*x
^4+b*x^2+a)^(1/2)-3/32*e*a*b^2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^
2+a)^(1/2))+1/35*d/c*x*(c*x^4+b*x^2+a)^(1/2)*b^2+1/160*g*b^2*x^4/c*(c*x^4+b
*x^2+a)^(1/2)+1/10*g*a^2/c*(c*x^4+b*x^2+a)^(1/2)+1/9*f*c*x^7*(c*x^4+b*x^2+a
)^(1/2)+10/63*f*b*x^5*(c*x^4+b*x^2+a)^(1/2)+11/45*f*x^3*(c*x^4+b*x^2+a)^(1/
2)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cgx^7 + cfx^6 + (ce + bg)x^5 + (cd + bf)x^4 + (be + ag)x^3 + aex + (bd + af)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*g*x^7 + c*f*x^6 + (c*e + b*g)*x^5 + (c*d + b*f)*x^4 + (b*e + a*
g)*x^3 + a*e*x + (b*d + a*f)*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2 + cx^4\right)^{\frac{3}{2}} \left(d + ex + fx^2 + gx^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)*(d + e*x + f*x**2 + g*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + e*x + d), x)
```

3.104 $\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=505

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}f-2bf+5cd\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx}\cdots}{\cdots}$$

```
[Out] ((5*b*c*d - 2*b^2*f + 6*a*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt
[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])
/(16*c^2) + (x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) +
(g*(a + b*x^2 + c*x^4)^(3/2))/(6*c) - ((b^2 - 4*a*c)*(2*c*e - b*g)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)) - (a^(1/4)
*(5*b*c*d - 2*b^2*f + 6*a*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)
*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d - 2*b*f + 3*Sqrt[a]*Sqrt[c]*f)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqr
t[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.278505, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1673, 1176, 1197, 1103, 1195, 1247, 640, 612, 621, 206}

$$\frac{x\sqrt{a+bx^2+cx^4}\left(6acf-2b^2f+5bcd\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6acf-2b^2f+5bcd\right)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\cdots\right.\right.}{15c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b*c*d - 2*b^2*f + 6*a*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt
[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])
/(16*c^2) + (x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) +
(g*(a + b*x^2 + c*x^4)^(3/2))/(6*c) - ((b^2 - 4*a*c)*(2*c*e - b*g)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)) - (a^(1/4)
*(5*b*c*d - 2*b^2*f + 6*a*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
```

```
- b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)
*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d - 2*b*f + 3*Sqrt[a]*Sqrt[c]*f)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqr
t[a + b*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
```

```
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(d+ex+fx^2+gx^3\right)\sqrt{a+bx^2+cx^4}\,dx = \int \left(d+fx^2\right)\sqrt{a+bx^2+cx^4}\,dx + \int x\left(e+gx^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{x\left(5cd+bf+3cfx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{1}{2}\,\mathrm{Subst}\left(\int (e+gx)\sqrt{a+bx+cx$$

$$= \frac{x\left(5cd+bf+3cfx^2\right)\sqrt{a+bx^2+cx^4}}{15c} + \frac{g\left(a+bx^2+cx^4\right)^{3/2}}{6c} + \frac{\left(\sqrt{a}\left(b+2\right.\right.}{}$$

$$= \frac{\left(5bcd-2b^2f+6acf\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(2ce-bg)\left(b+2cx^2\right)\sqrt{a+bx^2}}{16c^2}$$

$$= \frac{\left(5bcd-2b^2f+6acf\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(2ce-bg)\left(b+2cx^2\right)\sqrt{a+bx^2}}{16c^2}$$

$$= \frac{\left(5bcd-2b^2f+6acf\right)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(2ce-bg)\left(b+2cx^2\right)\sqrt{a+bx^2}}{16c^2}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.026, size = 1585, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*g*(c*x^4+b*x^2+a)^(3/2)/c-1/8*g*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*g*b^
2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8*g*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))*a+1/32*g*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))+1/5*f*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*f*b/c*x*(c*x^4+b*x^2+a)^(1/
2)-1/60*f*b/c*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)
^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^
2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/5*f*a^2*2^(1/2)/(((-4*a*c+b^2)^(1
/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*Ellipti
cF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2))+1/5*f*a^2*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4
-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(
((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))+1/15*f*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1
/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*b^2/c*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)
^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/15*f*a*
2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*b^2/c*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*e*x^2*(c*x^4+b*x^2
+a)^(1/2)+1/8*e/c*(c*x^4+b*x^2+a)^(1/2)*b+1/4*e/c^(1/2)*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16*e/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))*b^2+1/3*d*x*(c*x^4+b*x^2+a)^(1/2)+1/6*d*a*2^(1/2)/(((-4*
a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-1/6*d*b*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+
b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2
)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/6*d*b*
a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(gx^3 + fx^2 + ex + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx^2+cx^4}\left(d+ex+fx^2+gx^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)*(d + e*x + f*x**2 + g*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d), x)
```

3.105 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=359

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}-$$

```
[Out] (g*Sqrt[a + b*x^2 + c*x^4])/(2*c) + (f*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)) - (a^(1/4)*f*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 +
c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4
])
```

Rubi [A] time = 0.158558, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1197, 1103, 1195, 1247, 640, 621, 206}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+f\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (g*Sqrt[a + b*x^2 + c*x^4])/(2*c) + (f*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*
Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)) - (a^(1/4)*f*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 +
c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4
])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+fx^2}{\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{x\left(e+gx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{e+gx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}f\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}f}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+}}$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}} - \frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{c}\right.}{}$$

Mathematica [C] time = 1.42207, size = 526, normalized size = 1.47

$$\frac{-i\sqrt{2}\sqrt{c}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(f\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*S
qrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*(2*Sqrt[c]*g*(a + b*x^2 + c*x^4) + (2*c*e - b*g)*Sqrt[a
+ b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*
c^(3/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.023, size = 453, normalized size = 1.3

$$\frac{g}{2c}\sqrt{cx^4+bx^2+a}-\frac{bg}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}-\frac{af\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(\sqrt{-4\,ac+b^2}-b\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-\cdots}\right.}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*g*(c*x^4+b*x^2+a)^(1/2)/c-1/4*g*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))-1/2*f*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((
-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4
*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))/c^(1/2)+1/4*d*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a
*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex + fx^2 + gx^3}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.106 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=447

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}f\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(bd-}{a^{3/4}\left(b^2-4ac\right)}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - (Sqrt[c]*(b*d - 2*a*f)*x*Sqrt[a + b*x^2 + c*x^4])/(a*
(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(b*d - 2*a*f)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2
- 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d - Sqrt[a]*f)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2
*Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.272621, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1673, 1178, 1197, 1103, 1195, 1247, 636}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(bd-2af)E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d-\sqrt{a}f\right)F\left(2\text{t}\right.}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - (Sqrt[c]*(b*d - 2*a*f)*x*Sqrt[a + b*x^2 + c*x^4])/(a*
(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(b*d - 2*a*f)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2
- 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d - Sqrt[a]*f)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
```

```
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2
*Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right) - \frac{\int \frac{a(2cd-bf}{\sqrt{a}}}{a(b}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{be-2ag+(2ce-bg)x^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{c}(bd-2af)\right)\int \frac{1-}{\sqrt{a+b}}}{\sqrt{a}\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{be-2ag+(2ce-bg)x^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}(bd-2af)x\sqrt{a+bx}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\right.}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.039, size = 1005, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -g/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+f*(-2*c*(-1/(4*a*c-b^2)*x^
3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)-1/4*b/(4*a*c-b^2)*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2
)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e/(c*x^4+b*x^2+a)^(1/2)*(2*c*x^2+b)
/(4*a*c-b^2)+d*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)
/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/
2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2
)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(
1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right)}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d)/(c^2*x^8 + 2*b*c
*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{gx^3+fx^2+ex+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.107 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx$

Optimal. Leaf size=680

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(6a^{3/2}\sqrt{c}f-3\sqrt{a}b\sqrt{c}d+abf-10acd+2b^2d\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^(3/2)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(3*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^(3/2)) + (4*(2*c*e - b*g)*(b + 2*c*x^2))/(3*(b^2 - 4*a
*c)^2*Sqrt[a + b*x^2 + c*x^4]) + (x*(2*b^4*d - 17*a*b^2*c*d + 20*a^2*c^2*d
+ a*b^3*f + 4*a^2*b*c*f + c*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x
^2))/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(2*b^3*d -
16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(b^2 -
 4*a*c)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(2*b^3*d - 16*a*b*c*d + a*b^2
*f + 12*a^2*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(3*a^(7/4)*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4
)*(2*b^2*d - 3*Sqrt[a]*b*Sqrt[c]*d - 10*a*c*d + a*b*f + 6*a^(3/2)*Sqrt[c]*f
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
6*a^(7/4)*(b - 2*Sqrt[a]*Sqrt[c])*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.514804, antiderivative size = 680, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1178, 1197, 1103, 1195, 1247, 638, 613}

$$\frac{x\left(cx^2\left(12a^2cf+ab^2f-16abcd+2b^3d\right)+4a^2bcf+20a^2c^2d-17ab^2cd+ab^3f+2b^4d\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}\left(12a^2cf\right.}{3a^2\left(b^2-4ac\right)^2\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^(3/2)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(3*(b^2 - 4*a*c)
```

```
*(a + b*x^2 + c*x^4)^(3/2)) + (4*(2*c*e - b*g)*(b + 2*c*x^2))/(3*(b^2 - 4*a
*c)^2*Sqrt[a + b*x^2 + c*x^4]) + (x*(2*b^4*d - 17*a*b^2*c*d + 20*a^2*c^2*d
+ a*b^3*f + 4*a^2*b*c*f + c*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x
^2))/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(2*b^3*d -
16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(b^2 -
 4*a*c)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(2*b^3*d - 16*a*b*c*d + a*b^2
*f + 12*a^2*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(3*a^(7/4)*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4
)*(2*b^2*d - 3*Sqrt[a]*b*Sqrt[c]*d - 10*a*c*d + a*b*f + 6*a^(3/2)*Sqrt[c]*f
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
6*a^(7/4)*(b - 2*Sqrt[a]*Sqrt[c])*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
```

```
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx = \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx + \int \frac{x\left(e+gx^2\right)}{\left(a+bx^2+cx^4\right)^{5/2}}\,dx$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{e+gx}{\left(a+bx+cx^2\right)^{5/2}}\,dx, x, x^2\right) - \frac{\int \frac{-2b^2d+1}{}}{}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{x\left(2b^4d-17ab^2cd\right.}{}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{4(2ce-bg)\left(b\right.}{3\left(b^2-4ac\right)^2\sqrt{a+}}$$

$$= \frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{4(2ce-bg)\left(b\right.}{3\left(b^2-4ac\right)^2\sqrt{a+}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] $Aborted
```

Maple [B] time = 0.051, size = 1395, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x)
```

```
[Out] -1/3*g*(8*b*c^2*x^6+12*b^2*c*x^4+12*a*b*c*x^2+3*b^3*x^2+8*a^2*c+2*a*b^2)/(c
*x^4+b*x^2+a)^(3/2)/(16*a^2*c^2-8*a*b^2*c+b^4)+f*((2/3/c/(4*a*c-b^2)*x^3+1/
3*b/(4*a*c-b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+x^2*b/c+a/c)^2-2*c*(-1/6*
(12*a*c+b^2)/a/(4*a*c-b^2)^2*x^3-1/6*(4*a*c+b^2)*b/a/(4*a*c-b^2)^2/c*x)/((x
^4+x^2*b/c+a/c)*c)^(1/2)+1/4*(-1/3/a*b/(4*a*c-b^2)-1/3*(4*a*c+b^2)*b/a/(4*a
*c-b^2)^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2
)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/6*c*(12*a*c+b^2)/(4*a*c-b^2)^2*2^(1/2)
/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+
b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+1/3
*e*(16*c^3*x^6+24*b*c^2*x^4+24*a*c^2*x^2+6*b^2*c*x^2+12*a*b*c-b^3)/(c*x^4+b
*x^2+a)^(3/2)/(16*a^2*c^2-8*a*b^2*c+b^4)+d*((-1/3/a*b/(4*a*c-b^2)/c*x^3+1/3
*(2*a*c-b^2)/a/(4*a*c-b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+x^2*b/c+a/c)^2
-2*c*(1/3*b*(8*a*c-b^2)/(4*a*c-b^2)^2/a^2*x^3-1/6*(20*a^2*c^2-17*a*b^2*c+2*
b^4)/a^2/(4*a*c-b^2)^2/c*x)/((x^4+x^2*b/c+a/c)*c)^(1/2)+1/4*(2/3*(5*a*c-b^2
)/a^2/(4*a*c-b^2)-1/3*(20*a^2*c^2-17*a*b^2*c+2*b^4)/a^2/(4*a*c-b^2)^2)*2^(1
/2)/(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))-1/3*b*c*(8*a*c-b^2)/(4*a*c-b^2)^2/a*2^(1/2)/(((-4*a*c+b
^2)^(1/2)-b)/a)^(1/2)*(4-2*((-4*a*c+b^2)^(1/2)-b)/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(
EllipticF(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b)/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(((-4*a*c+b^2)^(1/2)-b
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3 + fx^2 + ex + d}{\left(cx^4 + bx^2 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(gx^3+fx^2+ex+d\right)}{c^3x^{12}+3\,bc^2x^{10}+3\left(b^2c+ac^2\right)x^8+\left(b^3+6\,abc\right)x^6+3\,a^2bx^2+3\left(ab^2+a^2c\right)x^4+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + e*x + d)/(c^3*x^12 + 3*b*
c^2*x^10 + 3*(b^2*c + a*c^2)*x^8 + (b^3 + 6*a*b*c)*x^6 + 3*a^2*b*x^2 + 3*(a
*b^2 + a^2*c)*x^4 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{gx^3+fx^2+ex+d}{\left(cx^4+bx^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + e*x + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

3.108 $\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rubi [A] time = 0.0181643, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1588}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{ag-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.006, size = 18, normalized size = 1.

$$gx\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] g*x/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [A] time = 1.12618, size = 23, normalized size = 1.21

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Fricas [A] time = 1.21719, size = 39, normalized size = 2.05

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21693, size = 95, normalized size = 5.

$$\frac{\left(b^4g - 8\,ab^2cg + 16\,a^2c^2g\right)x}{32\left(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4\right)\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)*x/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 1
6*a^3*c^4)*sqrt(c*x^4 + b*x^2 + a))
```

3.109 $\int \frac{ag+ex-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi [A] time = 0.0666024, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1673, 1588, 12, 1107, 613}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+ex-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \int \frac{ex}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\int \frac{ag-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx\\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+e\int \frac{x}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx\\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.005, size = 52, normalized size = 0.9

$$\frac{4\,acgx - b^2gx + 2\,cex^2 + be}{4\,ac - b^2}\frac{1}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x+2*c*e*x^2+b*e)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [A] time = 1.23641, size = 69, normalized size = 1.21

$$-\frac{2\,cex^2 + be - \left(b^2g - 4\,acg\right)x}{\sqrt{cx^4 + bx^2 + a}\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*c*e*x^2 + b*e - (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4*
a*c))
```

Fricas [A] time = 1.33697, size = 173, normalized size = 3.04

$$-\frac{\sqrt{cx^4 + bx^2 + a}\left(2\,cex^2 - \left(b^2 - 4\,ac\right)gx + be\right)}{\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*e*x^2 - (b^2 - 4*a*c)*g*x + b*e)/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17025, size = 228, normalized size = 4.

$$-\frac{\left(\frac{2\left(b^2ce-4\,ac^2e\right)x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}-\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{b^3e-4\,abce}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{16\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*((2*(b^2*c*e - 4*a*c^2*e)*x/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
- (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*
c^4))*x + (b^3*e - 4*a*b*c*e)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))/sqr
t(c*x^4 + b*x^2 + a)
```

3.110 $\int \frac{ag+fx^3-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{f\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi [A] time = 0.0795246, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1673, 1588, 12, 1114, 636}

$$\frac{f\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ag+fx^3-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \int \frac{fx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\int \frac{ag-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+f\int \frac{x^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{1}{2}f\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right) \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{f\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.004, size = 53, normalized size = 0.9

$$\frac{4acgx - b^2gx - bfx^2 - 2af}{4ac - b^2}\frac{1}{\sqrt{cx^4 + bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x-b*f*x^2-2*a*f)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [A] time = 1.13281, size = 66, normalized size = 1.16

$$\frac{bfx^2 + 2af + (b^2g - 4acg)x}{\sqrt{cx^4 + bx^2 + a}(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*f*x^2 + 2*a*f + (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c))
```

Fricas [A] time = 1.30984, size = 171, normalized size = 3.

$$\frac{\sqrt{cx^4 + bx^2 + a}\left(bfx^2 + (b^2 - 4ac)gx + 2af\right)}{(b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*f*x^2 + (b^2 - 4*a*c)*g*x + 2*a*f)/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+f*x**3+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17742, size = 221, normalized size = 3.88

$$\frac{\left(\frac{\left(b^3f-4\,abcf\right)x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{2\left(ab^2f-4\,a^2cf\right)}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{16\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^3*f - 4*a*b*c*f)*x/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4) + (b^
4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))
*x + 2*(a*b^2*f - 4*a^2*c*f)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))/sqrt
(c*x^4 + b*x^2 + a)
```

3.111 $\int \frac{ag+ex+fx^3-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{-2af+x^2(2ce-bf)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)/((b^2 - 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0906941, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1673, 1588, 1247, 636}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{-2af+x^2(2ce-bf)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)/((b^2 - 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{ag+ex+fx^3-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \int \frac{x\left(e+fx^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\int \frac{ag-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx\\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{e+fx}{\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{be-2af+(2ce-bf)x^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.004, size = 63, normalized size = 0.9

$$\frac{4\,acgx-b^2gx-bfx^2+2\,cex^2-2\,af+be}{4\,ac-b^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (4*a*c*g*x-b^2*g*x-b*f*x^2+2*c*e*x^2-2*a*f+b*e)/(c*x^4+b*x^2+a)^(1/2)/(4*a*
c-b^2)
```

Maxima [A] time = 1.15474, size = 127, normalized size = 1.84

$$-\frac{\sqrt{cx^4+bx^2+a}\left(\left(2\,ce-bf\right)x^2+be-2\,af-\left(b^2g-4\,acg\right)x\right)}{\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*((2*c*e - b*f)*x^2 + b*e - 2*a*f - (b^2*g - 4*a*c*
g)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Fricas [A] time = 1.30622, size = 193, normalized size = 2.8

$$\frac{\sqrt{cx^4+bx^2+a}\left(\left(b^2-4\,ac\right)gx-\left(2\,ce-bf\right)x^2-be+2\,af\right)}{\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fric
as")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*((b^2 - 4*a*c)*g*x - (2*c*e - b*f)*x^2 - b*e + 2*a*
f)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+f*x**3+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18484, size = 262, normalized size = 3.8

$$\frac{\left(\frac{\left(b^3f-4\,abcf-2\,b^2ce+8\,ac^2e\right)x}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}\right)x+\frac{2\,ab^2f-8\,a^2cf-b^3e+4\,abce}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{8\,\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/8*(((b^3*f - 4*a*b*c*f - 2*b^2*c*e + 8*a*c^2*e)*x/(a*b^4*c^2 - 8*a^2*b^2*
c^3 + 16*a^3*c^4) + (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(a*b^4*c^2 - 8*a^2
*b^2*c^3 + 16*a^3*c^4))*x + (2*a*b^2*f - 8*a^2*c*f - b^3*e + 4*a*b*c*e)/(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))/sqrt(c*x^4 + b*x^2 + a)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
 If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
 9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
 MemberQ[{
       Exp,Log,
       Sin,Cos,Tan,Cot,Sec,Csc,
       ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
       Sinh,Cosh,Tanh,Coth,Sech,Csch,
       ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
 MemberQ[{
       Erf, Erfc, Erfi,
       FresnelS, FresnelC,
       ExpIntegralE, ExpIntegralEi, LogIntegral,
       SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
       Gamma, LogGamma, PolyGamma,
       Zeta, PolyLog, ProductLog,
       EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
 MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
 MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```